ADVANCED ELECTRICAL CIRCUIT

BETI 1333

STEP RESPONSE PARALLEL RLC CIRCUIT

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LESSON OUTCOME

At the end of this chapter, students are able:

- to describe second order step response parallel RLC circuit

SUBTOPICS

Step Response Parallel RLC Circuit

Application of Second Order Circuit

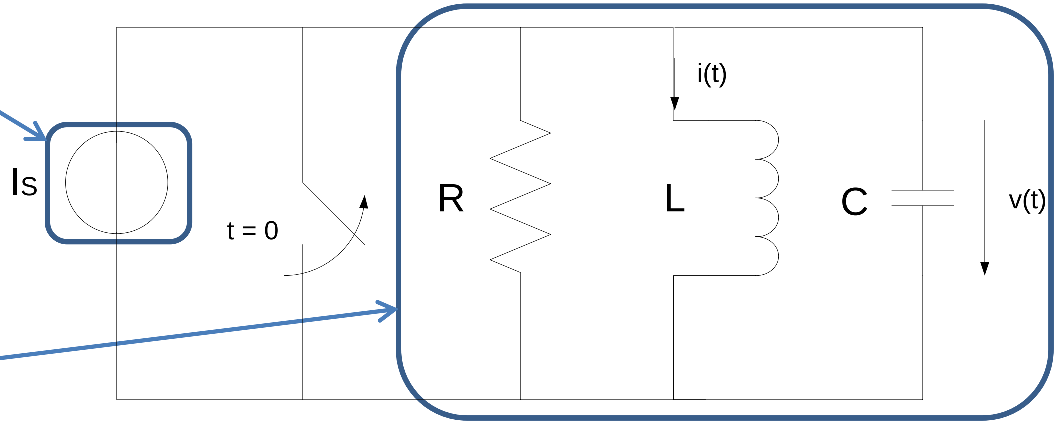
STEP RESPONSE PARALLEL RLC CIRCUIT

What?

1. Step function as excitation source

2. Contains several resistors, a capacitor and an inductor

3. Output response: Transient and steady-state response



STEP RESPONSE PARALLEL RLC CIRCUIT

Step response parallel RLC Circuit:

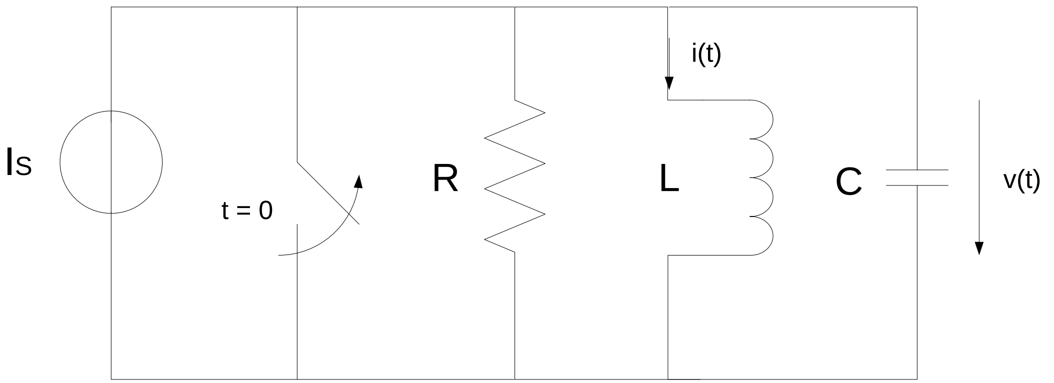


Figure 1

By applying Kirchhoff's Current Law:

$$I_R + I_L + I_C = I_S$$

$$\frac{v}{R} + i + C\frac{dv}{dt} = I_S$$

Second order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC} i = \frac{1}{LC} I_S$$

Output response:

$$i(t) = i_T(t) + i_{ss}(t)$$

Transient response

Steady-state response
iss(t) = i(∞)

STEP RESPONSE PARALLEL RLC CIRCUIT

Types of complete response of step response parallel RLC circuit:

1. Overdamped response ($\alpha > \omega_0$)

$$i(t) = \underbrace{A_1 e^{s_1 t} + A_2 e^{s_2 t}}_{\text{Transient response}} + \underbrace{i(\infty)}_{\text{Steady-state response}}$$

2. Critically damped response ($\alpha = \omega_0$)

$$i(t) = \underbrace{(A_1 + A_2 t)e^{-\alpha t}}_{\text{Transient response}} + \underbrace{i(\infty)}_{\text{Steady-state response}}$$

3. Underdamped response ($\alpha < \omega_0$)

$$i(t) = \underbrace{e^{-\alpha t}(A_1 cos\omega_d t + A_2 sin\omega_d t)}_{\text{Transient response}} + \underbrace{i(\infty)}_{\text{Steady-state response}}$$

Note:

1. A_1 and A_2can be deermined from the initial conditions namely $i(0)$ and $\frac{di(0)}{dt}$.

$$\frac{di(0)}{dt} = \frac{v(0)}{L}$$

2. $\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

α: Damping factor; ω_0: Undamped natural frequency

3. $s_{1,2} = -\alpha \pm \sqrt{-({\omega_0}^2 - \alpha^2)} = -\alpha \pm j\omega_d$

$s_{1,2}$: Roots of the characteristic equation

EXAMPLE 1

The switch in Figure 2 is closed at t = 0. Find i(t) for t > 0.

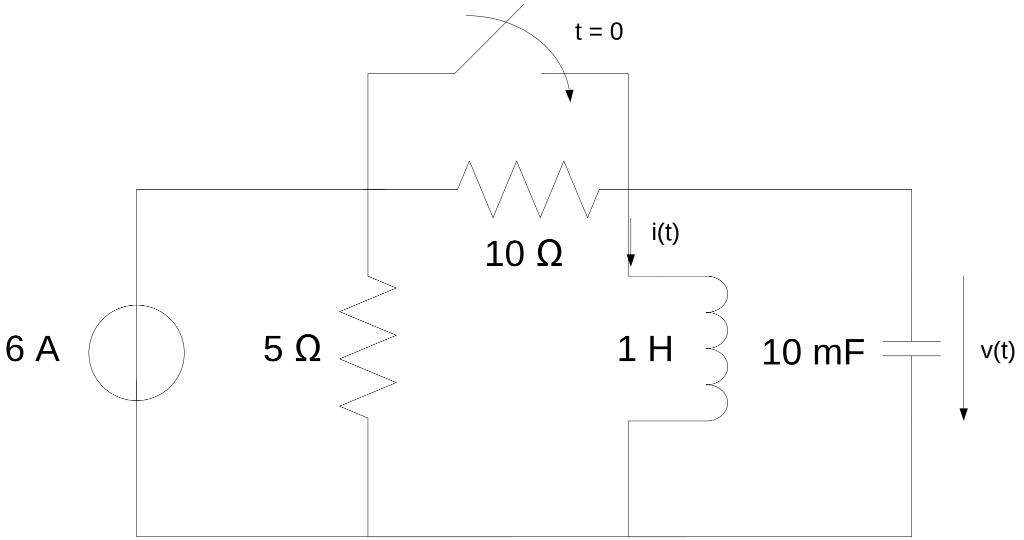


Figure 2

SOLUTION 1

Step 1: Find the initial current across inductor, i(0)

initial voltage across capacitor, v(0) when t < 0.

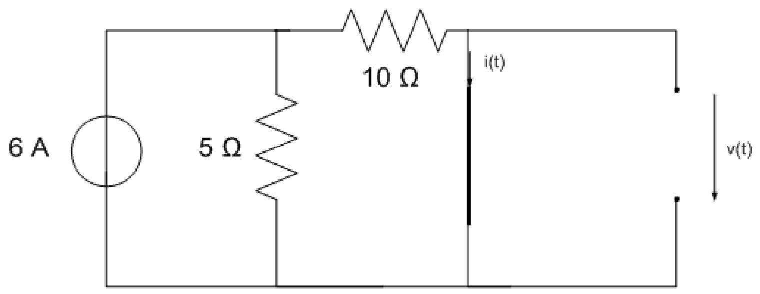


Figure 3

$$i(t) = i(0) = \frac{5\ \Omega}{(10+5)\Omega} * 6\ A = 2\ A$$

$$v(t) = v(0) = 0\ V$$

Tips 1:

When t < 0, capacitor acts like open circuit and inductor acts like short circuit.

SOLUTION 1

Step 2: Determine type of natural response or this circuit, when t > 0.

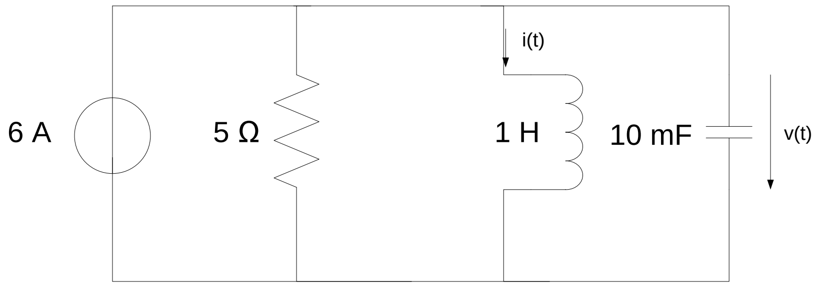


Figure 4

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)(0.01)} = 10$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1(0.01)}} = 10$$

$\alpha = \omega_0$ → Critically damped response

Complete current response for critically damped case:

$$i(t) = (A_1 + A_2 t)e^{-\alpha t} + i(\infty)$$

SOLUTION 1

Step 3: Determine the final value of current through inductor, i(∞).

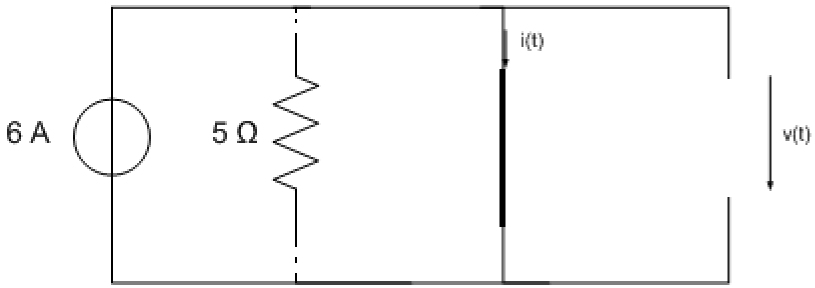


Figure 5

$$i(\infty) = 6\ A$$

Tips 2:

At dc steady-state, capacitor acts like open circuit and inductor acts like short circuit.

Tips 3:

Current will flow through less resistance. A 5 Ω resistor is short-circuited.

SOLUTION 1

Step 4: Determine A_1 and A_2 from initial conditions $i(0)$ and $\frac{di(0)}{dt}$, when t > 0.

$$i(0) = (A_1 + A_2(0))e^{-10(0)} + 6 = 2$$

$$A_1 + 6 = 2 \rightarrow A_1 = -4$$

$$\frac{di(0)}{dt} = \frac{v(0)}{L} = 0\ \frac{A}{s}$$

$$\frac{di}{dt} = A_2 e^{-10t} + (-10)(A_1 + A_2 t)e^{-10t}$$

$$\frac{di(0)}{dt} = A_2 e^{-10(0)} + (-10)(-4 + A_2(0))e^{-10(0)} = 0$$

$$\rightarrow A_2 = -40$$

Complete current response:

$$i(t) = 6 + (-4 - 40t)e^{-10t}\ A$$

EXAMPLE 2

The switch in Figure 6 is closed at t = 0. Find i(t) for t > 0.

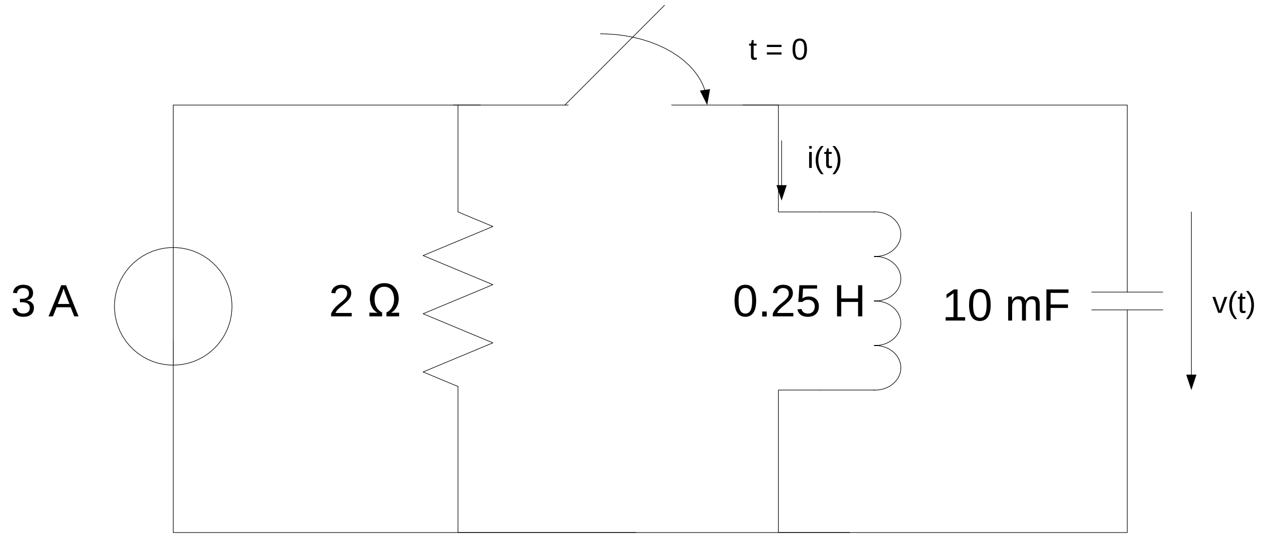


Figure 6

SOLUTION 2

Step 1: Find the initial current across inductor, i(0)

initial voltage across capacitor, v(0) when t < 0.

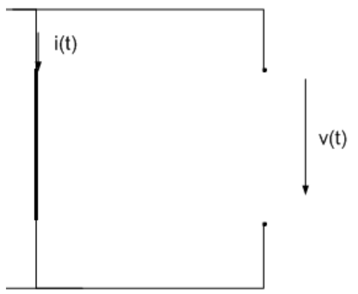


Figure 7

$i(t) = i(0) = 0\ A$

$v(t) = v(0) = 0\ V$

Tips 1:

When t < 0, capacitor acts like open circuit and inductor acts like short circuit.

SOLUTION 2

Step 2: Determine type of natural response or this circuit, when t > 0.

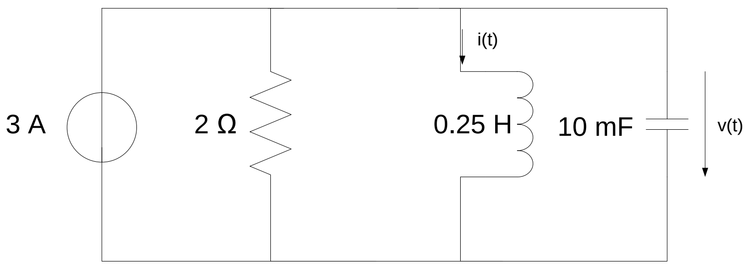


Figure 8

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2)(0.01)} = 25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25(0.01)}} = 20$$

$\alpha > \omega_0$ → Overdamped response

Complete current response for overdamped case:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + i(\infty)$$

SOLUTION 2

Step 3: Determine roots of the characteristic equation, $s_{1,2}$ when t > 0.

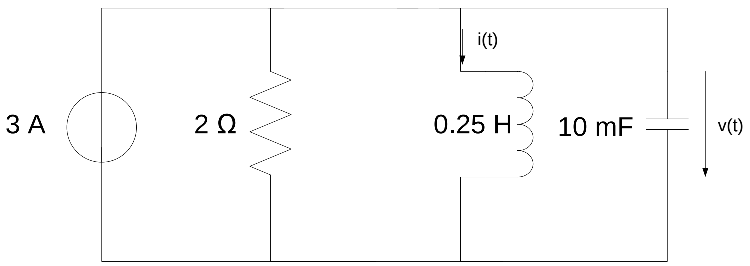


Figure 9

$$s_{1,2} = -\alpha \pm \sqrt{-(\omega_0{}^2 - \alpha^2)} = -25 \pm \sqrt{-(20^2 - 25^2)}$$
$$s_{1,2} = -10, -40$$

SOLUTION 2

Step 4: Determine the final value of current through inductor, i(∞).

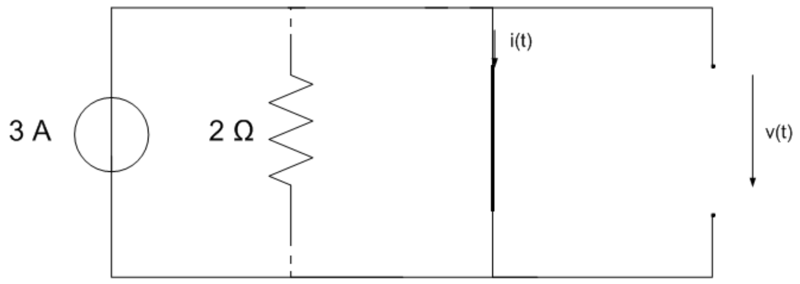


Figure 5

$i(\infty) = 3\ A$

Tips 2:

At dc steady-state, capacitor acts like open circuit and inductor acts like short circuit.

Tips 3:

Current will flow through less resistance. A 2 Ω resistor is short-circuited.

SOLUTION 2

Step 5: Determine A_1 and A_2 from initial conditions $i(0)$ and $\frac{di(0)}{dt}$, when t > 0.

$$i(0) = A_1 e^{-10(0)} + A_2 e^{-40(0)} + 3 = 0$$

$$A_1 + A_2 + 3 = 0 \quad \rightarrow A_1 = -3 - A_2$$

$$\frac{di(0)}{dt} = \frac{v(0)}{L} = 0 \ \frac{A}{s}$$

$$\frac{di}{dt} = -10A_1 e^{-10t} - 40A_2 e^{-40t}$$

$$\frac{di(0)}{dt} = -10A_1 e^{-10(0)} - 40A_2 e^{-40(0)} = 0$$

$$-10A_1 - 40A_2 = 0$$

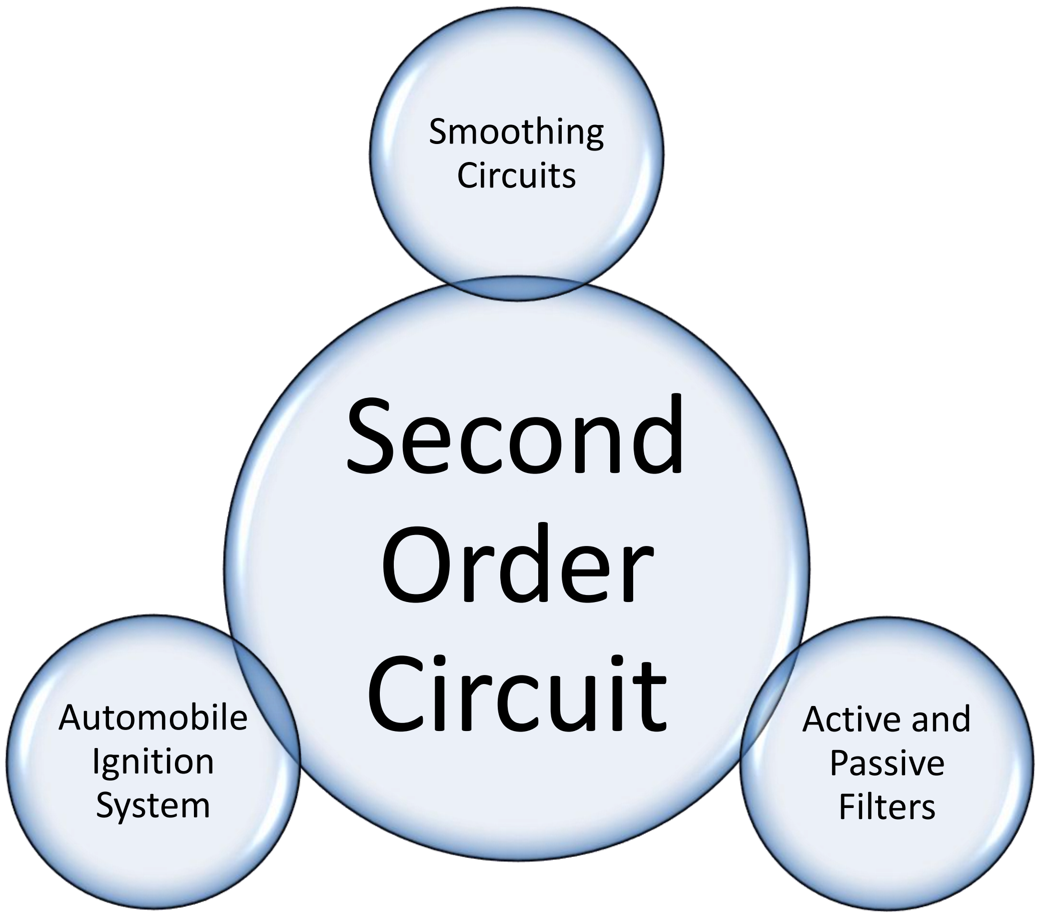
$$-10(-3 - A_2) - 40A_2 = 0 \quad \rightarrow A_2 = 1$$

$$\rightarrow A_1 = -4$$

Complete current response:

$$i(t) = 3 - 4e^{-10t} + e^{-40t} \ A$$

APPLICATION



SELF REVIEW QUESTIONS

1. The initial voltage in a step response parallel RLC circuit is found by:

   a) Replacing capacitor with open circuit

   b) Replacing inductor with open circuit

   c) Replacing capacitor with short circuit

   d) Replacing inductor with short circuit

2. The final current in a step response parallel RLC circuit is found by:

   a) Replacing capacitor with open circuit

   b) Replacing inductor with open circuit

   c) Replacing capacitor with short circuit

   d) Replacing inductor with short circuit

3. Which one is CORRECT about underdamped response:

   a) $\alpha < \omega_0$ b) $\alpha > \omega_0$

   c) $\alpha = \omega_0$ d) $\alpha = 0$

4. The output response of step response RLC circuit is transient and __________ response.

5. Given R = 4 Ω and C = 1 F. Find the value of L so that a parallel RLC circuit will produce critically damped response.

   a) 640 H b) 6.4 mH

   c) 64 H d) 640 mH

ANSWERS

1. a
2. d
3. a
4. steady-state
5. c