ADVANCED ELECTRICAL CIRCUIT

BETI 1333

SOURCE-FREE PARALLEL AND STEP RESPONSE SERIES RLC CIRCUIT

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LESSON OUTCOMES

At the end of this chapter, students are able:

- to describe second order source-free parallel RLC circuit
- to describe second order step response series RLC circuit

SUBTOPICS

- Source-free Parallel RLC Circuit
- Step Response Series RLC Circuit

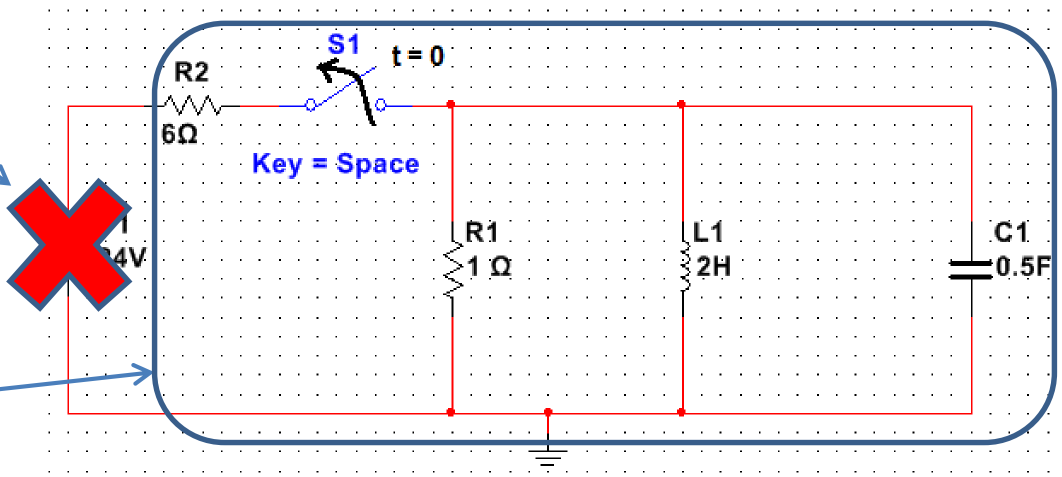
SOURCE-FREE PARALLEL RLC CIRCUIT

What?

1. No external sources of excitation

2. Contains several resistors, a capacitor and an inductor

3. Output response: Transient/natural response



SOURCE-FREE PARALLEL RLC CIRCUIT

Source-free parallel RLC Circuit:

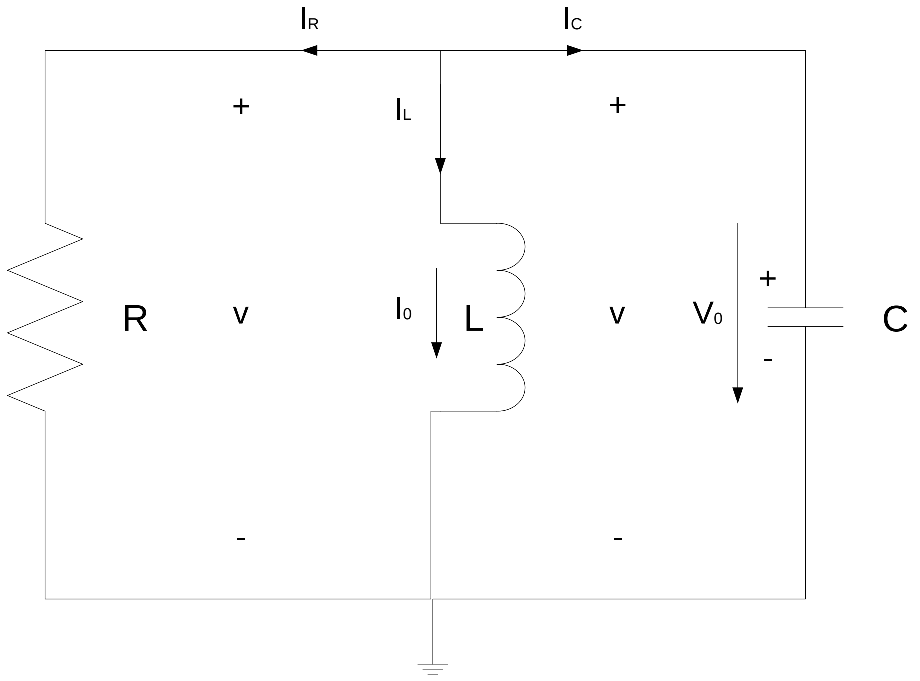


Figure 1

Assumption:

$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)dt$ and $v(0) = V_0$

Initial current through inductor

Initial voltage across capacitor

By applying Kirchhoff's Current Law:

$$I_R + I_L + I_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(t)dt + C\frac{dv}{dt} = 0$$

Second order differential equation:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

SOURCE-FREE PARALLEL RLC CIRCUIT

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Roots of the characteristic equation

$$\alpha = \frac{1}{2RC}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Damping factor

Undamped natural frequency

<u>Types of natural response of source-free parallel RLC circuit:</u>

1. Overdamped response ($\alpha > \omega_0$)

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

2. Critically damped response ($\alpha = \omega_0$)

$$v(t) = (A_1 + A_2 t)e^{-\alpha t}$$

3. Underdamped response ($\alpha < \omega_0$)

$$v(t) = e^{-\alpha t}(A_1 cos\omega_d t + A_2 sin\omega_d t)$$

<u>Note:</u> A_1 and A_2 can be determined from the initial conditions namely $v(0)$ and $\frac{dv(0)}{dt}$.

$$\frac{dv(0)}{dt} = -\frac{(V_0 + RI_0)}{RC}$$

EXAMPLE 1

The switch in Figure 2 is opened at t = 0. Find v(t) for t > 0.

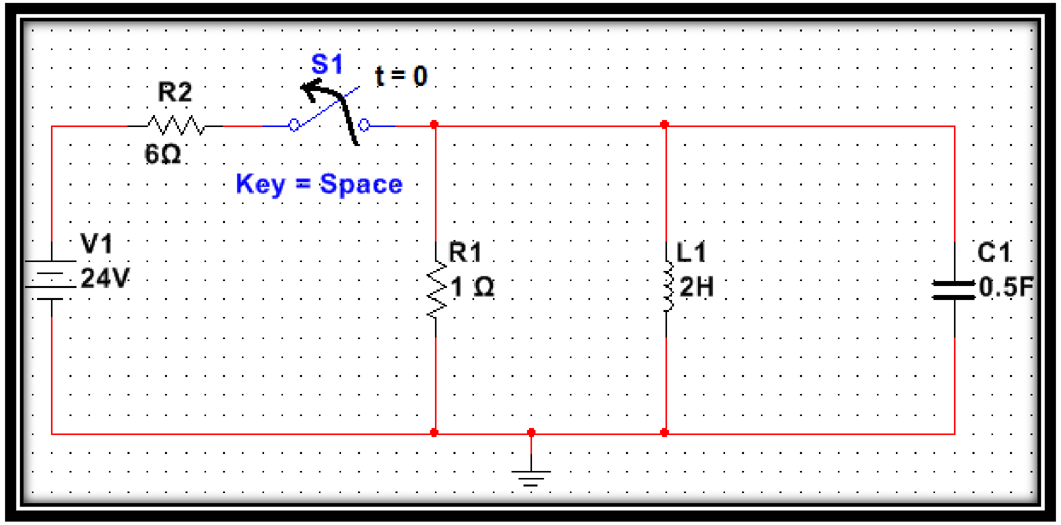


Figure 2

SOLUTION 1

Step 1: Find initial voltage across capacitor, V_0 and initial current across inductor, I_0 when t < 0.

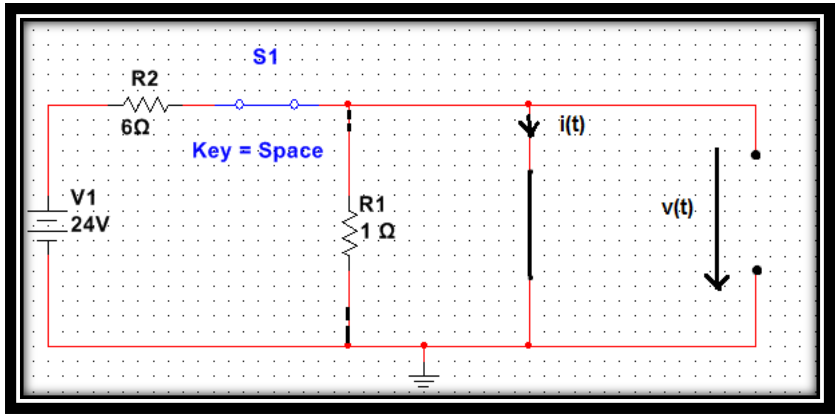


Figure 3

$$v(t) = V_0 = 0V$$

$$i(t) = I_0 = \frac{V_1}{R_2} = \frac{24V}{6\Omega} = 4A$$

Tips 1:

When t < 0, capacitor acts like an open circuit and inductor acts like a short circuit.

Tips 2:

Current will flow through inductor, which is less resistance than R_1. R_1 is short-circuited.

SOLUTION 1

Step 2: Determine type of natural response of this circuit, when t > 0.

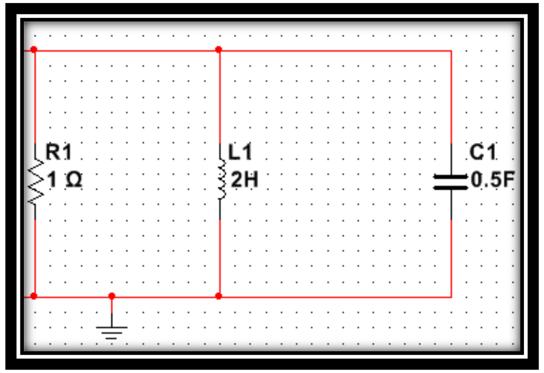


Figure 4

$$\alpha = \frac{1}{2RC} = \frac{1}{2(1)(0.5)} = 1, \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 * 0.5}} = 1$$

$\alpha = \omega_0 = 1$ → Critically damped response

Voltage response for critically damped case:
$v(t) = (A_1 + A_2 t)e^{-t}$, where $\alpha = 1$

SOLUTION 1

<u>Step 3:</u> Determine A_1 and A_2 from initial conditions $v(0)$ and $\frac{dv(0)}{dt}$, when t > 0.

$$v(0) = \left(A_1 + A_2(0)\right)e^{-0} = 0 \ \rightarrow A_1 = 0$$

$$\frac{dv(0)}{dt} = -\frac{(V_0 + RI_0)}{RC} = -\frac{(0 + 1 * 4)}{1 * 0.5} = -8\frac{V}{s}$$

By differentiating voltage response:

$$\frac{dv}{dt} = -(A_1 + A_2 t)e^{-t} + A_2 e^{-t}$$

$$\frac{dv(0)}{dt} = -(0 + A_2(0))e^{-0} + A_2 e^{-0} = -8 \ \rightarrow A_2 \ = -8$$

Voltage response:

$v(t) = 8te^{-t}$ V

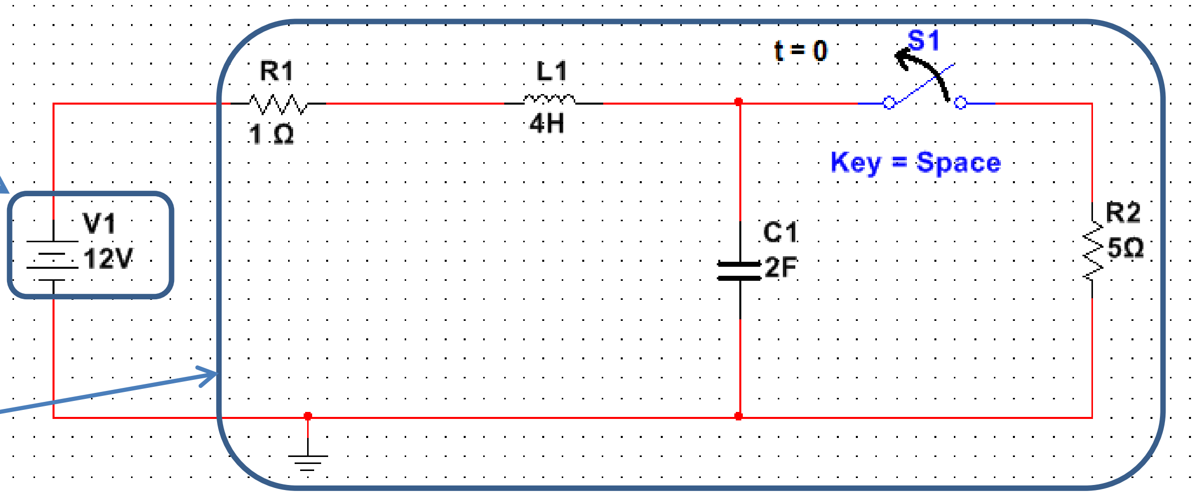
STEP RESPONSE SERIES RLC CIRCUIT

What?

1. Step function as excitation source

2. Contains several resistors, a capacitor and an inductor

3. Output response: Transient and steady-state response



STEP RESPONSE SERIES RLC CIRCUIT

Step response series RLC Circuit:

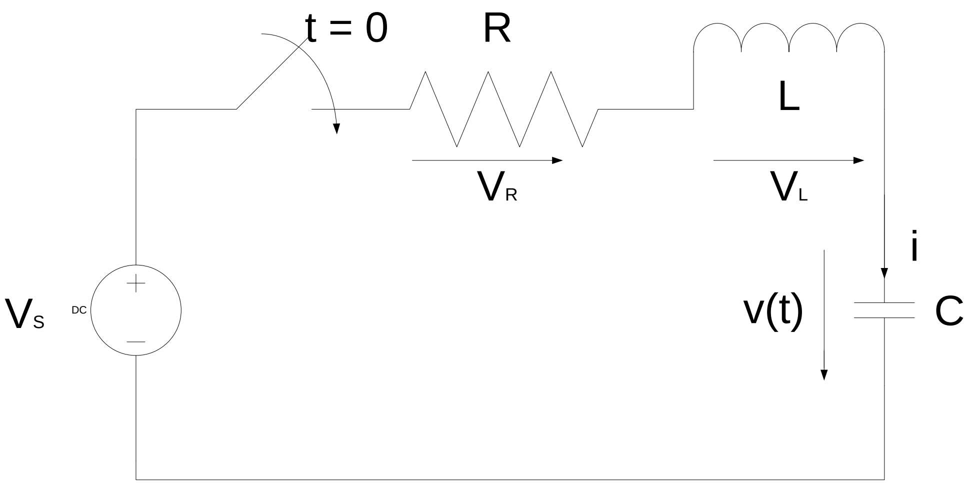


Figure 5

By applying Kirchhoff's Voltage Law:

$$V_R + V_L + V_C = V_S$$

$$iR + L\frac{di}{dt} + v = V_S$$

Second order differential equation:

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}V_S$$

Output response:

$$v(t) = v_T(t) + v_{SS}(t)$$

Transient response

Steady-state response $v_{ss}(t) = v(\infty)$

STEP RESPONSE SERIES RLC CIRCUIT

Types of complete response of step response series RLC circuit:

1. Overdamped response ($\alpha > \omega_0$)

$$v(t) = \underbrace{A_1 e^{s_1 t} + A_2 e^{s_2 t}}_{\text{Transient response}} + \underbrace{v(\infty)}_{\text{Steady-state response}}$$

2. Critically damped response ($\alpha = \omega_0$)

$$v(t) = \underbrace{(A_1 + A_2 t)e^{-\alpha t}}_{\text{Transient response}} + \underbrace{v(\infty)}_{\text{Steady-state response}}$$

3. Underdamped response ($\alpha < \omega_0$)

$$v(t) = \underbrace{e^{-\alpha t}(A_1 cos\omega_d t + A_2 sin\omega_d t)}_{\text{Transient response}} + \underbrace{v(\infty)}_{\text{Steady-state response}}$$

Note:

1. A_1 and A_2 can be determined from the initial conditions namely $v(0)$ and $\frac{dv(0)}{dt}$.

$$\frac{dv(0)}{dt} = \frac{i(0)}{C}$$

2. $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

α: Damping factor

ω_0: Undamped natural frequency

3. $s_{1,2} = -\alpha \pm \sqrt{-({\omega_0}^2 - \alpha^2)} = -\alpha \pm j\omega_d$

$s_{1,2}$: Roots of the characteristic equation

EXAMPLE 2

The switch in Figure 6 is opened at t = 0. Find v(t) for t > 0.

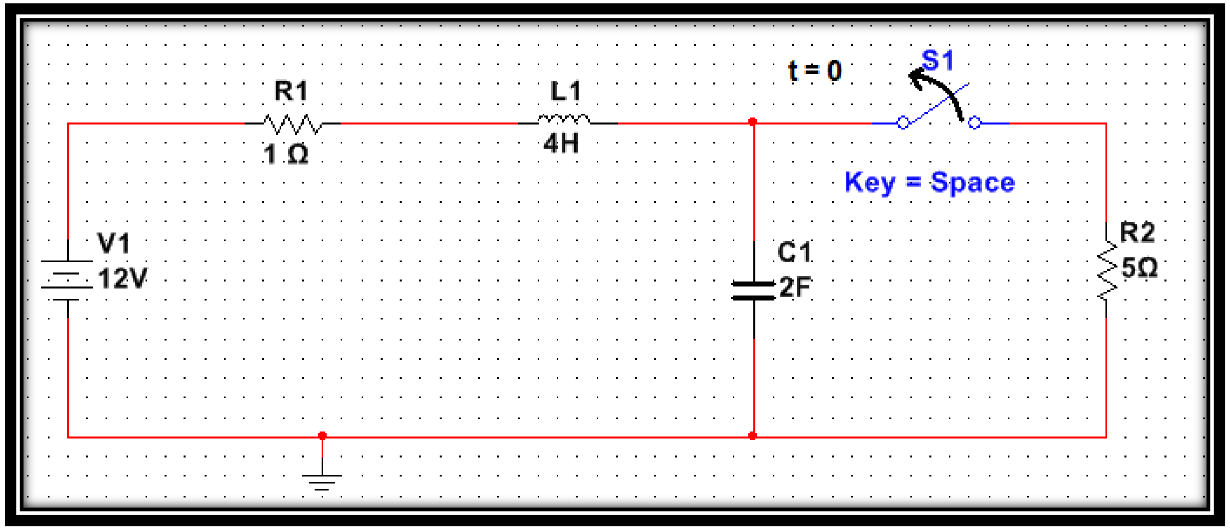


Figure 6

SOLUTION 2

Step 1: Find initial voltage across capacitor, V_0 and initial current across inductor, I_0 when t < 0.

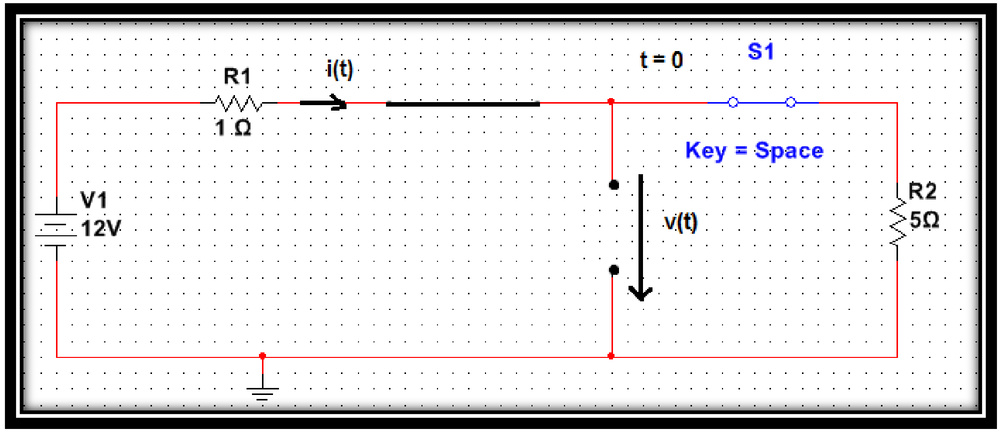


Figure 7

$$v(t) = V_0 = \frac{5\Omega}{(5+1)\Omega} * 12V = 10V$$

$$i(t) = I_0 = \frac{V_1}{R_1 + R_2} = \frac{12V}{6\Omega} = 2A$$

Tips 1:

When t < 0, capacitor acts like an open circuit and inductor acts like a short circuit.

SOLUTION 2

Step 2: Determine type of natural response of this circuit, when t > 0.

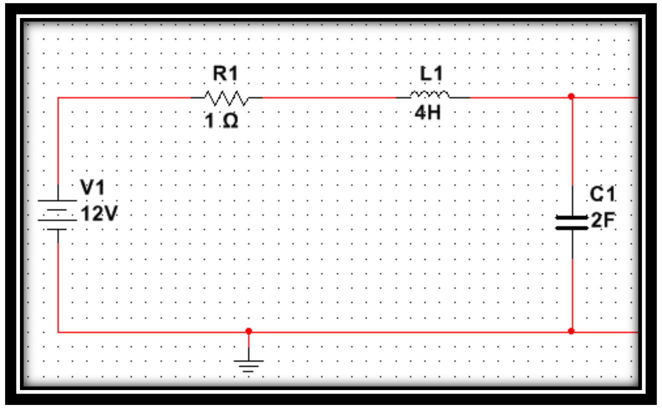


Figure 8

$\alpha = \frac{R}{2L} = \frac{1}{2(4)} = \frac{1}{8},$ $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4*2}} = \frac{1}{\sqrt{8}}$

$\alpha < \omega_0$ → Underdamped response

Complete voltage response for underdamped case:

$v(t) = (A_1 cos\omega_d t + A_2 sin\omega_d t)e^{-\alpha t} + v(\infty)$

SOLUTION 2

Step 3: Determine the final value of voltage across capacitor, v(∞).

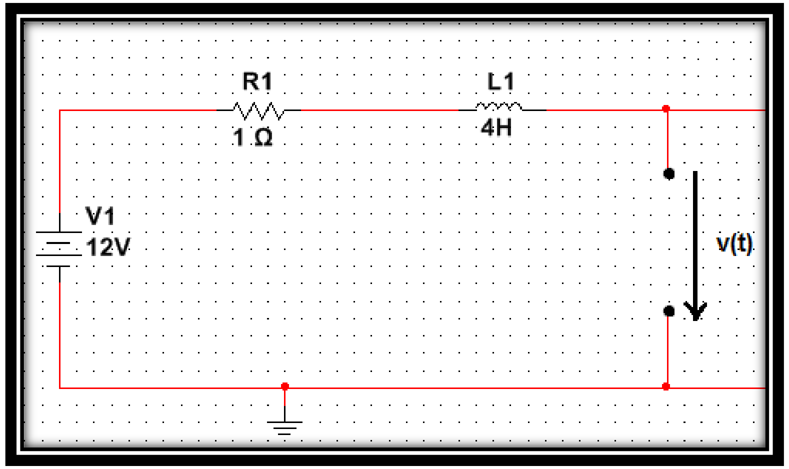


Figure 9

$v(t) = v(\infty) = 12V$

Tips 2:

At dc steady-state, capacitor is an open circuit.

SOLUTION 2

Step 3: Find ω_d.

$$s_{1,2} = -\alpha \pm \sqrt{-({\omega_0}^2 - \alpha^2)} = -\alpha \pm j\omega_d$$

$$s_{1,2} = -\frac{1}{8} \pm \sqrt{-\left(\frac{1}{\sqrt{8}}\right)^2 + \left(\frac{1}{8}\right)^2}$$

$$s_{1,2} = -0.125 \pm j0.331 \rightarrow \omega_d = 0.331$$

SOLUTION 2

<u>Step 4:</u> Determine A_1 and A_2 from initial conditions $v(0)$ and $\frac{dv(0)}{dt}$, when t > 0.

$$v(0) = (A_1\cos(0.331 * 0) + A_2\sin(0.331 * 0))\,e^{-0.125(0)} + 12V = 10V$$

$$A_1 + 12V = 10V \;\; \rightarrow \; A_1 = -2$$

$$\frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{2}{2} = 1\frac{V}{s}$$

$$\frac{dv}{dt} = -0.125(A_1 cos0.331t + A_2 sin0.331t)e^{-0.125t} + (0.331)(-A_1 sin0.331t + A_2 cos0.331t)e^{-0.125t}$$

$$\frac{dv(0)}{dt} = -0.125(A_1\cos(0.331 * 0) + A_2\sin(0.331 * 0))\,e^{-0.125(0)}$$

$$+(0.331)(-A_1\sin(0.331 * 0) + A_2\cos(0.331 * 0))e^{-0.125(0)} = 1 \;\rightarrow A_2 = 2.266$$

Voltage response:

$$v(t) = 12 + (-2cos0.331t + 2.266sin0.331t)e^{-0.125t}V$$

SELF REVIEW QUESTIONS

1. Name 3 types of natural response in a source-free parallel RLC circuit.

   Answer: ___________

2. Given R = 4 Ω, C = 0.05 F and L = 1 mH. Find damping factor, α for this parallel RLC circuit.

   a) 0.05   b) 0.5

   c) 5   d) 50

3. Name the behaviour of a step response RLC circuit.

   Answer: ____________

4. The undamped natural frequency of a step response series RLC circuit is equal to 10. Determine the value of L required if C = 0.4 F.

   a) 0.25 mH   b) 2.5 mH

   c) 250 mH   d) 25 mH

5. State type of natural response for a step response series RLC circuit when damping factor is equal to the undamped natural frequency.

   Answer: ___________

ANSWERS

1. Overdamped response, critically damped response, underdamped response
2. c
3. Transient and steady-state response
4. d
5. Critically damped response