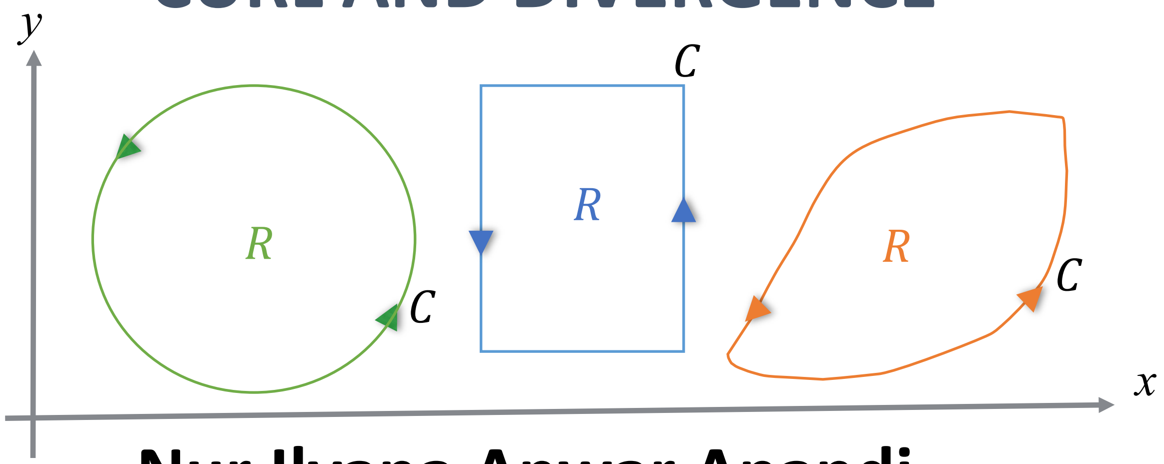
BEKG 2433
ENGINEERING MATHEMATICS 2

GREEN'S THEOREM
CURL AND DIVERGENCE



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Lesson Outcomes

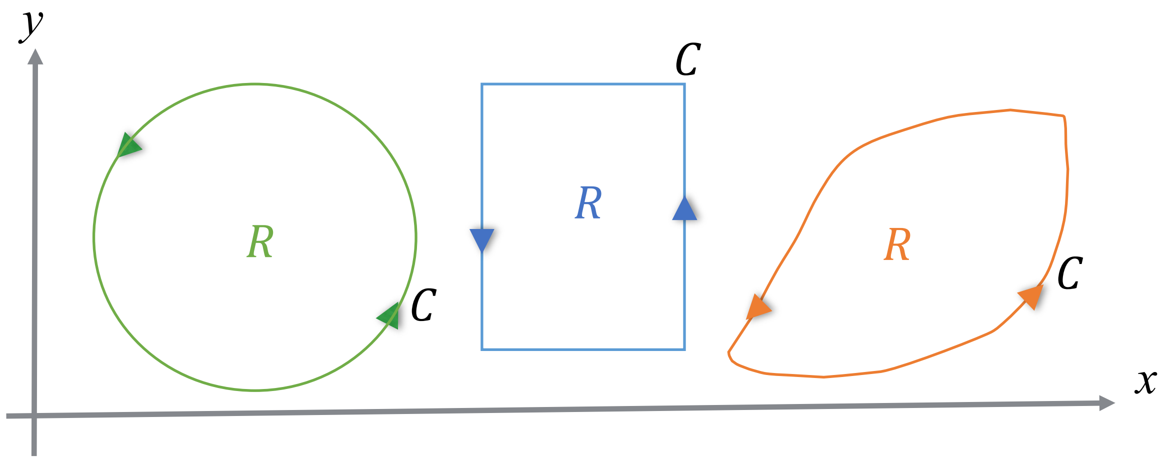
Upon completion of this lesson, students should be able to:

- Calculate the divergence of vector fields.
- Calculate the curl of vector fields.
- State and evaluate the Green’s Theorem.

Introduction Green's Theorem

Green's theorem gives us a way of evaluating the line integral of a smooth vector field, $\boldsymbol{F}$ around a simple closed curve, C. Let $\boldsymbol{R}$ be a connected plane region whose boundary is a simple, closed piecewise smooth curve C oriented counterclockwise or positive orientation.

Here is a sketch of simple closed curve C and $\boldsymbol{R}$ be the region enclosed by the curve.



Green's Theorem

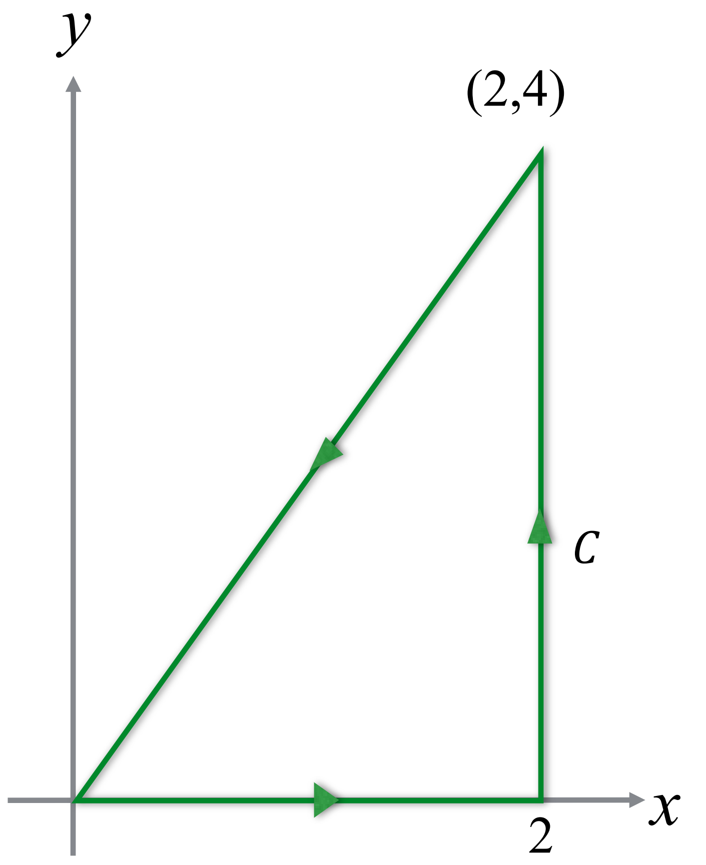
Let C is the closed curve and R be the region enclosed by the curve. If $f(x,y)$ and $g(x,y)$ have continuous first partial derivatives on some open set containing $\boldsymbol{R}$, then

$$\int_{C} f(x,y)dx + g(x,y)dy = \iint_{R} \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

Here is the alternate notations when working with the line integrals in which the curve C assumption that satisfies the condition of Green's Theorem

$$\oint_{C} f(x,y)dx + g(x,y)dy = \iint_{R} \left(g_x - f_y\right) dA$$

Remarks: $dA = dxdy = dydx = rdrd\theta$



Example 11.1:

Use the Green's Theorem to evaluate

$$\int_C x^2 y\, dx + 3x dy$$

over the triangular path for the Figure.

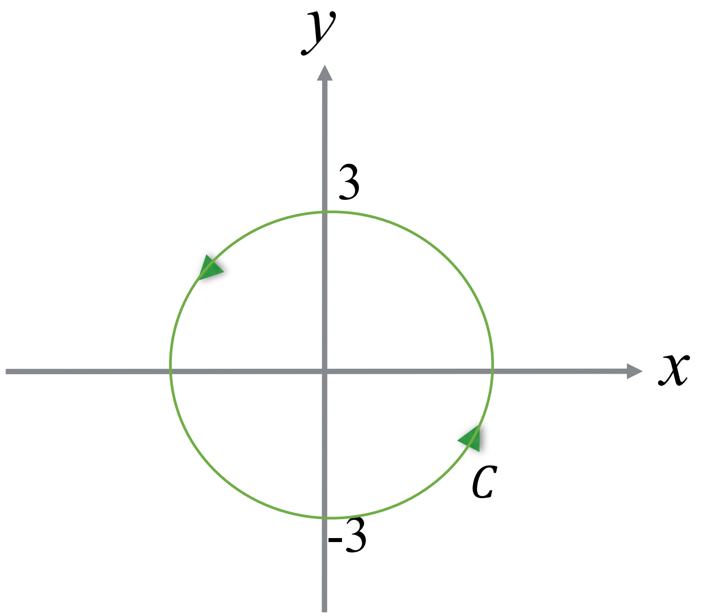
Solution:

The triangle formed by the lines $y = 2x$, $x = 2$ and $y = 0$.

Since $f(x, y) = x^2 y$ and $g(x, y) = 3x$. Hence, apply the Green's Theorem

$$\int_C x^2 y\, dx + 3x dy = \iint_R \left(\frac{\partial(3x)}{\partial x} - \frac{\partial(x^2 y)}{\partial y}\right) dA = \int_0^2 \int_0^{2x} (3 - x^2)\, dy\, dx$$

$$= \int_0^2 (3 - x^2)\, [y]_0^{2x} dx = \int_0^2 2x(3 - x^2)\, dx = \left[3x^2 - \frac{x^4}{2}\right]_0^2 = 4$$



Example 11.2:

Evaluate by using Green's Theorem

$$\int_C y\,dx + 4x dy\,,$$

where C is the positively oriented circle $x^2 + y^2 = 9$.

Solution:

Since $f(x, y) = y$ and $g(x, y) = 4x$. Hence, apply the Green's Theorem

$$\int_C y\,dx + 4x dy = \iint_R \left(\frac{\partial(4x)}{\partial x} - \frac{\partial(y)}{\partial y}\right) dA$$

$$= \int_0^{2\pi}\int_0^3 (4-1)\, r dr d\theta = \int_0^{2\pi} \left[\frac{3r^2}{2}\right]_0^3 d\theta = \left[\frac{27}{2}\theta\right]_0^{2\pi} = 27\pi$$

Exercise 11.1:

1) Evaluate by using Green's Theorem $\int_C (x^2y+1)\,dx + x^2dy$, over the triangle with formed by the lines $y=0$, $x=1$ and $y=2x$ the positive orientation.
2) Evaluate by using Green's Theorem $\int_C (e^x+5x^3)\,dx + (2+xy)dy$, over the positively oriented triangle formed by the lines $y=0$, $y=3-x$ and $y=2x$.
3) Find $\int_C (1-y^3)\,dx + x^3dy$ where C is the positively oriented circle of radius 2 centered at the origin.

[Ans:1) $\frac{5}{6}$ 2) 2 3) 2π]

Green's Theorem : Work done by a force

This method allows us to evaluate either a line integral around a closed path C or a do uble integral over the enclosed region, R. Thus, able to see a relationship between ce rtain kinds of line integrals on closed curve and its double integrals.

If $\boldsymbol{F}(x, y) = f(x, y)\mathbf{i} + g(x, y)\mathbf{j}$ is the force acting on a particle moving along the arc KL of the curve C, then the line integral

$$\int_C \boldsymbol{F}.d\boldsymbol{r} = \int_K^L \boldsymbol{F}.d\boldsymbol{r} = \int_C f(x, y)dx + g(x, y)dy$$

represents the work done in moving the particle from point K to point L.

Vector form of Green's Theorem is given as

$$\int_C \boldsymbol{F}.d\boldsymbol{r} = \iint_R (\nabla \times \mathbf{F}).\mathbf{k}\, dxdy$$

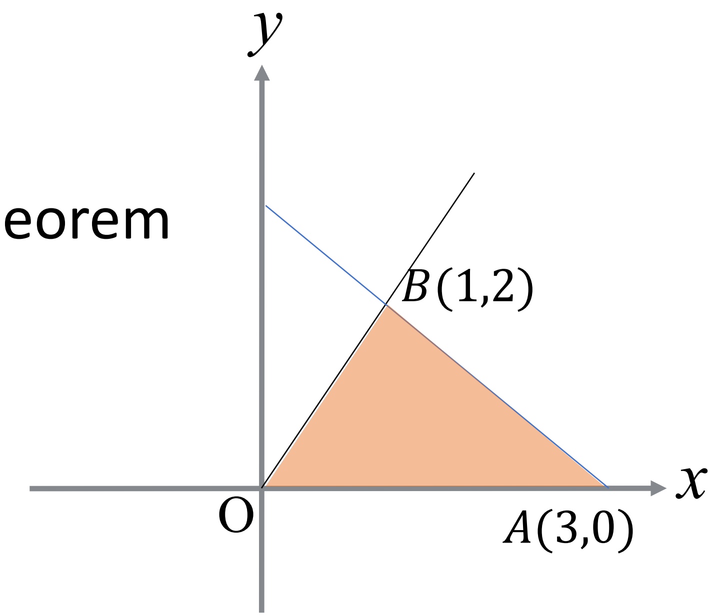
where $\boldsymbol{r} = x\mathbf{i} + y\mathbf{j}$,$\mathbf{k}$ is the unit vector along z −axis.

Example 11.3:

Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ along the plane bounded by the points $(0,0)$, $(3,0)$ and $(1,2)$ where $\boldsymbol{F}(x,y) = (2y + x^3)\mathbf{i} + (4x + y^2)\mathbf{j}$.

Solution:

Since $f(x,y) = 2y + x^3$ and $g(x,y) = 4x + y^2$. By the Green's Theorem



$$\int_C (2y + x^3)\,dx + (4x + y^2)dy$$

$$= \iint_R \left(\frac{\partial(4x + y^2)}{\partial x} - \frac{\partial(2y + x^3)}{\partial y}\right) dA$$

$$= \iint_R (4 - 2)\,dxdy = 2\,(Area\ of\ \Delta OAB) = 6$$

Example 11.4:

Find the work done by the force field $\mathbf{F}(x,y) = (e^{7x} - y^3)\mathbf{i} + (\cos 2y + x^3)\mathbf{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.

Solution:

Since $f(x,y) = e^{7x} - y^3$ and $g(x,y) = \cos 2y + x^3$. Hence, apply the Green's Theorem

$$\int_C (e^{7x} - y^3)\,dx + (\cos 2y + x^3)dy = \iint_R \left(\frac{\partial(\cos 2y + x^3)}{\partial x} - \frac{\partial(e^{7x} - y^3)}{\partial y}\right) dA$$

$$= \iint_R (3x^2 + 3y^2)\,dA = 3\iint_R (x^2 + y^2)\,dA = \int_0^{2\pi}\int_0^1 (r^2)\,r\,dr\,d\theta$$

$$= 3\int_0^{2\pi} \frac{1}{4}\,d\theta = \frac{3\pi}{2}$$

Converted to polar coordinates

Example 11.5:

Consider the vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$ on the disk region $R\{(x, y): x^2 + y^2 \leq a\}$ and is curve C the boundary of R in positive direction. Evaluate $\oint_C \boldsymbol{F}.d\boldsymbol{r}$ using the Green's Theorem.

Solution:

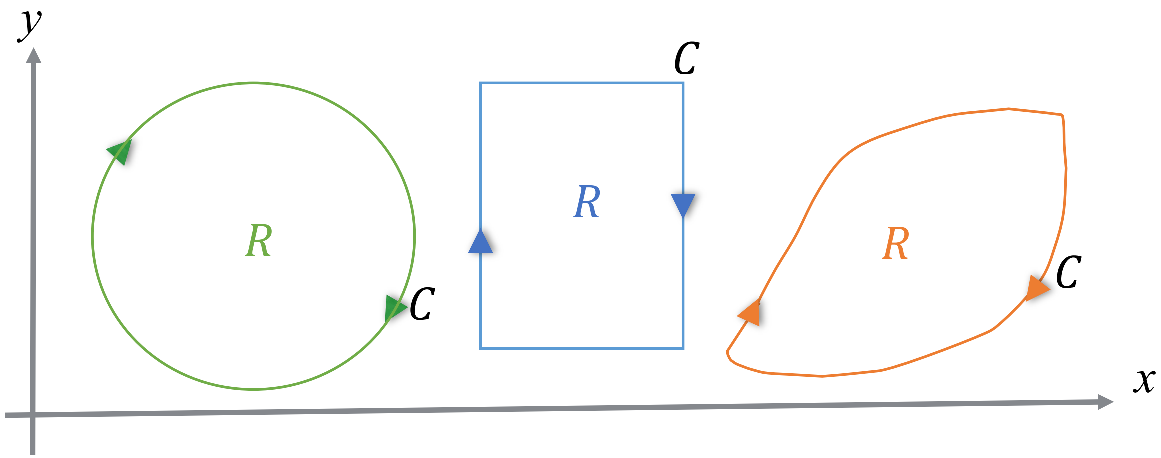
Since $f(x, y) = -y$ and $g(x, y) = x$ with $f_y = -1$ and $g_x = 1$, respectively.
Hence, apply the Green's Theorem

$$\int_C (-y)\,dx + xdy = \iint_R \left(\frac{\partial(x)}{\partial x} - \frac{\partial(-y)}{\partial y}\right) dA = \iint_R (1-(-1))\,dA$$

$$= \int_0^{2\pi}\int_0^{a} 2\,rdrd\theta = 2a^2\pi$$

Utilizing Green's Theorem

If the closed curve runs clockwise or in a negative orientation,



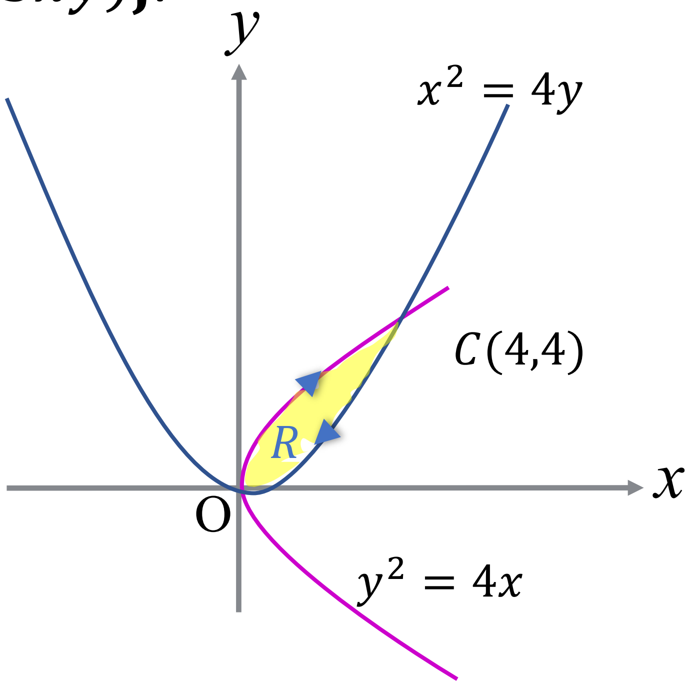
then allow us to switch the sign

$$\int_C f(x,y)dx + g(x,y)dy = -\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

Remarks: $dA = dxdy = dydx = rdrd\theta$

Example 11.6:

Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ along C is the clockwise oriented boundary of the region bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$ where $\boldsymbol{F}(x,y) = (5x-y)\mathbf{i} + (3xy)\mathbf{j}$.



Solution:

Since $f(x,y) = 5x - y$ and $g(x,y) = 3xy$. By the Green's Theorem

$$\int_C (5x-y)\,dx + (3xy)dy$$

$$= -\iint_R \left(\frac{\partial(3xy)}{\partial x} - \frac{\partial(5x-y)}{\partial y}\right) dA$$

$$= -\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} (3y+1)\,dxdy = -\frac{512}{15}$$

Exercise 11.2:

1) Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ along C is the clockwise oriented boundary of the region bounded by the triangle with formed by the lines $y = 0$, $x = 1$ and $y = 2x$ where
$$\boldsymbol{F}(x,y) = (x^2y + 1)\mathbf{i} + x^2\mathbf{j}.$$
2) Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ along the plane bounded by the lines $y = 0$, $y = 2x$ and $y = 3 - x$ where
$$\boldsymbol{F}(x,y) = (e^x + 5x^3)\mathbf{i} + (2 + xy)\mathbf{j}.$$
3) Find the work done by the force field $\mathbf{F}(x,y) = (e^{7x} - y^3)\mathbf{i} + (x^3)\mathbf{j}$ on a particle that travels once around the circle $x^2 + y^2 = 2$ in the counterclockwise direction.

[Ans:1) $-\frac{5}{6}$ 2) -2 3) -6π]

Recall Vector Fields Operations

Expand the operation of a vector with 'del' operator where notation for gradient, ∇ :

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Divergence of vector field $\mathbf{F}(x, y, z)$:

$$\nabla \cdot \mathbf{F}(x, y, z) = \frac{\partial}{\partial x}\big(f(x, y, z)\big) + \frac{\partial}{\partial y}\big(g(x, y, z)\big) + \frac{\partial}{\partial z}\big(h(x, y, z)\big)$$

A curl of vector field $\mathbf{F}(x, y, z)$:

$$\nabla \times \mathbf{F}(x, y, z) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M(x, y, z) & N(x, y, z) & P(x, y, z) \end{vmatrix}$$

Curl of vector field, F

Recall the ∇ operator defined as

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}.$$

If $\boldsymbol{F}(x,y,z) = f(x,y,z)\mathbf{i} + g(x,y,z)\mathbf{j} + h(x,y,z)\mathbf{k}$, then the curl of $\mathbf{F}$, (denoted by $\nabla \times \mathbf{F}$) can be written as:

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix} = \left|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{matrix}\right| \begin{matrix} \mathbf{i} & \mathbf{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ f & g \end{matrix}$$

$$= \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right)\mathbf{i} + \left(\frac{\partial f}{\partial z} - \frac{\partial h}{\partial x}\right)\mathbf{j} + \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)\mathbf{k}$$

Example 11.7:

Calculate the curl of $\boldsymbol{F}$ if

$$\boldsymbol{F}(x,y,z) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$$

Solution:

$$\text{curl}\,\boldsymbol{F} = \nabla \times \boldsymbol{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2z & 3x & 5y \end{vmatrix}$$

$$= \left(\frac{\partial(5y)}{\partial y} - \cancelto{0}{\frac{\partial(3x)}{\partial z}}\right)\mathbf{i} + \left(\frac{\partial(2z)}{\partial z} - \cancelto{0}{\frac{\partial(5y)}{\partial x}}\right)\mathbf{j} + \left(\frac{\partial(3x)}{\partial x} - \cancelto{0}{\frac{\partial(2z)}{\partial y}}\right)\mathbf{k}$$

$$= 5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

Example 11.8:

Given that

$$\boldsymbol{F}(x,y,z) = yx^2\mathbf{i} + 2y^3z\mathbf{j} + 3z\mathbf{k}$$

Find the curl of $\boldsymbol{F}$ at the point $(2,-1,\pi)$.

Solution:

$$\text{curl } \boldsymbol{F} = \nabla \times \boldsymbol{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yx^2 & 2y^3z & 3z \end{vmatrix}$$

$$= \left(\frac{\partial(3z)}{\partial y}^{0} - \frac{\partial(2y^3z)}{\partial z}\right)\mathbf{i} + \left(\frac{\partial(yx^2)}{\partial z}^{0} - \frac{\partial(3z)}{\partial x}^{0}\right)\mathbf{j} + \left(\frac{\partial(2y^3z)}{\partial x}^{0} - \frac{\partial(yx^2)}{\partial y}\right)\mathbf{k}$$

$$= -2y^3\mathbf{i} - x^2\mathbf{k}$$

$$\nabla \times \boldsymbol{F}\Big|_{(2,-1,\pi)} = -2(-1)^3\mathbf{i} - 2^2\mathbf{k} = 2\mathbf{i} - 4\mathbf{k}$$

Let $\boldsymbol{F}$ represents the velocity field of a flowing fluid and curl $\boldsymbol{F}$ is the tendency of particles at the point (x, y, z) to rotate about the axis that points in the direction of curl $\boldsymbol{F}$.

If $\boldsymbol{F}$ is a conservative vector field, then curl $\boldsymbol{F} = \mathbf{0}$, which its fluid is called irrotational.

Example 11.9:

Determine if vector $\boldsymbol{F}(x, y, z) = x^2\mathbf{i} + 3y\mathbf{j} + (5z - 2)\mathbf{k}$ is conservative?

Solution:

$$\text{curl } \boldsymbol{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2 & 3y & 5z - 2 \end{vmatrix}$$

$$= \left(\frac{\partial(5z - 2)}{\partial y} - \frac{\partial(4y)}{\partial z}\right)\mathbf{i} + \left(\frac{\partial(x^2)}{\partial z} - \frac{\partial(5z - 2)}{\partial x}\right)\mathbf{j} + \left(\frac{\partial(4y)}{\partial x} - \frac{\partial(x^2)}{\partial y}\right)\mathbf{k}$$

$$= 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} = \mathbf{0}$$

Hence, $\boldsymbol{F}(x, y, z) = x^2\mathbf{i} + 3x\mathbf{j} + (5z - 2)\mathbf{k}$ is conservative since curl $\boldsymbol{F} = \mathbf{0}$.

Exercise 11.3:

1) Determine the curl of $\boldsymbol{F}$ if

$$\boldsymbol{F}(x,y,z) = (x^2 - y)\mathbf{i} + 4z\mathbf{j} + x^2\mathbf{k}$$

2) Compute curl of $\boldsymbol{F}(x,y,z) = \cos 3x\,\mathbf{i} + (2y - y^2)\mathbf{j} + (z - 6x^3)\mathbf{k}$ at point $(1, -\pi, 12)$.
3) Determine if vector $\boldsymbol{F}(x,y,z) = \sin x\,\mathbf{i} + y^3\mathbf{j} + (3 + e^{-5z})\mathbf{k}$ is conservative?
4) Determine if vector

$$\boldsymbol{F}(x,y,z) = \left(2y^4 + \frac{3x^2y}{z^2}\right)\mathbf{i} + \left(8xy - 2 + \frac{x^3}{z^2}\right)\mathbf{j} + \left(e^2 - \frac{x^3y}{z^3}\right)\mathbf{k}$$

is conservative?

[Ans: 1) $-4\mathbf{i} - 2x\mathbf{j} + \mathbf{k}$ 2) 18 3) Yes, $\boldsymbol{F}$ is conservative.4) Not conservative since curl $\boldsymbol{F} \neq \mathbf{0}$]

Divergence of vector field, $\mathbf{F}$

Recall the ∇ operator defined as

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}.$$

If $\boldsymbol{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$, then the divergence of $\mathbf{F}$, (denoted by $\nabla \cdot \mathbf{F}$) can be written as:

$$\text{div}\,\mathbf{F} = \nabla \cdot \mathbf{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle f, g, h \rangle = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

Example 11.10:

Determine the divergence of

i. $\boldsymbol{F}(x,y,z) = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$.

ii. $\boldsymbol{F}(x,y,z) = (xy^3z^2)\mathbf{i} + (\sin x + y^3)\mathbf{j} + (xyz)\mathbf{k}$ at the point (2,1,0).

Solution:

$$\nabla \cdot \mathbf{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

i. $\nabla \cdot \mathbf{F} = \frac{\partial(2x)}{\partial x} + \frac{\partial(3y)}{\partial y} + \frac{\partial(4z)}{\partial z} = 2 + 3 + 4 = 9$

ii. $\nabla \cdot \mathbf{F} = \frac{\partial(xy^3z^2)}{\partial x} + \frac{\partial(\sin x + y^3)}{\partial y} + \frac{\partial(xyz)}{\partial z} = y^3z^2 + 3y^2 + xy$

$\nabla \cdot \mathbf{F}|_{(2,1,0)} = (1)^3(0)^2 + 3(1)^2 + (2)(1) = 5$

Exercise 11.4:

1) Determine the divergence of

i. $\boldsymbol{F}(x,y,z) = 7x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$.

ii. $\boldsymbol{F}(x,y,z) = xyz\mathbf{i} + 2xyz\mathbf{j} - 3xyz\mathbf{k}$.

iii. $\boldsymbol{F}(x,y,z) = (xy^3z - e^z)\mathbf{i} + (2y^2 - \sin z)\mathbf{j} + (e^{xyz})\mathbf{k}$ at the point $(0,1,5)$.

2) Verify $\text{div}\,(\text{curl}\,\boldsymbol{F}) = 0$ for the vector field $\boldsymbol{F}(x,y,z) = xz\mathbf{i} + x^3z\mathbf{j} + yz\mathbf{k}$.

[Ans: 1)i. 10 ii. $yz + 2xz - 3xy$ iii. 9]

Application of Green's Theorem

Recall that the area, A of a region R with the following double integral

$$A = \iint_R \quad dA$$

If closed curve C is the boundary of the region R; it allow us to use Green's Theorem in reverse to compute the region R by evaluating the following integrals

$$A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - ydx$$

Properties of Curl and Divergence

Recall that a gradient vector with φ as a function of three variables, grad φ and has continuous second order partial derivatives is denoted as $\nabla\varphi = \frac{\partial\varphi}{\partial x}\boldsymbol{i} + \frac{\partial\varphi}{\partial y}\boldsymbol{j} + \frac{\partial\varphi}{\partial z}\boldsymbol{k}$.

The curl of its gradient is

$$\nabla \times (\nabla\varphi) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} & \frac{\partial\varphi}{\partial z} \end{vmatrix} = \left(\frac{\partial^2\varphi}{\partial y\partial z} - \frac{\partial^2\varphi}{\partial z\partial y}\right)\mathbf{i} + \left(\frac{\partial^2\varphi}{\partial z\partial x} - \frac{\partial^2\varphi}{\partial x\partial z}\right)\mathbf{j} + \left(\frac{\partial^2\varphi}{\partial x\partial y} - \frac{\partial^2\varphi}{\partial y\partial x}\right)\mathbf{k}$$

Since φ has continuous second order partial derivatives, where any form of multiple derivative expression is $\frac{\partial^2\varphi}{\partial y\partial z} = \frac{\partial^2\varphi}{\partial z\partial y}$. Hence, $\nabla \times (\nabla\varphi) = (0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k} = \mathbf{0}$

Thus, the curl of its gradient is the zero vector.

Properties of Curl and Divergence

If a conservative vector field, $\mathbf{F}$ can be written as the gradient of a function, $\mathbf{F} = \nabla\varphi$.
The curl of any conservative vector field $\mathbf{F}$ is zero vector. $\nabla \times (\mathbf{F}) = \mathbf{0}$

The divergence of a curl is dot product of the two vector:

$$\nabla \cdot (\nabla \times \mathbf{F}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle 0,0,0 \rangle = (0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k} = \mathbf{0}$$

Hence, the divergence of a curl is zero vector.

The relationship between the curl and the divergence is given by the following fact

$$\text{div}\,(\text{curl}\,\boldsymbol{F}) = 0.$$

Example 11.11:

Verify $\text{div}\,(\text{curl}\,\boldsymbol{F}) = 0$ for the vector field $\boldsymbol{F}(x,y,z) = yx^2\mathbf{i} + 2y^3z\mathbf{j} + 3z\mathbf{k}$.

Solution:

First compute the curl

$$\text{curl}\,\boldsymbol{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yx^2 & 2y^3z & 3z \end{vmatrix} = -2y^3\mathbf{i} - x^2\mathbf{k}$$

Now compute the divergence of it

$$\text{div}\,(\text{curl}\,\boldsymbol{F}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle -2y^3, 0, -x^2 \rangle = 0$$

Exercise 11.5:

1. If $\boldsymbol{F}(x,y,z) = x^2\mathbf{i} + xye^x\mathbf{j} + \sin z\,\mathbf{k}$, find div $(\text{curl}\,\boldsymbol{F}) = 0$.
2. Verify div $(\text{curl}\,\boldsymbol{F}) = 0$ for the vector field $\boldsymbol{F}(x,y,z) = yx^3\mathbf{i} - 5y^3z\mathbf{j} + \sin z\,\mathbf{k}$.

[Ans: 1. 0 2. 0]

Reference

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THANK YOU