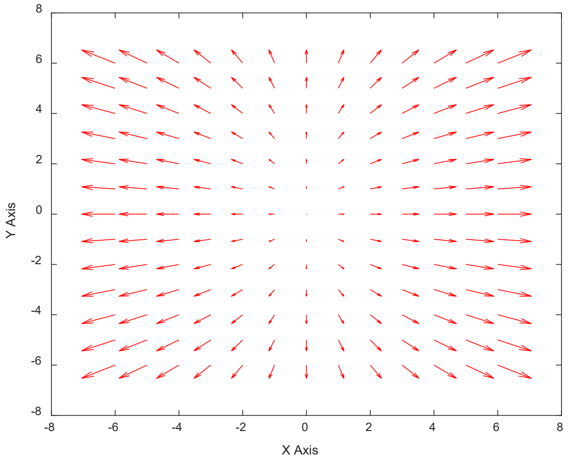
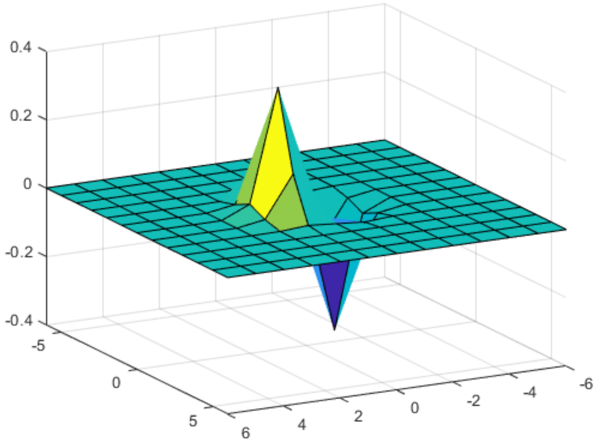
BEKG 2433
ENGINEERING MATHEMATICS 2

Week 10: VECTOR FIELDS & LINE INTEGRAL





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Lesson Outcomes

Upon completion of this lesson, students should be able to:

- define vector fields.
- evaluate the line integrals.
- evaluate work done by a vector field.

Introduction: Vector Fields

A vector field on two-dimensional (2D) space is a function $\mathbf{F}$ that assigns to each point (x, y) a 2D vector given by

$$\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$$

or in a simpler form

$$\mathbf{F}(x, y) = \langle M(x, y), N(x, y)\rangle.$$

Here, the function $\mathbf{F}(x, y)$ has vector components that depend on the coordinates x and y. $M(x, y)$ and $N(x, y)$ are an ordinary scalar functions or called scalar fields (depends on x and y) that determine the components of the vectors at each point.

Introduction: Vector Fields

A vector field on three-dimensional (3D) space is a function $\mathbf{F}$ that assigns to each point (x, y, z) a 3D vector given by

$$\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$$

or

$$\mathbf{F}(x, y, z) = \langle M(x, y, z), N(x, y, z), P(x, y, z) \rangle.$$

A possible practical interpretation of these vector fields such as

1) effects of a force being exerted through space;
2) direct representation of physical motion.

Vector Fields

First, we learn a general concept of a vector field and general ways to express the function.

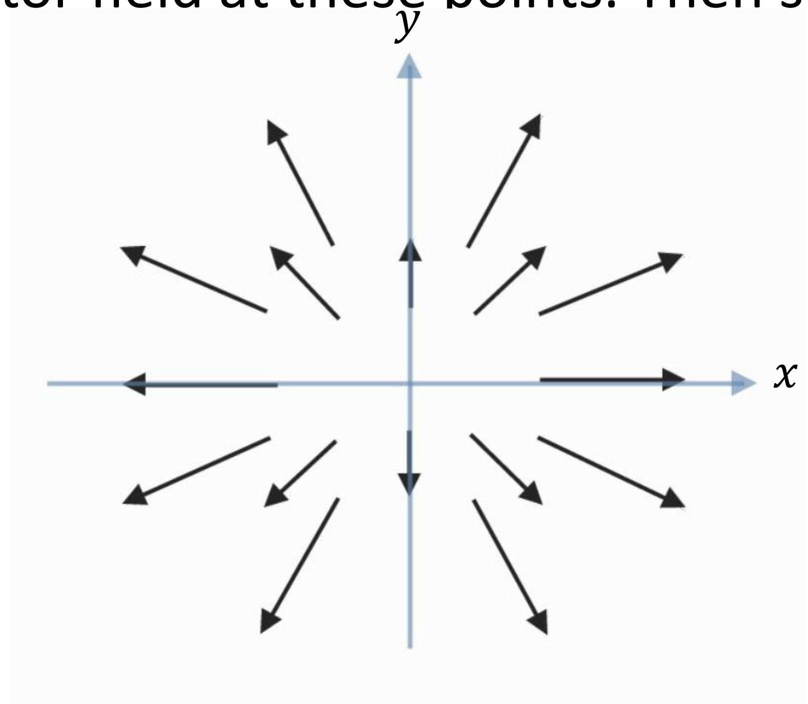
Example 10.1:

Given the vector field $\mathbf{F}(x, y) = x\boldsymbol{i} + y\boldsymbol{j}$.

By choosing various points (x, y), evaluate the vector field at these points. Then sketch the vector field $\mathbf{F}$.

Solution:

(x,y)	$x\boldsymbol{i} + y\boldsymbol{j}$	(x,y)	$x\boldsymbol{i} + y\boldsymbol{j}$
$(0,1)$	$\boldsymbol{j}$	$(0,-1)$	$-\boldsymbol{j}$
$(1,2)$	$\boldsymbol{i} + 2\boldsymbol{j}$	$(-1,2)$	$-\boldsymbol{i} + 2\boldsymbol{j}$
$(1,0)$	$\boldsymbol{i}$	$(-1,0)$	$-\boldsymbol{i}$
$(2,1)$	$2\boldsymbol{i} + \boldsymbol{j}$	$(2,-1)$	$2\boldsymbol{i} - \boldsymbol{j}$



Vector Fields

Recall operation of vector function,

Given that two vector functions for a single variable, t :

$$\mathbf{F}(t) = x_1(t)\boldsymbol{i} + y_1(t)\boldsymbol{j} + z_1(t)\boldsymbol{k} \quad \text{and} \quad \boldsymbol{G}(t) = x_2(t)\boldsymbol{i} + y_2(t)\boldsymbol{j} + z_2(t)\boldsymbol{k}$$

Product of scalar, α and vector:

$$\alpha\boldsymbol{F}(t) = \alpha x_1(t)\boldsymbol{i} + \alpha y_1(t)\boldsymbol{j} + \alpha z_1(t)\boldsymbol{k}$$

Vector Sum:

$$\mathbf{F}(t) + \mathbf{G}(t) = \big(x_1(t) + x_2(t)\big)\boldsymbol{i} + \big(y_1(t) + y_2(t)\big)\boldsymbol{j} + \big(z_1(t) + z_2(t)\big)\boldsymbol{k}$$

Product of a scalar function, $f(t)$ and a Vector Function:

$$f(t)\mathbf{F}(t) = f(t)x_1(t)\boldsymbol{i} + f(t)y_1(t)\boldsymbol{j} + f(t)z_1(t)\boldsymbol{k}.$$

Vector Fields Operations

Expand the operation of a vector with 'del' operator where notation for gradient, ∇ :

$$\mathbf{\nabla} = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Divergence of vector field $\mathbf{F}(x, y, z)$:

$$\mathbf{\nabla} \cdot \mathbf{F}(x, y, z) = \frac{\partial}{\partial x}\big(M(x, y, z)\big) + \frac{\partial}{\partial y}\big(N(x, y, z)\big) + \frac{\partial}{\partial z}\big(P(x, y, z)\big)$$

A gradient vector with φ as a function of three variables, grad φ or denoted as:

$$\mathbf{\nabla}\varphi = \frac{\partial \varphi}{\partial x}\boldsymbol{i} + \frac{\partial \varphi}{\partial y}\boldsymbol{j} + \frac{\partial \varphi}{\partial z}\boldsymbol{k}.$$

A curl of vector field $\mathbf{F}(x, y, z)$:

$$\mathbf{\nabla} \times \mathbf{F}(x, y, z) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M(x, y, z) & N(x, y, z) & P(x, y, z) \end{vmatrix}$$

Gradient Fields and Potential Functions

The vector field, $\mathbf{F}$, is called the gradient field for the scalar function, φ, is given by $\mathbf{F} = \boldsymbol{\nabla}\varphi$. The scalar function, φ, which is called a potential function for $\mathbf{F}$.

Example 10.2:

Find the gradient field for the potential function $\varphi(x, y, z) = x^2 + y^2 + z^2$.

Solution:

$$\text{grad } \varphi = \boldsymbol{\nabla}\varphi = \frac{\partial \varphi}{\partial x}\boldsymbol{i} + \frac{\partial \varphi}{\partial y}\boldsymbol{j} + \frac{\partial \varphi}{\partial z}\boldsymbol{k} = 2x\boldsymbol{i} + 2y\boldsymbol{j} + 2z\boldsymbol{k}$$

Gradient Fields and Potential Functions

Example 10.3:

Find the gradient field, grad φ, for the potential function $\varphi(x, y, z) = xyz$.

Solution:

$$\text{grad } \varphi = \nabla\varphi = \frac{\partial\varphi}{\partial x}\boldsymbol{i} + \frac{\partial\varphi}{\partial y}\boldsymbol{j} + \frac{\partial\varphi}{\partial z}\boldsymbol{k} = yz\boldsymbol{i} + xz\boldsymbol{j} + xy\boldsymbol{k}$$

Exercise 10.1:

Find the gradient field, grad φ for the following potential function.

1. $\varphi(x, y, z) = yz + x^2$
2. $\varphi(x, y, z) = e^z \sin(2x + y)$
3. $\varphi(x, y, z) = \ln xyz$

[Ans: 1. $2x\boldsymbol{i} + z\boldsymbol{j} + y\boldsymbol{k}$ 2. $2e^z \cos(2x + y)\,\boldsymbol{i} + e^z \cos(2x + y)\,\boldsymbol{j} + e^z \sin(2x + y)\,\boldsymbol{k}$ 3. $\frac{1}{x}\boldsymbol{i} + \frac{1}{y}\boldsymbol{j} + \frac{1}{z}\boldsymbol{k}$]

Line Integrals

The line integral is a single integral over a curve in 3D space that can be interpreted as the area under a curve C.

An important application of line integrals is the computation of the work done as a variable force moves along an arbitrary path.

The idea of line integral involves

- Integrate over a curve (instead of integrating over an interval $[a, b]$)
- Involve scalar fields or vector fields
- Solve problems involving fluid flow, forces, electricity and magnetism

Line Integrals

Let C be a smooth curve in 2D spaces where both $f(x, y)$ and $g(x, y)$ is continuous on some open region containing the curve C.
Then define parametrically $x = x(t)$ and $y = y(t)$ for $a \leq t \leq b$.
Hence, the line integral of $f(x, y)$ and $g(x, y)$ along C is denoted by

$$\int_C f(x, y)\, dx = \int_a^b f(x(t), y(t))\, x'(t)dt$$

$$\int_C g(x, y)\, dy = \int_a^b g(x(t), y(t))\, y'(t)dt$$

Line Integrals

Example 10.4:

Evaluate

$$\int_C 3xy\,dx + 2(x^2 + y^2)dy$$

over the circular arc given by $x = \cos t$ and $y = \sin t$ for $0 \leq t \leq \frac{\pi}{2}$.

Solution:

From $x'(t) = \frac{dx}{dt} = -\sin t$ and $y'(t) = \frac{dy}{dt} = \cos t$.

Solution continued:

Hence perform the line integrals

$$\int_C 3xy\,dx = \int_0^{\frac{\pi}{2}} 3\cos t \sin t\,(-\sin t)dt = -\int_0^{\frac{\pi}{2}} 3\cos t \sin^2 t\,dt$$

and

$$\int_C 2(x^2 + y^2)\,dy = \int_0^{\frac{\pi}{2}} 2(\cos^2 t + \sin^2 t)\,(\cos t)dt$$

Thus

$$\int_C 3xy\,dx + 2(x^2 + y^2)dy = [-\sin^3 t + 2\sin t]_0^{\frac{\pi}{2}} = 1$$

Line Integrals

Example 10.5:

Evaluate

$$\int_C 3xy\,dx + 2(x^2 + y^2)dy$$

over the circular arc given by $x = t$ and $y = \sqrt{1 - t^2}$ for $0 \leq t \leq 1$.

Solution:

From $x'(t) = \frac{dx}{dt} = \frac{d}{dt}(t) = 1$ and $y'(t) = \frac{d}{dt}\left(\sqrt{1 - t^2}\right) = -\frac{2t}{2\sqrt{1-t^2}}$.

Solution continued:

Hence perform the line integrals

$$\int_C 3xy\,dx = \int_0^1 3(t)\sqrt{1-t^2}\,dt = -\int_0^1 3t(1-t^2)^{1/2}\,dt$$

and

$$\int_C 2(x^2+y^2)\,dy = \int_0^1 2(t^2+(1-t^2))\left(-\frac{t}{\sqrt{1-t^2}}\right)dt = -\int_0^1 2t(1-t^2)^{-1/2}\,dt$$

Thus

$$\int_C 3xy\,dx + 2(x^2+y^2)dy = \left[-2(1-t^2)^{\frac{3}{2}} + (1-t^2)^{\frac{1}{2}}\right]_0^1 = -1$$

Line Integrals for arc length

Let C be a smooth curve in 3D spaces and $f(x, y, z)$ be continuous on some open region containing the curve C.
Then define parametrically $x = x(t)$, $y = y(t)$ and $z = z(t)$ or write in a vector form $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ for $a \leq t \leq b$.
If s is the arc length of the curve measured from $t = a$ to $t = b$, then

$$\frac{ds}{dt} = \|\mathbf{r}'(t)\| = \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}.$$

Thus, the line integral of $f(x, y, z)$ along C with respect to s is denoted by

$$\int_C f(x, y, z)\, ds = \int_a^b f(x(t), y(t), z(t))\, ds = \int_a^b f\left(x(t), y(t), z(t)\right)\|\boldsymbol{r}'(t)\|dt\,.$$

Line Integrals

Example 10.6:

Evaluate

$$\int_C 3(x+yz)\,ds$$

over the line from $P(1,1,2)$ and $Q(0,2,1)$.

Solution:

Compute $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \langle 0,2,1\rangle - \langle 1,1,2\rangle = \langle -1,1,-1\rangle$

Use it and define some vector and its first derivative

$\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle = \langle 1,1,2\rangle + t\langle -1,1,-1\rangle = \langle 1-t, 1+t, 2-t\rangle$ for $0 \le t \le 1$ and

$$\mathbf{r}'(t) = \langle \frac{d}{dt}(1-t), \frac{d}{dt}(1+t), \frac{d}{dt}(2-t)\rangle = \langle -1,1,-1\rangle$$

Solution continued:

Thus

$$\frac{ds}{dt} = \|\mathbf{r}'(t)\| = \sqrt{(-1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$$

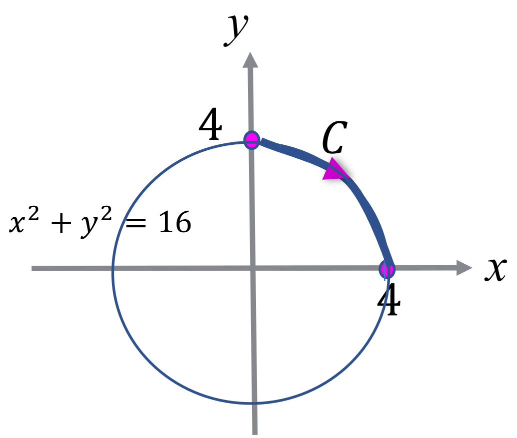
From $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle = \langle 1 - t, 1 + t, 2 - t \rangle$ where $0 \le t \le 1$

Hence perform the line integrals

$$\int_C 3(x + yz)\, ds = \int_0^1 3((1 - t) + (1 + t)(2 - t))\sqrt{3}dt = 3\sqrt{3}\int_0^1 t^2 - 2t + 3\, dt$$

Thus

$$\int_C 3(x + yz)\, ds = 3\sqrt{3}\left[\frac{t^3}{3} - t^2 + 3t\right]_0^1 = 7\sqrt{3}$$



Line Integrals

Example 10.7:

Evaluate $\int_C 2y\, ds$ where C is the quarter-circle $x^2 + y^2 = 16$ from $(0,4)$ and $(4,0)$.

Solution:

Let $x(t) = 4\sin t$ and $y(t) = 4\cos t$.

$\mathbf{r}(t) = \langle 4\sin t, 4\cos t \rangle$ for $0 \le t \le \frac{\pi}{2}$. So $\mathbf{r}'(t) = \langle 4\cos t, -4\sin t \rangle$

$$\|\mathbf{r}'(t)\| = \sqrt{(4\cos t)^2 + (-4\sin t)^2} = 4$$

$$\int_C 2y\, ds = 2\int_0^{\frac{\pi}{2}} 4\cos t\, \|\mathbf{r}'(t)\| dt = 32[\sin t]_0^{\frac{\pi}{2}} = 32$$

Exercise 10.2:

1) Evaluate $\int_C \ 6xy + 12y \, ds$ on the following lines.
   a)The line from $P(1,0,0)$ to $Q(0,1,1)$.
   b)The line from $Q(0,1,1)$ to $P(1,0,0)$.
2) Evaluate

$$\int_C (x + yz) \, ds$$

   over the line from $P(1,2,1)$ and $Q(2,1,0)$.

3) Evaluate $\int_C \ 3y \, ds$ where C is the semi-circle $x^2 + y^2 = 4$ from $(0, -2)$ and $(2,0)$.

[Ans: 1) a)$7\sqrt{3}$ b) $7\sqrt{3}$ 2) $\frac{7\sqrt{3}}{3}$ 3) -24]

Line Integrals in Vector form

Let C be a smooth curve in 3D spaces with $M(x,y,z)$, $N(x,y,z)$ and $P(x,y,z)$ being continuous on some open region containing the curve C.
The line integrals along C can be written in vector field notation, given by

$$\mathbf{F}(x,y,z) = M(x,y,z)\mathbf{i} + N(x,y,z)\mathbf{j} + P(x,y,z)\mathbf{k}$$

Then define parametrically $x = x(t)$, $y = y(t)$ and $z = z(t)$ for $a \leq t \leq b$, so that the curve C can be expressed in terms of position vector

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

and the derivatives

$$\mathbf{r}'(t) = \frac{d\mathbf{r}(t)}{dt} = \frac{d}{dt}(x(t))\mathbf{i} + \frac{d}{dt}(y(t))\mathbf{j} + \frac{d}{dt}(z(t))\mathbf{k}.$$

Vector Line Integral

Let $\mathbf{F}$ be continuous vector field on a region containing a smooth oriented curve C parameterized by arc length. Let $\mathbf{T}$ be the unit tangent vector of each point of C consistent with the orientation. The line integral of $\mathbf{F}$ over C is

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds$$

Since $\frac{ds}{dt} = \|\mathbf{r}'(t)\|$ implies $\frac{ds}{\|\mathbf{r}'(t)\|} = dt$.

Hence,

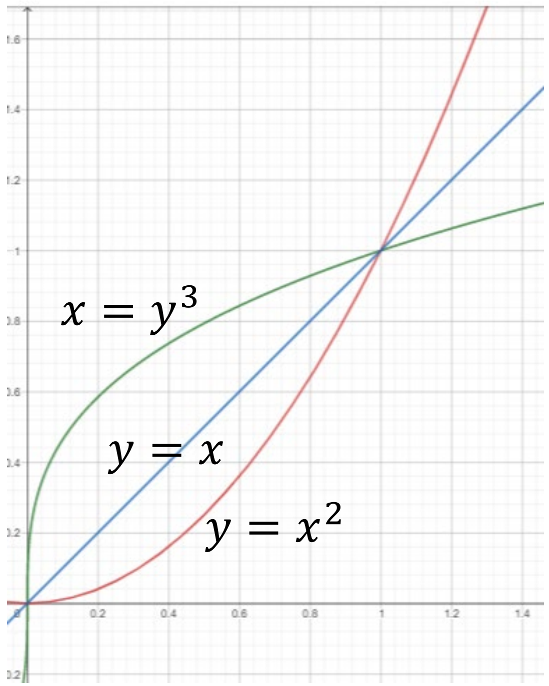
$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_C \mathbf{F}\cdot\frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}\,ds = \int_C \mathbf{F}\cdot\mathbf{r}'(t)\,dt$$

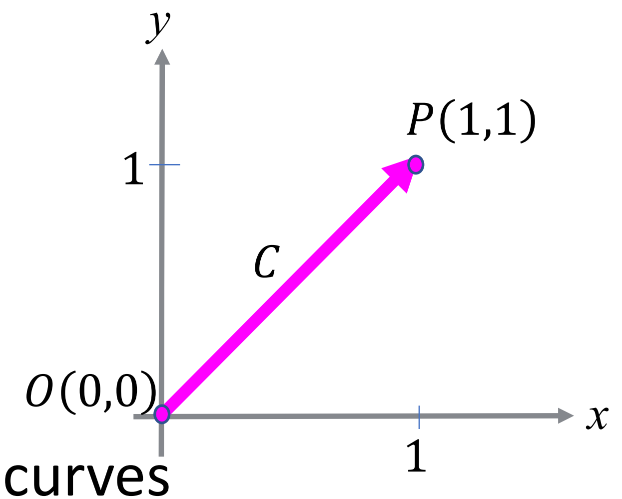
Note that reversing the orientation of a curve reverses the sign of the line integral of a vector line integral.

Line Integrals in Vector form

A vector field $\boldsymbol{F}(x, y)$ is said to be conservative if there exists a differentiable function $\varphi(x, y)$ such that the gradient of $\varphi(x, y)$ is $\boldsymbol{F}(x, y) = \nabla\varphi(x, y)$.
The function $\varphi(x, y)$ is called the scalar potential function for $\boldsymbol{F}(x, y)$

Example 10.8:
Let $\boldsymbol{F}(x, y) = 2y\mathbf{i} + 3x\mathbf{j}$.
Evaluate the line integral over the following curves
a) Curve is the path for a line segment from $(0,0)$ to $(1,1)$.
b) Along the parabola from $(0,0)$ to $(1,1)$.
c) Along the cubic from $(0,0)$ to $(1,1)$.





Line Integrals in Vector form

Example 10.8:

Let $\boldsymbol{F}(x, y) = 2y\mathbf{i} + 2x\mathbf{j}$. Evaluate the line integral over the following curves

a) Curve is the path for a line segment from $(0,0)$ to $(1,1)$.

Solution:

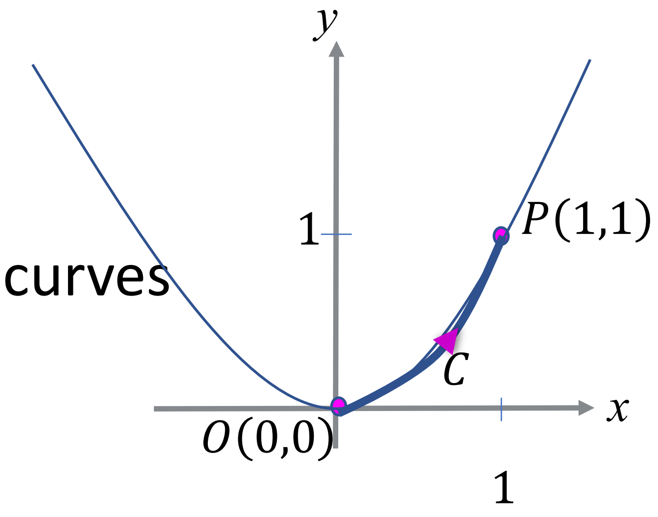
Line segment $y = x$ implies that parametric form is $x = t$ and $y = t$ for $0 \leq t \leq 1$, gives a position vector

$$\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + 0\mathbf{k}$$

From $\boldsymbol{F}(x, y) = 2y\mathbf{i} + 3x\mathbf{j}$. Implies $\boldsymbol{F}(x(t), y(t)) = 2t\mathbf{i} + 2t\mathbf{j}$.

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r} = \int_0^1 \mathbf{F}(x(t), y(t)) \cdot \mathbf{r}'(t)dt = \int_0^1 (2t\mathbf{i} + 2t\mathbf{j}) \cdot (\mathbf{i} + \mathbf{j})dt = \int_0^1 4t\, dt = 2$$

Line Integrals in Vector form



Example 10.8:

Let $\boldsymbol{F}(x, y) = 2y\mathbf{i} + 2x\mathbf{j}$. Evaluate the line integral over the following curves

b) Along the parabola from $(0,0)$ to $(1,1)$.

Solution:

Parabola $y = x^2$ implies that parametric form is $x = t$ and $y = t^2$ for $0 \le t \le 1$, gives a position vector $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$

From $\boldsymbol{F}(x, y) = 2y\mathbf{i} + 2x\mathbf{j}$. Implies $\boldsymbol{F}(x(t), y(t)) = 2t^2\mathbf{i} + 2t\mathbf{j}$.

$$\int_C \mathbf{F}(x,y)\cdot d\mathbf{r} = \int_0^1 \mathbf{F}\big(x(t), y(t)\big)\cdot \mathbf{r}'(t)dt = \int_0^1 (2t^2\mathbf{i} + 2t\mathbf{j})\cdot(\mathbf{i} + 2t\mathbf{j})dt$$

$$= \int_0^1 6t^2\,dt = 2$$

Line Integrals in Vector form

Example 10.8:

Let $\boldsymbol{F}(x,y) = 2y\mathbf{i} + 2x\mathbf{j}$. Evaluate the line integral over the following curves

c) Along the cubic from $(0,0)$ to $(1,1)$.

Solution:

Cubic $x = y^3$ implies that the parametric form is $y = t$ and $x = t^3$ for $0 \le t \le 1$, gives a position vector

$$\mathbf{r}(t) = t^3\mathbf{i} + t\mathbf{j}$$

From $\boldsymbol{F}(x,y) = 2y\mathbf{i} + 2x\mathbf{j}$. Implies $\boldsymbol{F}(x(t),y(t)) = 2t\mathbf{i} + 2t^3\mathbf{j}$.

$$\int_C \mathbf{F}(x,y)\cdot d\mathbf{r} = \int_0^1 \mathbf{F}(x(t),y(t))\cdot \mathbf{r}'(t)dt = \int_0^1 (2t\mathbf{i} + 2t^3\mathbf{j})\cdot(3t^2\mathbf{i} + \mathbf{j})dt = \int_0^1 8t^3\,dt$$

$$= 1$$

Vector Line Integrals

Example 10.9:

Let $\boldsymbol{F}(x,y) = (y-x)\mathbf{i} + x\mathbf{j}$. Evaluate the line integral of $\boldsymbol{F}$ on the path C from $K(0,2)$ to $L(2,0)$ via two-line segments through $M(0,0)$.

Solution:

C consists of two line segments:

i) From K to M, C_1 :$\mathbf{r}(t) = 0\mathbf{i} + (2-t)\mathbf{j}$, and $\mathbf{r}'(t) = -\mathbf{j}$ for $0 \le t \le 2$,

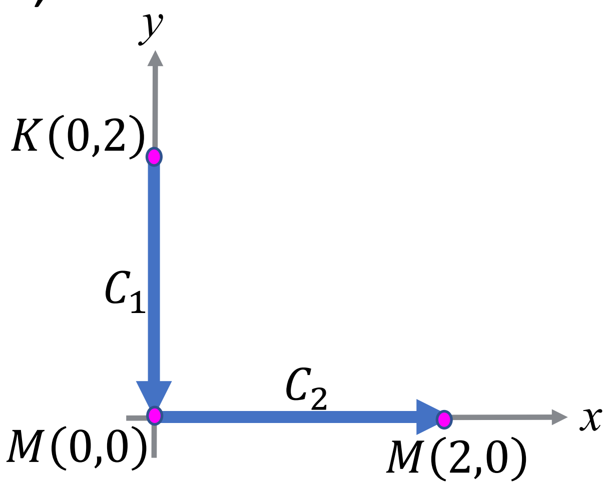
with $\boldsymbol{F}(x,y) = (y-x)\mathbf{i} + x\mathbf{j}$, can be written as

$$\mathbf{F}\big(x(t), y(t)\big) = (2-t)\mathbf{i}.$$

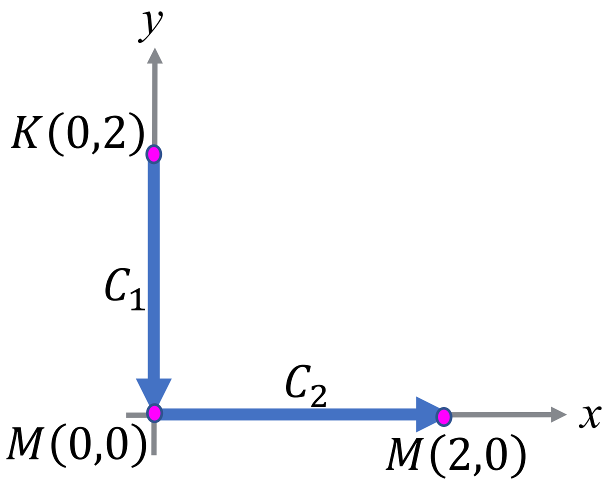
ii) From L to M, C_2 :$\mathbf{r}(t) = t\mathbf{i} + 0\mathbf{j}$, and $\mathbf{r}'(t) = \mathbf{i}$ for $0 \le t \le 2$,

with $\boldsymbol{F}(x,y) = (y-x)\mathbf{i} + x\mathbf{j}$, can be written as

$$\mathbf{F}\big(x(t), y(t)\big) = -t\mathbf{i} + t\mathbf{j}.$$



Solution Example 10.9 :



$$\int_C \mathbf{F}\cdot\mathbf{r}'(t)dt = \int_{C_1} \mathbf{F}\cdot\mathbf{r}'(t)dt + \int_{C_2} \mathbf{F}\cdot\mathbf{r}'(t)dt$$

$$= \int_0^2 \langle 2-t, 0\rangle\cdot\langle 0,-1\rangle dt + \int_0^2 \langle -t, t\rangle\cdot\langle 1,0\rangle dt$$

$$= \int_0^2 0\,dt + \int_0^2 -t\,dt = \left[-\frac{t^2}{2}\right]_0^2 = -2$$

Vector Line Integral

Application: A common application of line integrals of vector fields is computing the work done in moving an object in a force field such as a gravitational or electric field.

Work Done in a Force Field

Let $\mathbf{F}(x, y, z)$ be a continuous force field in the 3D region. The work done, denoted by W, in moving an object along with C in the positive direction for $a \leq t \leq b$, is given by

$$W = \int_C \mathbf{F}(x, y, z) \cdot d\mathbf{r} = \int_a^b \mathbf{F}(x(t), y(t), z(t)) \cdot \mathbf{r}'(t) dt$$

where

$$d\mathbf{r}(t) = dx(t)\mathbf{i} + dy(t)\mathbf{j} + dz(t)\mathbf{k}.$$

Line Integrals in Vector form

Example 10.10:

Find the work done by a force $\boldsymbol{F}(x,y,z)=2xz\mathbf{i}+yz\mathbf{j}+(y-2x)\mathbf{k}$ where C is the curve $x=t^2-1, y=2t, z=t$ for $0\le t\le 1$.

Solution:

Let $\boldsymbol{F}\cdot \mathrm{d}\mathbf{r}=2xzdx+yzdy+(y-2x)dz$ and the derivatives of x,y,z with respect to t are given by $dx=2tdt, dy=2dt, dz=dt$.

Thus, the required work done is $W=\int_C \mathbf{F}\cdot d\mathbf{r}$

$$=\int_C 2xzdx+yzdy+(y-2x)dz=\int_0^1\left[2(t^2-1)(t)(2t)+2t^2(2)+\left(2t-2(t^2-1)\right)\right]dt$$

$$=\int_0^1 4t^4-2t^2+2t+2\,dt=\left[4\frac{t^5}{5}-2\frac{t^3}{3}+t^2+2t\right]_0^1=\frac{47}{15}$$

Exercise 10.3:

1. Given the force field $\boldsymbol{F}(x, y) = 2\mathbf{i} + x\mathbf{j}$ and C is the portion of $y = x^3$ from $(0,0)$ to $(1,1)$. Find the work required to move an object on the given oriented curve.
2. Evaluate the line integral of $\boldsymbol{F}(x, y) = (y - x)\mathbf{i} + x\mathbf{j}$ on the path C from $P(0,1)$ to $Q(1,0)$ via two-line segments through origin.
3. Evaluate the line integral of $\boldsymbol{F}(x, y) = (y - x)\mathbf{i} + x\mathbf{j}$ on the path C from $P(0,1)$ to $Q(2,0)$ via two-line segments through origin.

[Ans: 1.$\frac{11}{4}$2. $-\frac{1}{2}$ 3. -2]

Reference

1)Weir M. D., Hass J., Giordano F. R. (2017), Thomas's Calculus, 14th Edition, Pearson-Addison Wesley

2) Fehribach, J. D. (2020). Multivariable and Vector Calculus. In Multivariable and Vector Calculus. De Gruyter.

3) Stroud, K. A., & Booth, D. J. (2020). Engineering mathematics. Bloomsbury Publishing.

THANK YOU