ENGINEERING MATHEMATICS 1
BMFG 1313
NUMERICAL DIFFERENTIATION

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Lesson Outcomes

Upon completion of this lesson, the student should be able to:

- Find the first and second derivatives of a function by using forward, backward and central differencing approximation.
- Find the first and second derivatives of a function by using high accuracy differentiation formula.

Numerical Differentiation
(Estimating the derivative of a function at a specific point)

Forward, Backward, Central Difference Approximation of First Derivatives	Forward, Backward, Central Difference Approximation of Second Derivatives

Taylor Series Expansion / Interpolation

First Derivatives:

Forward D.A. Backward D.A.	Accuracy $O(h)$:
Centered D.A. High Accuracy F.D.A. High Accuracy B.D.A.	Accuracy $O(h^2)$:
High Accuracy C.D.A.	Accuracy $O(h^4)$:

Second Derivatives:

Forward D.A. Backward D.A.	Accuracy $O(h)$:
Centered D.A. High Accuracy F.D.A. High Accuracy B.D.A.	Accuracy $O(h^2)$:
High Accuracy C.D.A.	Accuracy $O(h^4)$:

6.4 Introduction to Numerical Differentiation

Why we need numerical differentiation?

Given a complicated function

i.e. $f(x) = \left[\cos(-9x^5 + e^{-2x})e^{x^2+4}\right]$

Evaluate $f''(-3.2)$.

Method 1

Step 1: Find the derivative of f' followed by f''.

Step 2: Evaluate $f''(-3.2)$.

Which one is faster and easier?

Method 2

Method 2

Step 1: Construct some points from $f(x)$, e.g. $f(-5)$, $f(-3)$, $f(-1)$

Step 2: Evaluate $f''(-3.2)$ by numerical differentiation.

6.4 Introduction to Numerical Differentiation

Taylor Series Expansion

A one-dimensional Taylor series is an expansion of a real function $f(x)$ about a point $x = a$:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$$

$$+\frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

It is used to approximate the first and the second derivatives.

6.4 Introduction to Numerical Differentiation

Derivation of Formulas:

Taylor's Series:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1}-x_i) + \frac{f''(x_i)}{2!}(x_{i+1}-x_i)^2 + \frac{f^{(3)}(x_i)}{3!}(x_{i+1}-x_i)^3 + \cdots \qquad (1)$$

First Order FDA:

Let $x_i = x$ and $x_{i+1} = x + h$ in Eqn (1),

$$f(x+h) = f(x) + f'(x)(h) + \frac{f''(x)}{2!}(h)^2 + \frac{f^{(3)}(x)}{3!}(h)^3 + \cdots \qquad (2)$$

$\therefore f(x+h) \approx f(x) + f'(x)(h)$ and hence,

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

6.4 Introduction to Numerical Differentiation

First Order BDA:

Replace h with $-h$ in Eqn (2),

$$f(x-h) = f(x) - hf'(x) + h^2\frac{f''(x)}{2!} - h^3\frac{f^{(3)}(x)}{3!} + \cdots \qquad (3)$$

$\therefore f(x-h) \approx f(x) - hf'(x)$ and hence,

$$f'(x) = \frac{f(x)-f(x-h)}{h}$$

First Order CDA:

Cancel out the term of $\frac{f''(x)}{2!}(h)^2$ in Equations (2) & (3): Eqn (2) – Eqn (3):

$$f(x+h) - f(x-h) = \left[f(x) + f'(x)(h) + h^2\frac{f''(x)}{2!} + h^3\frac{f^{(3)}(x)}{3!} + \cdots\right]$$

$$-\left[f(x) - hf'(x) + h^2\frac{f''(x)}{2!} - h^3\frac{f^{(3)}(x)}{3!} + \cdots\right] \qquad (4)$$

$\therefore f(x+h) - f(x-h) \approx 2hf'(x)$ and hence,

$$f'(x) = \frac{f(x+h)-f(x-h)}{2h}$$

6.4 Introduction to Numerical Differentiation

First Order HAFDA:

$$f(x+h) = f(x) + f'(x)(h) + \frac{f''(x)}{2!}(h)^2 + \frac{f^{(3)}(x)}{3!}(h)^3 + \cdots \quad (2)$$

Replace h with $2h$ in Eqn (2),

$$f(x+2h) = f(x) + f'(x)(2h) + \frac{f''(x)}{2!}(2h)^2 + \frac{f^{(3)}(x)}{3!}(2h)^3 + \cdots$$

$$f(x+2h) = f(x) + 2hf'(x) + 4h^2\frac{f''(x)}{2!} + 8h^3\frac{f^{(3)}(x)}{3!} + \cdots \quad (5)$$

Cancel out the term of $\frac{f''(x)}{2!}(h)^2$ in Equations (2) & (5): 4Eqn (2) – Eqn (5):

$$4f(x+h) - f(x+2h) \approx 3f(x) + 2hf'(x) - 4h^3\frac{f^{(3)}(x)}{3!} + \cdots \quad (6)$$

$\therefore\ 4f(x+h) - f(x+2h) \approx 3f(x) + 2hf'(x)$ and hence,

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

6.4 Introduction to Numerical Differentiation

First Order HABDA:

Replace h with $-h$ in Eqn (6):

$$4f(x-h)-f(x-2h)\approx 3f(x)-2hf'(x)+4h^3\frac{f^{(3)}(x)}{3!}+\cdots \qquad (7)$$

$\therefore\ 4f(x-h)-f(x-2h)\approx 3f(x)-2hf'(x)$ and hence,

$$f'(x)=\frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$$

6.4 Introduction to Numerical Differentiation

First Order HACDA:

Replace h with $-h$ in Eqn (5):

$$f(x-2h) = f(x) - 2hf'(x) + 4h^2\frac{f''(x)}{2!} - 8h^3\frac{f^{(3)}(x)}{3!} + \cdots \tag{8}$$

Cancel out the terms of $\frac{f''(x)}{2!}$ and $\frac{f^{(3)}(x)}{3!}$ in Equations (2,3,5,8):

$$8f(x+h) - 8f(x-h) - f(x+2h) + f(x-2h) \approx 8\left[f(x) + hf'(x) + h^2\frac{f''(x)}{2!} + h^3\frac{f^{(3)}(x)}{3!} + \cdots\right]$$

$$-8\left[f(x) - hf'(x) + h^2\frac{f''(x)}{2!} - h^3\frac{f^{(3)}(x)}{3!} + \cdots\right] - \left[f(x) + 2hf'(x) + 4h^2\frac{f''(x)}{2!} + 8h^3\frac{f^{(3)}(x)}{3!} + \cdots\right]$$

$$+\left[f(x) - 2hf'(x) + 4h^2\frac{f''(x)}{2!} - 8h^3\frac{f^{(3)}(x)}{3!} + \cdots\right]$$

$$\therefore 8f(x+h) - 8f(x-h) - f(x+2h) + f(x-2h) \approx 12hf'(x) \tag{9}$$

and hence,

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

6.5 First Order Derivatives

List of formulas derived in Section 6.4 for the first order of derivatives:

Forward D.A. $[O(h)]$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

Backward D.A. $[O(h)]$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

Centered D.A. $[O(h^2)]$

6.5 First Order Derivatives

High Accuracy Forward D.A. $[O(h^2)]$

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

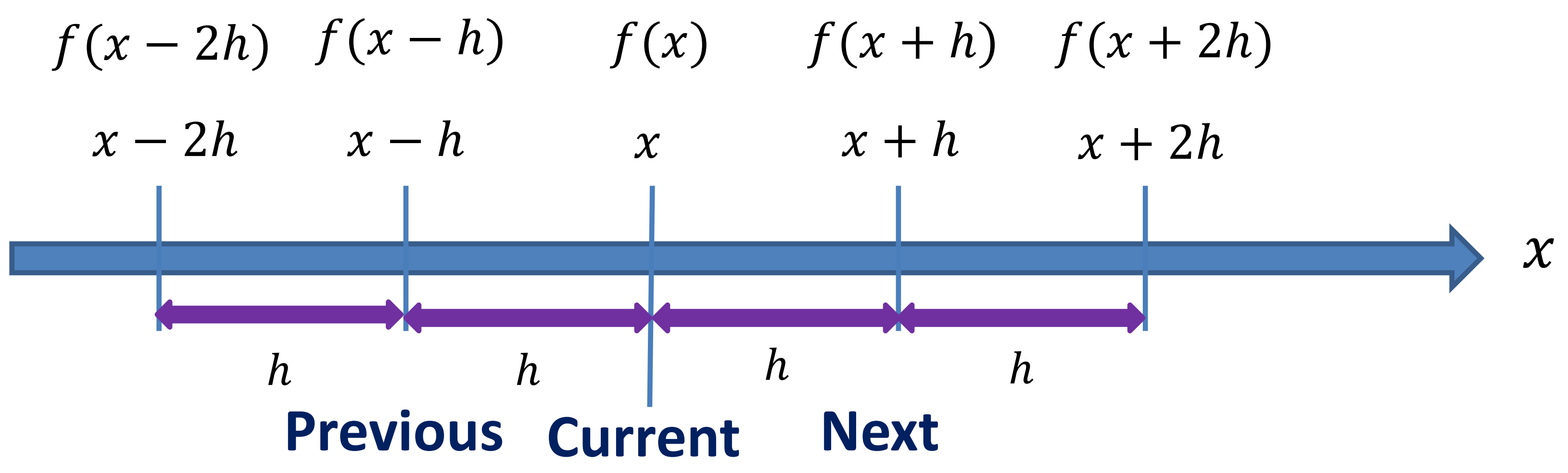
High Accuracy Backward D.A. $[O(h^2)]$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

High Accuracy Centered D.A. $[O(h^4)]$

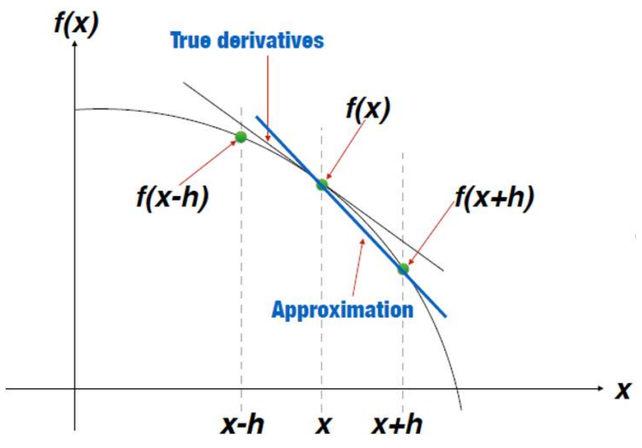
6.5 First Order Derivatives

Time Line:

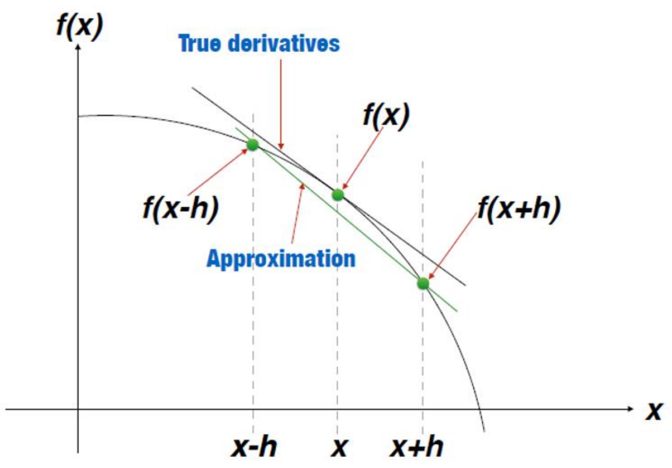


Step size: h

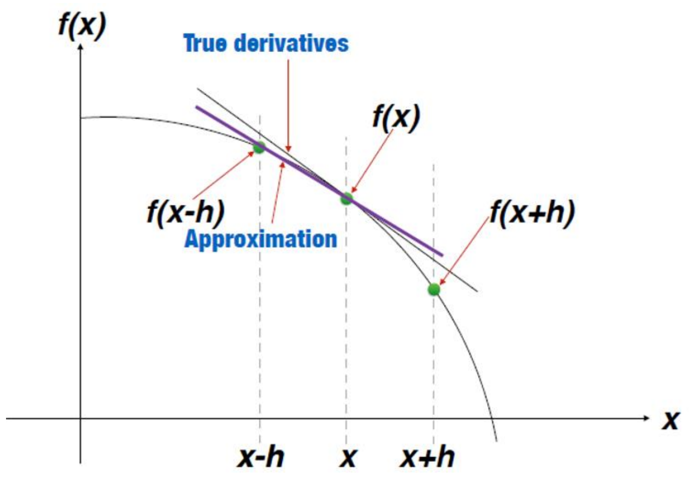
6.5 First Order Derivatives



Forward Difference Approximation with accuracy of $O(h)$



Centered Difference Approximation with accuracy of $O(h^2)$



Backward Difference Approximation with accuracy of $O(h)$

6.5 First Order Derivatives

Example 1:

Estimate the first derivative of a function

$$f(x) = 0.13x^5 - 0.75x^3 + 0.12x^2 - 0.5x + 1$$

at x = 0.5 with a step size of $h = 0.25$ and $h = 0.50$ using:

(a) forward and backward difference approximations of $O(h)$

(b) centered difference approximation of $O(h^2)$ to estimate the first derivative of

(c) Note that the derivative can be calculated directly as

$$f'(x) = 0.65x^4 - 2.25x^2 + 0.24x - 0.5$$

and the true value as $f'(0.5) = -0.901875$. Hence, calculate the percent relative error for the cases above. Carry six decimal places along the computation.

6.5 First Order Derivatives

Solution (a):

- For h = 0.25, the function $f(x) = 0.13x^5 - 0.75x^3 + 0.12x^2 - 0.5x + 1$ can be employed to obtain:

  x – h = 0.25 → $f(x - h)$ = 0.870908

  x = 0.5 → $f(x)$ = 0.690313

  x + h = 0.75 → $f(x + h)$ = 0.406943

- For h = 0.50, the values are:

  x – h = 0 → $f(x - h)$ = 1.000000

  x = 0.5 → $f(x)$ = 0.690313

  x + h = 1 → $f(x + h)$ = 0

- These values can be used to compute the forward, backward and centered difference approximation.

6.5 First Order Derivatives

Solution (a):

For *h =0.25,*

Forward difference approximation:

$$f'(0.5) \approx \frac{f(x+h)-f(x)}{h} = \frac{0.406943-0.690313}{0.25} = -1.133480$$

Backward difference approximation:

$$f'(0.5) \approx \frac{f(x)-f(x-h)}{h} = \frac{0.690313-0.870908}{0.25} = -0.722380$$

For *h =0.5,*

Forward difference approximation:

$$f'(0.5) \approx \frac{f(x+h)-f(x)}{h} = \frac{0-0.690313}{0.5} = -1.380626$$

Backward difference approximation:

$$f'(0.5) \approx \frac{f(x)-f(x-h)}{h} = \frac{0.690313-1.000000}{0.5} = -0.619374$$

6.5 First Order Derivatives

Solution (b):

For centered divided difference,

When $h = 0.25$:

$$f'(0.5) \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{0.406943 - 0.870908}{2(0.25)} = -0.927930$$

When $h = 0.5$:

$$f'(0.5) \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{0 - 1.00000}{2(0.5)} = -1.00000$$

6.5 First Order Derivatives

Solution (c):(percent relative error)

h	Method	Approximate value	Percent relative error
0.25	Forward	-1.133480	$\frac{\lvert -0.901875-(-1.133480)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{25.680388\%}$
	Backward	-0.722380	$\frac{\lvert -0.901875-(-0.722380)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{19.902426\%}$
	Centered	-0.927930	$\frac{\lvert -0.901875-(-0.927930)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{2.888981\%}$
0.5	Forward	-1.380626	$\frac{\lvert -0.901875-(-1.380626)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{53.083964\%}$
	Backward	-0.619374	$\frac{\lvert -0.901875-(-0.619374)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{31.323742\%}$
	Centered	-1.00000	$\frac{\lvert -0.901875-(-1.000000)\rvert}{\lvert -0.901875\rvert}\times 100\% = \mathbf{10.880111\%}$

Lower value of *h* gives better approximation. From the result above, centered diff. approx. gives the best estimation.

6.5 First Order Derivatives

Example 2:

Use high accuracy formula to estimate the first derivative of:

$$f(x) = 0.13x^5 - 0.75x^3 + 0.12x^2 - 0.5x + 1$$

at $x = 0.5$ and step size of $h = 0.25$.

6.5 First Order Derivatives

Solution:

$x - 2h = 0 \quad \rightarrow \quad f(x-2h) = 1$

$x - h = 0.25 \quad \rightarrow \quad f(x-h) = 0.8709$

$x = 0.5 \quad \rightarrow \quad f(x) = 0.6903$

$x + h = 0.75 \quad \rightarrow \quad f(x+h) = 0.4069$

$x + 2h = 1 \quad \rightarrow \quad f(x+2h) = 0$

Forward difference approximation, $O(h^2)$

$$f'(0.5) \approx \frac{1}{2h}[4f(x+h) - f(x+2h) - 3f(x)]$$

$$= \frac{1}{2(0.25)}[4(0.4069) - 0 - 3(0.6903)] = -0.8866$$

6.5 First Order Derivatives

Solution:

Backward difference approximation, $O(h^2)$

$$f'(0.5) \approx \frac{1}{2h}[3f(x) - 4f(x-h) + f(x-2h)]$$

$$= \frac{1}{2(0.25)}[3(0.6903) - 4(0.8709) + 1)] = -0.8254$$

Centred difference approximation, $O(h^4)$

$$f'(0.5) \approx \frac{1}{12h}[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)]$$

$$= \frac{1}{12(0.25)}[-0 + 8(0.4069) - 8(0.8709) + 1] = -0.904$$

6.5 First Order Derivatives

Exercise 6.4

1) Estimate the first derivative of a function

$$f(t) = 3\sin 2t$$

at t = 0.5 with a step size of $h = 0.25$ using forward and backward difference approximation of $O(h)$ and centred difference approximation of $O(h^2)$.

[Ans: Forward $O(h)$=1.8724, Backward $O(h)$=4.3444, Centered $O(h^2)$=3.1084]

2) Use high accuracy formula of $O(h^2)$ and $O(h^4)$ to estimate the first derivative of

$$f(t) = e^{-\sin t}$$

at $t = 1$ using step size of 0.25.

[Ans: Forward O(h2) =-0.2274; Backward O(h2) =-0.2216; Centered O(h4) =-0.2331]

6.5 First Order Derivatives

Exercise 6.4

3) Given the following data

t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
X	0.231	0.451	0.511	0.623	0.687	0.731	0.882	0.903	0.922	0.987

Find the first derivative using high accuracy formula of $O(h^2)$ and $O(h^4)$ at t = 0.6, with h = 0.2.

[Ans: Forward $O(h^2)$ =1.08; Backward $O(h^2)$ =0.38; Centered $O(h^4)$ =0.71]

6.5 First Order Derivatives

Exercise 6.4:

4) A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time, t is in seconds and the distance x is in feet.

Time, t	0	3	6	9	12
Distance, x	0	225	412	685	946

Estimate the **velocity** of the car when $t = 6$ seconds using high accuracy centered difference approximation. Take $h = 3$.

[Ans: 75.9444ft/s]

6.6 Second Order Derivatives

Derivation of Formulas:

Second Order FDA:

Cancel out the term of $hf'(x)$ in Equations (2) & (5): Eqn (5) – 2Eqn (2):

$$f(x+2h) - 2f(x+h) = \left[f(x) + 2hf'(x) + 4h^2\frac{f''(x)}{2!} + 8h^3\frac{f^{(3)}(x)}{3!} + \cdots\right]$$

$$-2\left[f(x) + f'(x)(h) + \frac{f''(x)}{2!}(h)^2 + \frac{f^{(3)}(x)}{3!}(h)^3 + \cdots\right]$$

$$\therefore f(x+2h) - 2f(x+h) \approx -f(x) + h^2 f''(x) \qquad (10)$$

and hence,

$$f''(x) = \frac{f(x) - 2f(x+h) + f(x+2h)}{h^2}$$

Second Order BDA:

Replace h with $-h$ in Eqn (10),

$$f(x-2h) - 2f(x-h) \approx -f(x) + h^2 f''(x)$$

and hence,

$$f''(x) = \frac{f(x-2h) - 2f(x-h) + f(x)}{h^2}$$

6.6 Second Order Derivatives

Second Order CDA:

Cancel out the term of $hf'(x)$ in Equations (1) & (2): Eqn (1) + Eqn (2):

$$f(x+h)+f(x-h)=\left[f(x)+f'(x)(h)+h^2\frac{f''(x)}{2!}+h^3\frac{f^{(3)}(x)}{3!}+\cdots\right]$$

$$+\left[f(x)-hf'(x)+h^2\frac{f''(x)}{2!}-h^3\frac{f^{(3)}(x)}{3!}+\cdots\right]$$

$$\therefore f(x+h)+f(x-h)\approx 2f(x)+h^2f''(x) \qquad (11)$$

and hence,

$$f''(x)=\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$$

Second Order HAFDA:

Cancel out the term of $hf'(x)$ in $f(x+h), f(x+2h)$ and $f(x+3h)$

Second Order HABDA:

Replace h with $-h$ in the equation yield from **Second Order HAFDA.**

Second Order HACDA:

Cancel out the term of $hf'(x)$ in $f(x+h), f(x-h), f(x+2h)$ and $f(x-2h)$

6.6 Second Order Derivatives

List of formulas derived in previous slides for the second order of derivatives:

Forward D.A. $[O(h)]$

$$f''(x) = \frac{f(x) - 2f(x+h) + f(x+2h)}{h^2}$$

Backward D.A. $[O(h)]$

$$f''(x) = \frac{f(x-2h) - 2f(x-h) + f(x)}{h^2}$$

Centered D.A. $[O(h^2)]$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

6.6 Second Order Derivatives

High Accuracy Forward D.A. $[O(h^2)]$

$$f''(x) = \frac{2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)}{h^2}$$

$$f''(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

High Accuracy Backward D.A. $[O(h^2)]$

$$f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$$

High Accuracy Centered Difference $[O(h^4)]$

6.6 Second Order Derivatives

Example 1:

Use centered difference approximation to estimate the second derivative of:

$$f(x) = 0.13x^5 - 0.75x^3 + 0.12x^2 - 0.5x + 1$$

at $x = 0.5$ and step size of $h = 0.25$. Next, find the true value and the percent relative error.

6.6 Second Order Derivatives

Solution:

$x - h = 0.25 \quad \dashrightarrow \quad f(x-h) = 0.8709$

$x = 0.5 \quad \dashrightarrow \quad f(x) = 0.6903$

$x + h = 0.75 \quad \dashrightarrow \quad f(x+h) = 0.4069$

$$f''(x) \approx \frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)]$$

$$f''(0.5) = \frac{1}{0.25^2}[0.4069 - 2(0.6903) + 0.8709]$$

$$= -1.6448$$

$$f'(x) = 0.65x^4 - 2.25x^2 + 0.24x - 0.5$$

$$f''(x) = 2.6x^3 - 4.5x + 0.24$$

$$f'(0.5) = -1.685$$

Percentage of relative error $= \frac{|-1.685-(-1.6448)|}{|-1.685|} \times 100\% = 2.39\%$

6.6 Second Order Derivatives

Example 2:

Given the following data

t	1	2	3	4	5	6	7
$f(t)$	-1.8	-3.8	4.2	34.2	103	232.2	448.2

Calculate $f''(4)$ with step size of 1 by using forward and backward difference approximation of $O(h)$ and centered difference approximation of $O(h^2)$.

6.6 Second Order Derivatives

Solution:

$t - 2h = 2 \quad \longrightarrow \quad f(t-2h) = -3.8$

$t - h = 3 \quad \longrightarrow \quad f(t-h) = 4.2$

$t = 4 \quad \longrightarrow \quad f(t) = 34.2$

$t + h = 5 \quad \longrightarrow \quad f(t+h) = 103$

$t + 2h = 6 \quad \longrightarrow \quad f(t+2h) = 232.2$

- Forward difference approximation, $O(h)$

$$f'' \approx \frac{1}{h^2}[f(t) - 2f(t+h) + f(t+2h)]$$

$$f''(4) = \frac{1}{1^2}[34.2 - 2(103) + 232.2] = 60.4$$

6.6 Second Order Derivatives

Solution (cont.):

- Backward difference approximation, $O(h)$

$$f''(t) \approx \frac{1}{h^2}[f(t-2h) - 2f(t-h) + f(t)]$$

$$f''(4) = \frac{1}{1^2}[-3.8 - 2(4.2) + 34.2] = 22$$

- Centered difference approximation, $O(h^2)$

$$f''(t) \approx \frac{1}{h^2}[f(t+h) - 2f(t) + f(t-h)]$$

$$f''(4) \approx \frac{1}{1^2}[103 - 2(34.2) + 4.2] = 38.8$$

6.6 Second Order Derivatives

Example 3:

Use high accuracy difference approximation to estimate the second derivative of:

$$f(t) = 4e^{\sin 2t} - 1$$

at $t = 0.5$ and step size of $h = 0.1$.

Solution:

Compute the needed values of $f(t)$:

t	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(t)$	4.9045	6.0353	7.1960	8.2791	9.1587	9.7161	9.8685

6.6 Second Order Derivatives

Solution (cont.):

- Forward difference, $O(h^2)$

$$f''(t) \approx \frac{1}{h^2}[-f(t+3h)+4f(t+2h)-5f(t+h)+2f(t)]$$
$$f''(0.5) = -23.9665$$

- Backward difference, $O(h^2)$

$$f''(t) \approx \frac{1}{h^2}[2f(t)-5f(t-h)+4f(t-2h)-f(t-3h)]$$
$$f''(0.5) = -18.5345$$

- Centered difference, $O(h^4)$

$$f''(t) \approx \frac{1}{12h^2}[-f(t+2h)+16f(t+h)-30f(t)+16f(t-h)-f(t-2h)]$$
$$f''(0.5) = -20.4027$$

6.6 Second Order Derivatives

Exercise 6.5:

1) Given a function

$$f(t) = 2e^{\cos 3t}.$$

Use forward and backward difference approximation of $O(h)$ and centered difference approximation of $O(h^2)$ to approximate $f''(0.7)$ with step size of 0.1. Suppose the true value is 13.5807. Calculate the percent relative error for each method.

[Ans: Forward=10.36, ε= 23.72%; Backward=16.68, ε=22.82%; Centered=13.58, ε=0.0052%]

6.6 Second Order Derivatives

Exercise 6.5:

2) Given that

x	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(x)$	24.5325	29.9641	36.5982	44.7012	54.5982	66.6863	81.4509	99.4843

Use the high accuracy forward and backward difference approximation of $O(h^2)$ and centered difference approximations of $O(h^4)$ of the second derivatives formula to estimate $f''(2.0)$ using h = 0.1.

[Ans: Forward=208.42; Backward=211.91; Centered=218.3742]

6.6 Second Order Derivatives

Exercise 6.5:

3) A spring with spring constant k N/m is attached to a m kg mass with friction constant b Ns/m is forced to the right by an external force and the motion of the spring is governed by

$$F(t) = m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky$$

Given that the spring constant $k = 4$ N/m, the mass $m = 1$ kg and friction constant $b = 4$ Ns/m, estimate the external force, $F(t)$ for each of the time recorded as in Table by using appropriate 3-points difference formula.

t	0	0.5	1.0	1.5
y	2	2.5	3.6	5.2

6.6 Second Order Derivatives

Ans of Question 3: $*F(t)$ may have different possible values and it depends on the chosen method/combination in obtaining $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$. Give only 1 answer for each of the t value.

t	0	0.5	1.0	1.5
y	2	2.5	3.6	5.2
First order numerical diff.	FDA	FDA or CDA	CDA or BDA	BDA
$\frac{dy}{dt}$	0.4	1.7 or 1.6	2.7 or 2.8	3.7
Second order numerical diff.	FDA	FDA or CDA	CDA or BDA	BDA
$\frac{d^2y}{dt^2}$	2.4	2 or 2.4	2 or 2.4	2
$F(t)$	12	18.8 or 18.4 or 19.2	27.2 or 27.6 or 28	37.6