INTRODUCTION TO MECHANICAL ENGINEERING

BMCG 2423

STATICS : EQUILIBRIUM OF PARTICLE

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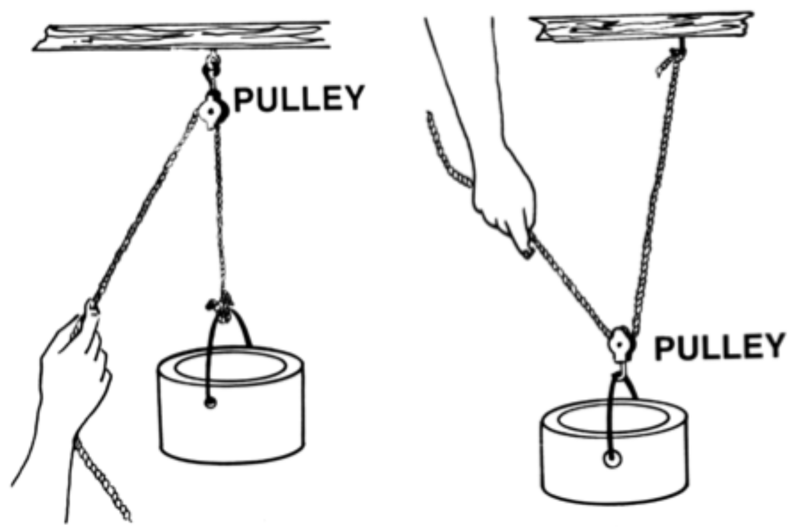
Lesson Outcome

At the end of lesson, students will be able to:

- sketch free body diagram (FDB).
- use equations of equilibrium in solving a 2-D problem.

Fundamental Quiz

- What is the **sum of forces** acting on a particle which is in **equilibrium** condition?
- What are the tension forces in the cables, if the pulleys are frictionless?



Applications

- What are the forces in cables of crane in carrying up the locomotive?

Applications

- For a given cable strength, what is the maximum weight of chandelier that can be hold?

Conditions for Equilibrium of Particle (2-D)

- Particles are at **equilibrium** if they are **at rest** or **moving with a constant velocity**.
- At these conditions, we can apply:

1) Newton's 1st law of motion, **$\Sigma F = 0$**

where **ΣF** is the vector sum of all the forces acting on the particle

2) Newton's 2nd law of motion, **$\Sigma F = ma$**

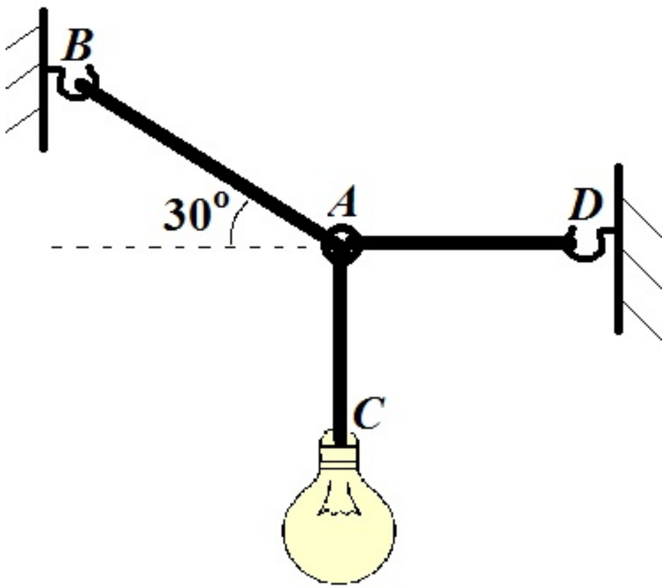
where the force fulfill Newton's 1st law of motion,

$$ma = 0$$

$$a = 0$$

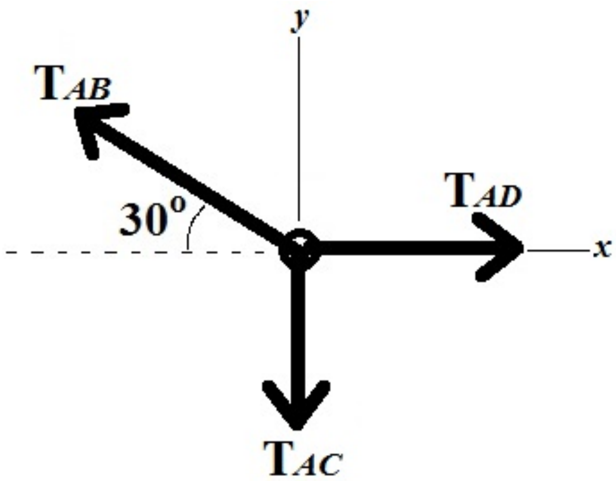
Equilibrium of Particle (2-D)

- The image shows an example of a 2-D or **coplanar** force system.
- If the whole lamp assembly is in equilibrium, then particle *A* (the ring) is also in equilibrium.
- The tensions in cables *AB*, *AC* and *AD* can be determined for a given weight of lamp at *C*.
- You need to learn how to draw a free body diagram (FBD) and apply equations of equilibrium.



Free Body Diagram (FBD)

- FBD is the most important thing before you start an analysis.
- You must understand the way to **draw** and **use** it.
- It shows a sketch that includes all **external forces** that are acting on a particle.
- The FBD assists us in writing the **equations of equilibrium** that are used to solve all the unknowns (it can be forces or angles).



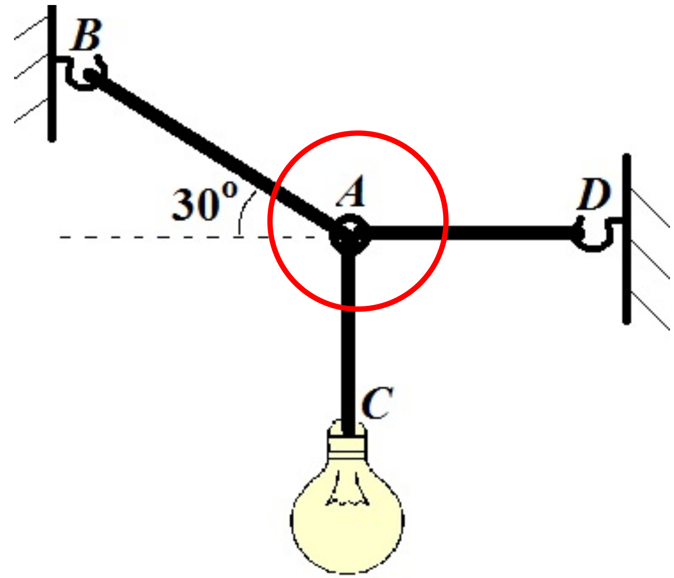
How to draw the FBD?

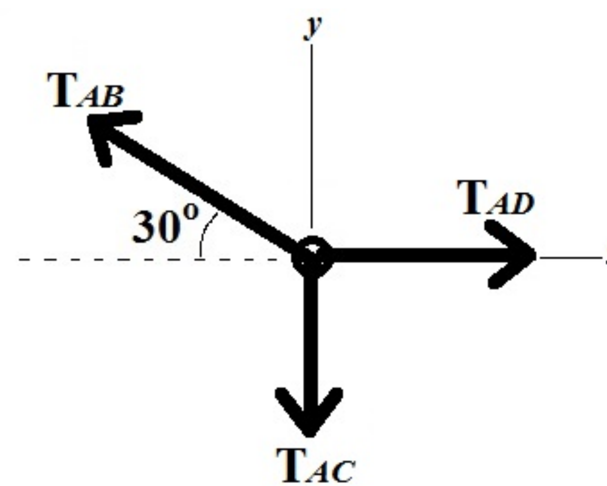
- **Isolate** / take out the particle from its surroundings.
- **Indicate all forces** that act on the particle.

Active force tends to move the particle.

Reactive force tends to resist the motion.

- **Identify** all **forces** and show all known **magnitudes** and **directions**.
- **Label** all unknown magnitudes and / or directions as **variables**.





Equations of Equilibrium (for 2-D)

- Since particle *A* is in **equilibrium**, the **total force** at *A* is **zero**. Therefore, tension force (T)

  $T_{AB} + T_{AC} + T_{AD} = 0$ or $\Sigma F = 0$

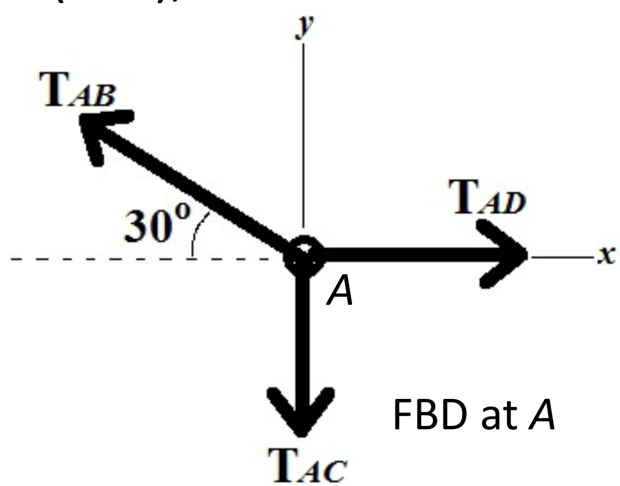
- In general, for a particle in equilibrium,

  $\Sigma F = 0$ or

  $\Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0 = 0 \boldsymbol{i} + 0 \boldsymbol{j}$ (vector equation)

  $\mathbf{\Sigma F_x = 0}$ and $\mathbf{\Sigma F_y = 0}$ (scalar form)

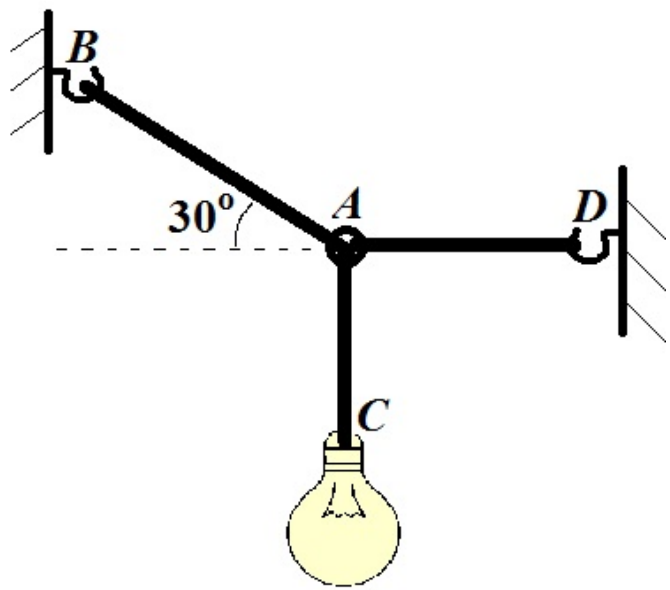
- The **two scalar equations of equilibrium** (EoE), as shown above can be used to solve for **two unknowns**.



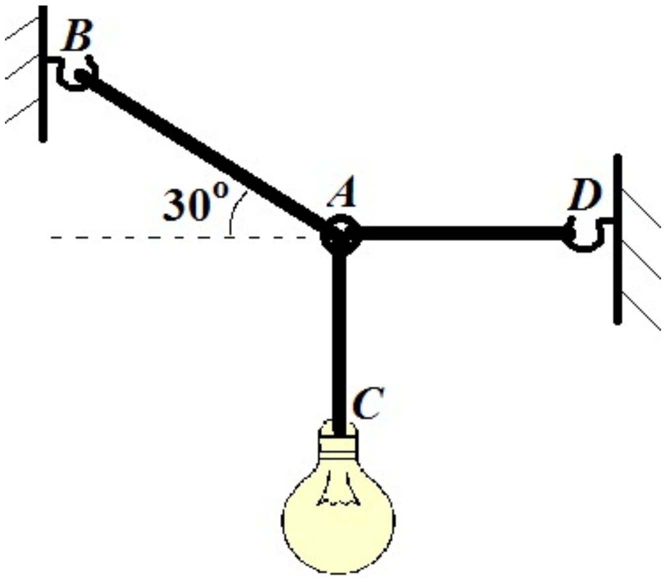
FBD at *A*

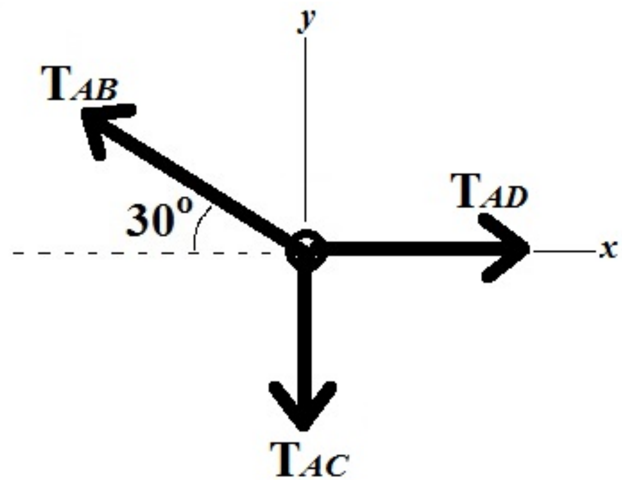
Example

Calculate the tensions in the cables if the weight of the lamp is 2 kg.



Example





Lamp mass = 2 kg

Lamp weight = 2 kg x 9.81 m/s^2 = 19.62 N

Use scalar EoE:

$+\rightarrow \Sigma F_x = T_{AD} - T_{AB} \cos 30° = 0$ ----------------(1)

$+\uparrow \Sigma F_y = T_{AB} \sin 30° - 19.62\text{ N} = 0$ ----------(2)

From (2); $\mathbf{T_{AB} = 39.24\ N}$

Substitute T_{AB} = 39.24 N in (1); $\mathbf{T_{AD} = 33.98\ N}$

Springs

- Spring force = spring constant * deformation

$$F = k * s$$

where deformation (s) can be either elongation or shortening.

- Example:

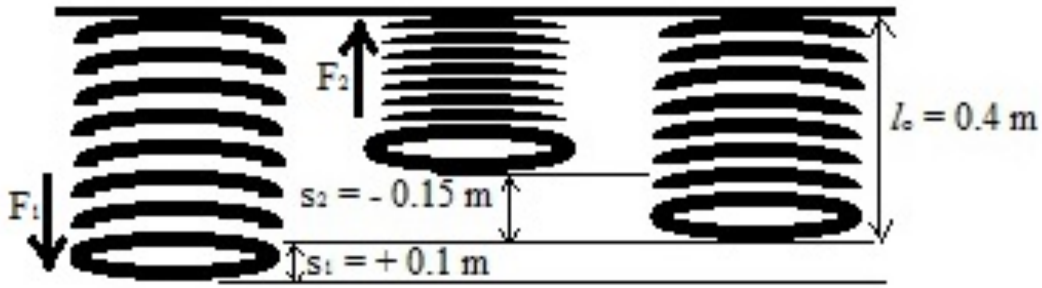
Given spring with **original length**, l_o = 0.4 m and **spring constant**, k = 200 N/m.

When the **elongation** $s_1 = +0.1$ m,

Force needed to **stretch** the spring, F_1 = (200 N/m)(0.1 m) = 20 N

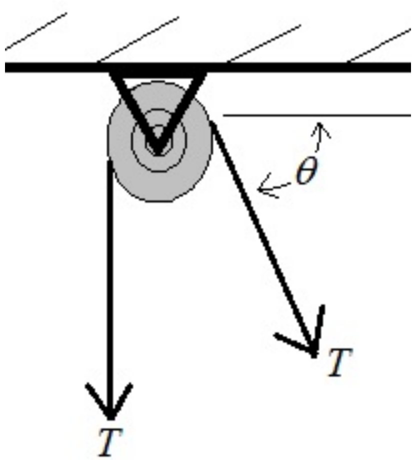
When the s**hortening** $s_2 = -0.15$ m,

Force needed to **compress** the spring, F_2 = (200 N/m)(0.15 m) = 30 N



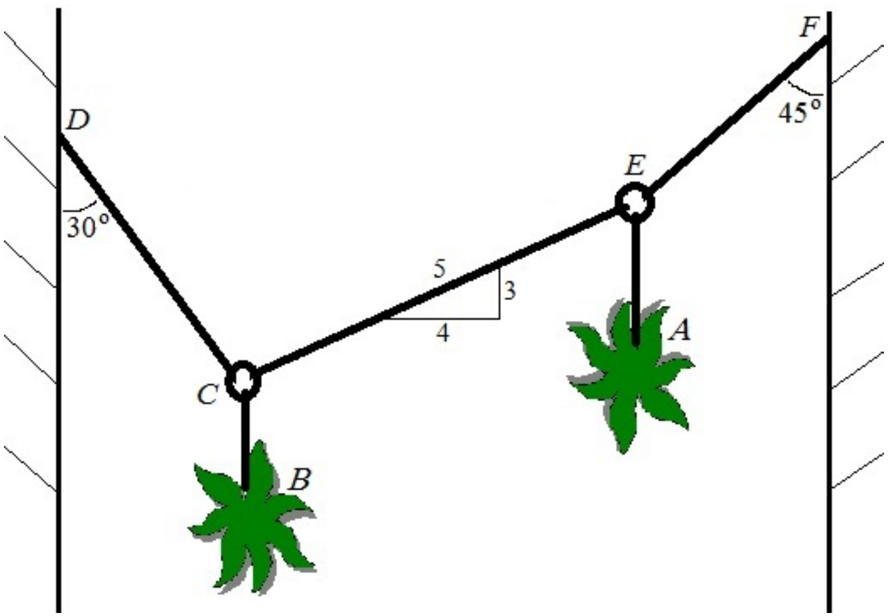
Cables and Pulleys

- Cables (or cords) are assumed to have negligible weight and they cannot stretch.
- A cable only supports tension (or pulling force) and acts in the direction of the cable.
- For any given angle θ, the cable is subjected to a constant tension *T* throughout its length.



Cable is in tension

Example



Given decoration *A* weighs 20 N as shown in figure.

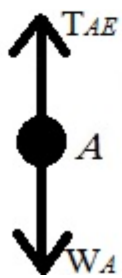
Find forces in the cables and weight of decoration *B*.

<u>Steps</u>

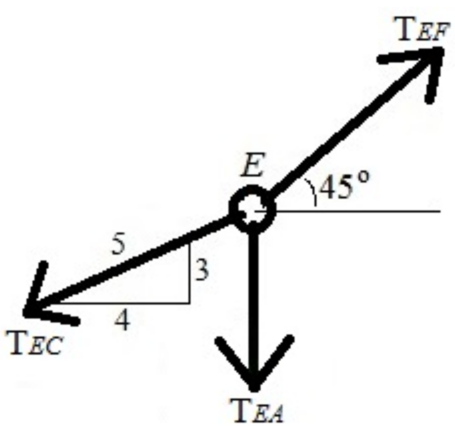
1. Sketch FBD for decoration at *A* and ring at *E*. Assume both items as particles.
2. Apply EoE at point *E* to solve for the unknowns.
3. Do this process again at C.

Example (continued)

FBD at *A*



FBD at *E*



Assume that all cables are in tension.

Given W_A = 20 N

<u>FBD *A*</u>

$+\uparrow \Sigma F_y = 0$; $T_{AE} - W_A = 0$

$T_{AE} = 20\text{ N} = T_{EA}$

<u>FBD *E*</u>

$+\rightarrow \Sigma F_x = 0$; $T_{EF} \cos 45° - T_{EC}\ (4/5) = 0$ -----------(1)

$+\uparrow \Sigma F_y = 0$; $T_{EF} \sin 45° - T_{EC}\ (3/5) - 20\text{ N} = 0$ --(2)

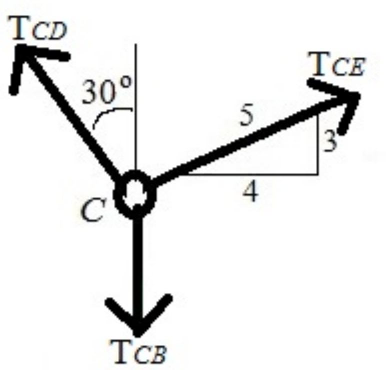
From (1); $T_{EF} = T_{EC}\ (4/5)/\cos 45°$ -----------------(3)

Substitute (3) in (2); **T_{EC} = 100 N** = T_{CE}

Substitute T_{EC} = 100 N in (3); **T_{EF} = 113.14 N**

Example (continued)

FBD at *C*



<u>FBD *C*</u>

$+\rightarrow \Sigma F_x = 0$; $T_{CE}(4/5) - T_{CD}\sin 30° = 0$ ------------(4)

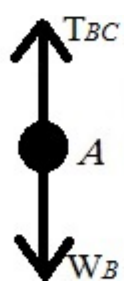
$+\uparrow \Sigma F_y = 0$; $T_{CE}(3/5) + T_{CD}\cos 30° - T_{CB} = 0$ ----(5)

From (4); $\mathbf{T_{CD} = 160\ N}$

Substitute $T_{CD} = 160$ N in (5);

$\mathbf{T_{CB} = 198.564\ N} = T_{BC}$

FBD at *B*

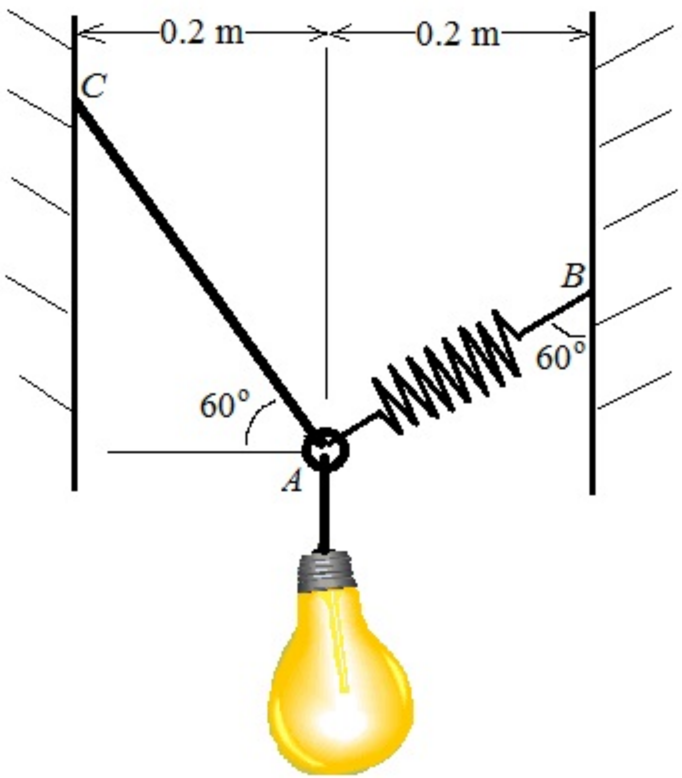


<u>FBD *B*</u>

$+\uparrow \Sigma F_y = 0$; $T_{BC} - W_B = 0$

$\mathbf{W_B = 198.56\ N}$

Example



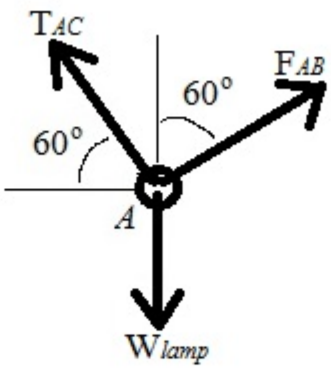
Given a lamp as shown in figure with mass 0.4 kg which is suspended by cable *AC* and spring *AB*.

Known that the un-stretched length of spring *AB* is 0.2 m.

Determine the spring constant k_{AB}.

Example (continued)

FBD at *A*



Given mass of lamp = 0.4 kg

Thus, weight of lamp, W_{lamp} = 0.4 kg x 9.81 m/s^2 = 3.924 N

<u>FBD *A*</u>

$+\rightarrow \Sigma F_x = 0; F_{AB} \sin 60° - T_{AC} \cos 60° = 0$ -----------------(1)

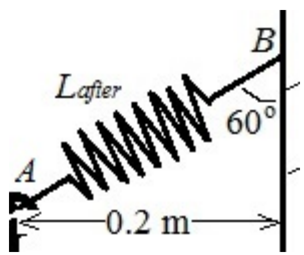
$+\uparrow \Sigma F_y = 0; F_{AB} \cos 60° + T_{AC} \sin 60° - 3.924 = 0$ ------(2)

Solve (1) and (2);

T_{AC} = 3.398 N

F_{AB} = 1.962 N

Spring *AB* geometry



Given unstretched spring length, $L_{initial}$ = 0.2 m

From spring *AB* geometry, L_{after} = 0.2 / (sin 60) = 0.231 m

Spring force, F = k * s

$F_{AB} = k_{AB} * (L_{after} - L_{initial})$

Thus, spring constant, **k_{AB} = 63.29 N/m**

End of Lesson

Recall:

- What are the conditions for equilibrium of particle?
- Can you mention two Newton's Laws of Motion?
- What is FBD?
- How to draw the FBD?
- What are two scalar equations of equilibrium?
- How many unknowns can be determined from the two scalar EoE?

References

- Hibbeler, R.C. and Yap, K.B., 2013, **Mechanics for Engineers – Statics**, Thirteenth SI Edition, Pearson, Singapore.