ENGINEERING DYNAMICS

KINEMATICS OF PARTICLES

CONTINUOUS MOTION

RECTILINEAR KINEMATICS: CONTINUOUS MOTION

The Objectives:
At the end of the course students will be able to find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path.

APPLICATIONS

In car racing, the team need to observe the motion of the racing car travels along a track. In order to simplify the analysis, can we assume the car together with the driver as a particle? What is quantities that can be use to analyze the motion of the at some instant?

An Overview of Mechanics

- **Mechanics**
  The study of how bodies react to forces acting on them.
  - **Statics**
    - The study of bodies in equilibrium.
  - **Dynamics**
    - **Kinematics**
      concerned with the geometric aspects of motion
    - **Kinetics**
      concerned with the forces causing the motion

RECTILINEAR KINEMATICS: CONTINUOUS MOTION

A particle *P* travels along a straight-line path from origin *O* . The path can be defined by the coordinate axis *s*.

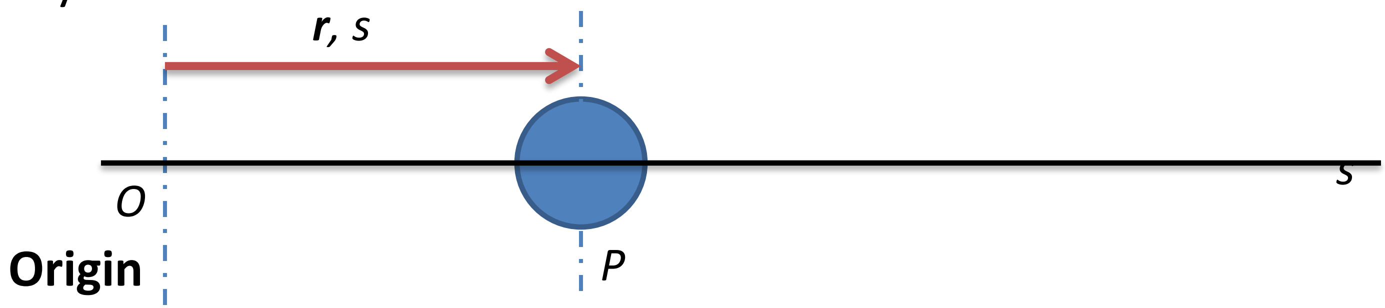

Figure 1 : Position

The Position

At any instant, the position is always measured from the origin.

The position can be represented as vector ***r*** , or the scalar *s* with the units is meter (m) or feet (ft).

The vector quantities ***r*** must have the magnitude and the direction.

Scalar quantities *s* can be represented either positive or negative value.

RECTILINEAR KINEMATICS: CONTINUOUS MOTION

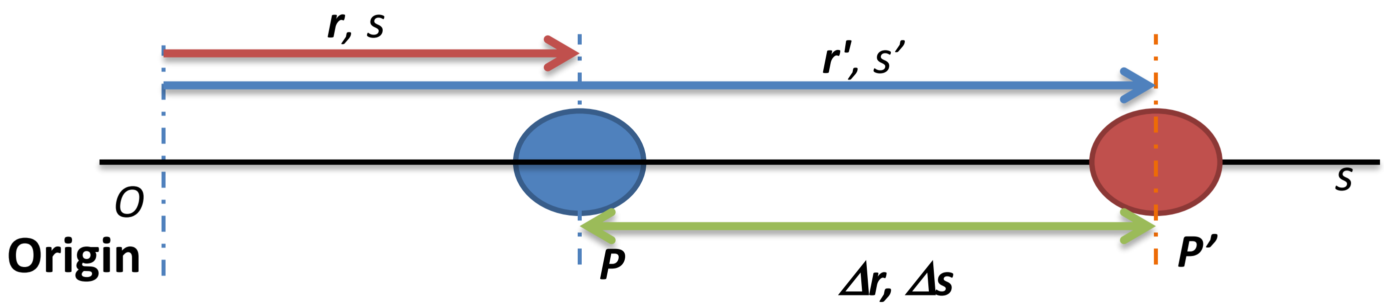

Figure 2 : Displacement

The particle moving from position *P* with the distant ***r*** to position *P*` with the distant ***r***` are measured from origin.

The displacement Δr, Δs

- The **displacement** of the particle when it moving from *P* to *P*` can be defined as its change in position.
- vector form, $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$; scalar form, $\Delta s = s' - s$.

The total distance traveled s_T

- The total distance traveled by the particle, sT, is a positive scalar that represents the total length of the path over which the particle travels.

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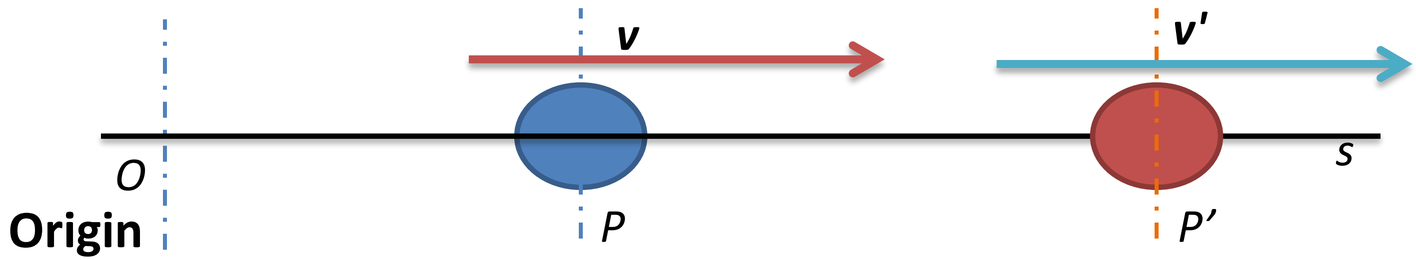

Figure 3 : Velocity

The particle moving from position *P* with the velocity ***v*** to position *P*` with the velocity ***v***`.

The definition	• Measure of the rate of change in the position of a particle. a vector quantity.
The unit	• m/s or ft/s.
The magnitude	• *speed*

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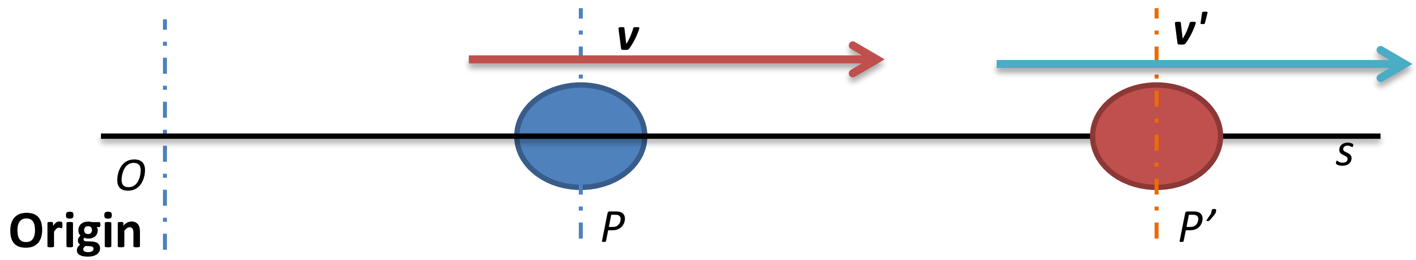

Figure 3 : Velocity

The average velocity

- $\boldsymbol{v}_{avg} = \Delta \boldsymbol{r} / \Delta t$

The instantaneous velocity

- $\boldsymbol{v} = d\boldsymbol{r} / dt$

Speed

- $v = ds / dt$

Average speed

- $(v_{sp})_{avg} = s_T / \Delta t$

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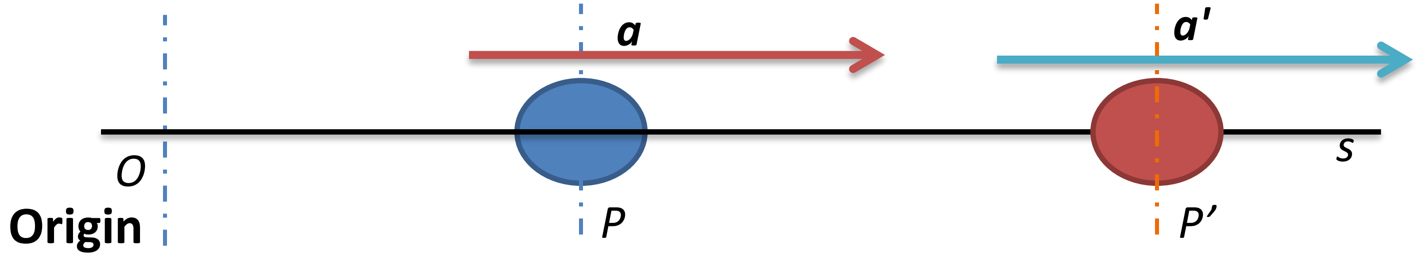

Figure 4 : Acceleration

Acceleration

the rate of change in the velocity of a particle.

vector quantity

m/s^2

ft/s^2

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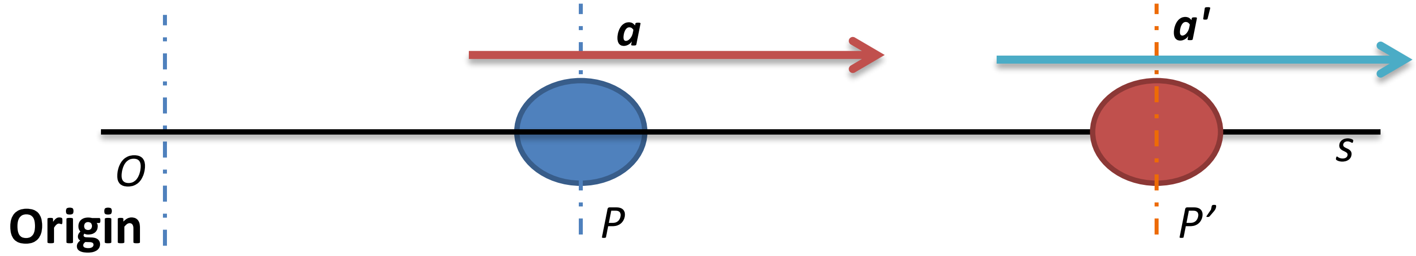

Figure 4 : Acceleration

The **instantaneous acceleration** is the time derivative of velocity.

Vector form:

$\mathbf{a} = d\mathbf{v} / dt$

Scalar form:

$a = dv / dt$
$= d^2s / dt^2$

Positive acceleration = speed increasing.

Negative acceleration = Speed decreasing.

Eliminate dt

$a\,ds = v\,dv$

SUMMARY OF KINEMATIC RELATIONS

Note that s_o and v_o represent
the initial position and velocity of the particle at t = 0.

Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad or \quad a = v\,dv/ds$$

Integrate acceleration for velocity and position.

Velocity

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\; dt \text{ or } \int_{v_o}^{v} v\,dv = \int_{s_o}^{s} a\; ds$$

Position

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\,dt$$

CONSTANT ACCELERATION

Special case

- Acceleration is constant ($a = a_c$)

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c \, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v \, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2)\, a_c t^2$$

$$\int_{v_o}^{v} v \, dv = \int_{s_o}^{s} a_c \, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

A common example : Gravity

- *A body freely falling toward earth.*
- $a_c = g = 9.81\ m/s^2$

EXAMPLE 1

Problem

- A particle travels along a straight line to the right with a velocity of $v = (4\ t - 3\ t^2)$ m/s where t is in seconds. Also, $s = 0$ when $t = 0$. Determine The position and acceleration of the particle when $t = 4$ s.

Strategy

- Establish the positive coordinate, s, in the direction the particle is traveling.
- Calculate the acceleration by take a derivative of the velocity function which is a function of time.
- To calculate the position, integrate the velocity function.

EXAMPLE 1

Solution (1^{st} Step)

- derivative of the velocity to determine the acceleration.

$$a = dv / dt \quad = d(4\,t - 3\,t^2) / dt = 4 - 6\,t$$

$\Rightarrow$ a = – 20 m/s^2 (or in the $\leftarrow$ direction) when t = 4 s

Solution (2^{nd} Step)

- Calculate the distance traveled in 4s.

$$v = ds / dt \quad \Rightarrow \quad ds = v\,dt \quad \Rightarrow$$

$$\Rightarrow \quad s - s_o = 2\,t^2 - t^3$$

$$\Rightarrow \quad s - 0 = 2(4)^2 - (4)^3$$

$$\Rightarrow \quad s = -32 \text{ m } (\text{ or } \leftarrow)$$