INTRODUCTION TO MECHANICAL ENGINEERING BMCG 2423

STATICS : FORCE VECTOR

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Lesson Outcome

At the end of lecture, students will be able to:

- Resolve a 2-D vector into x and y axis system.
- Determine the resultant force and its direction of coplanar forces.

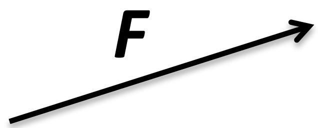
Scalar VS Vector

Problems in statics mechanics can be solved using either scalar or vector to represent the force.

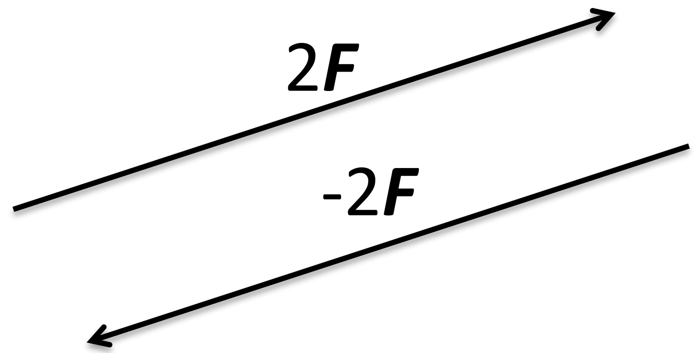
	Scalar	**Vector**
Examples:	mass, volume	force, velocity
Characteristics:	Has a magnitude (+ve or -ve)	Has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a "carrot"

Vector Operations

Multiplication and Division (Scalar)



Multiplied by a +ve & -ve scalar:



Division:

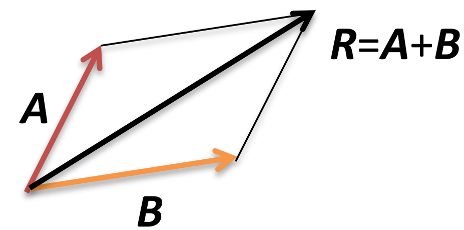
Note: The direction of the vector ***F* remain unchanged.

Vector Operations

Addition and Subtraction

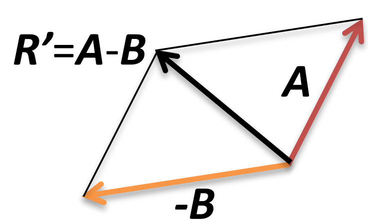
In general all vectors follow the **parallelogram law** of vector addition and subtraction.

Addition: $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$



Subtraction:

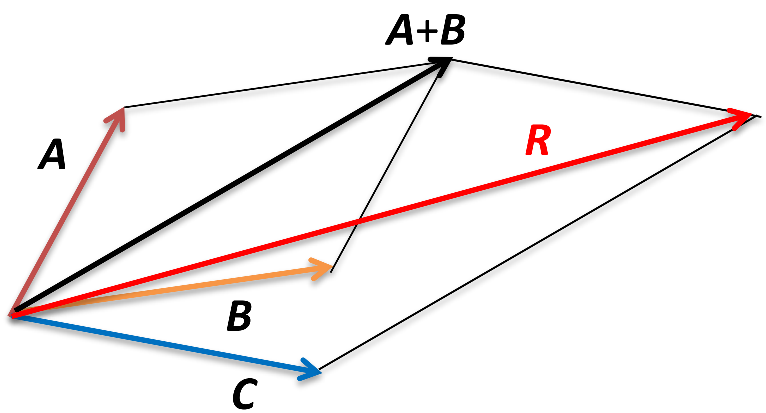
$\boldsymbol{R'} = \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$



Vector Operations

Addition and Subtraction

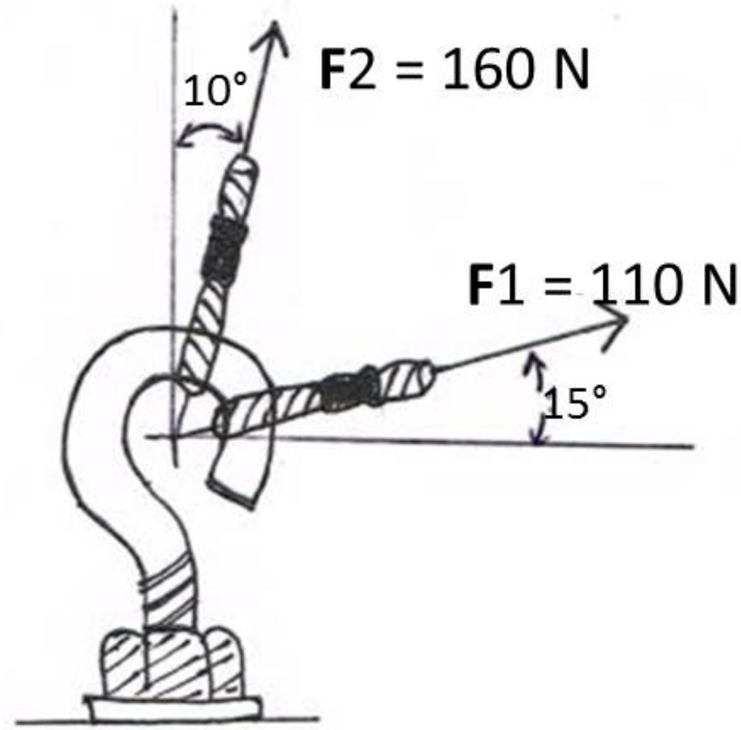
Addition and subtraction of several forces can be calculated using **parallelogram law** but could be difficult.



$$\boldsymbol{R} = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$$

Example: Resultant force

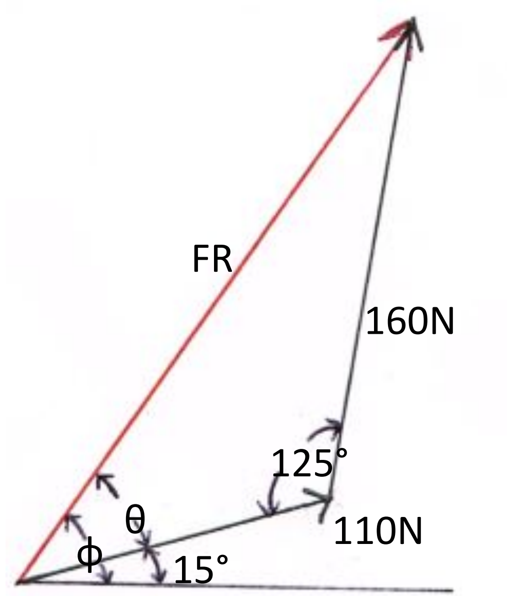
The screw eye below is subjected to two forces, **F**1 and **F**2. Determine the magnitude and direction of the resultant force.



Vector Addition of Forces

Example: Resultant force (continued)

Construct the vector triangle from the parallelogram law and solve resultant force using cosine law

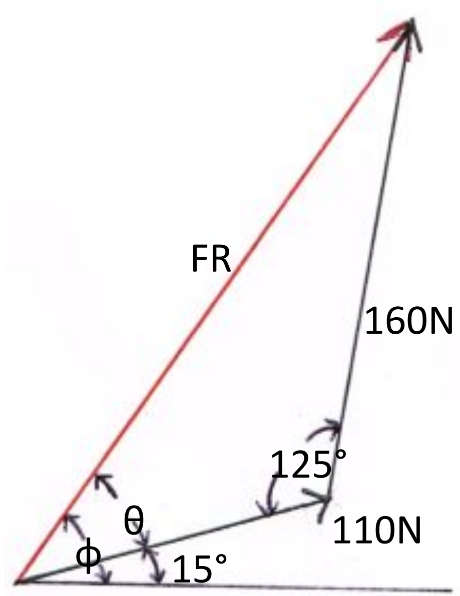


$$FR = \sqrt{(110\ N)^2 + (160\ N)^2 - 2(110\ N)(160\ N)\cos 125°}$$
$$= \sqrt{12\,100 + 25\,600 - 35\,200(-0.5736)} = 240.6\ N$$
$$= 241\ N$$

Vector Addition of Forces

Example: Resultant force (continued)

Apply sine law to determine angle, θ



$$\frac{160\ N}{\sin\theta} = \frac{240.6\ N}{\sin 125°}$$

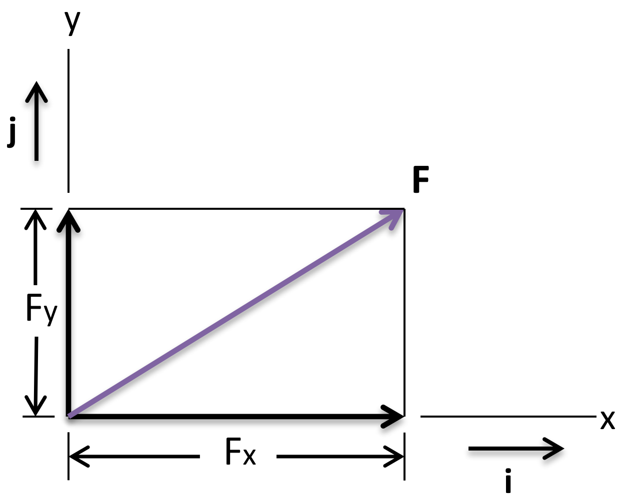
$$\sin\theta = \frac{160\ N(\sin 125°)}{240.6\ N}$$

$$\theta = 33.0°$$

Addition of Coplanar Forces

Resolution of Vector in Cartesian Notation

Resolution is a process of breaking up a vector into x and y axis system.

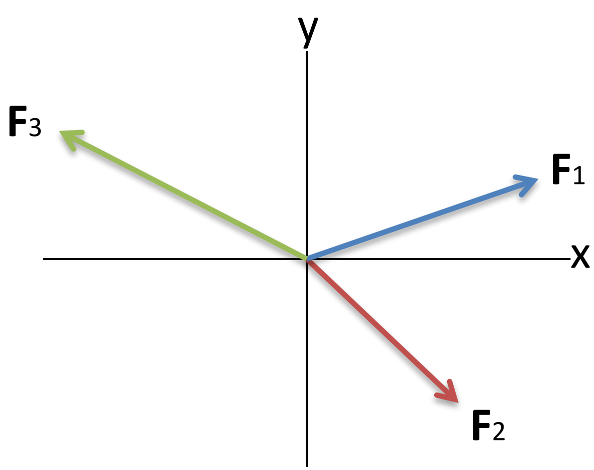


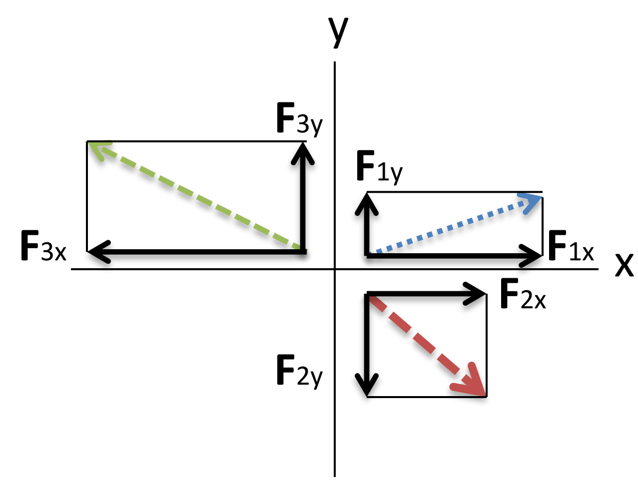
$\boldsymbol{F} = F_x\, \boldsymbol{i} + F_y\, \boldsymbol{j}$

- Break up vectors into x and y elements
- Each element has its magnitude and direction.
- Use the "unit vectors" ***i*** and ***j*** to represent the x and y axis.
- The x and y axes are always perpendicular to each other.

**Note: The process is like using the parallelogram law in reverse.

Addition of Coplanar Forces





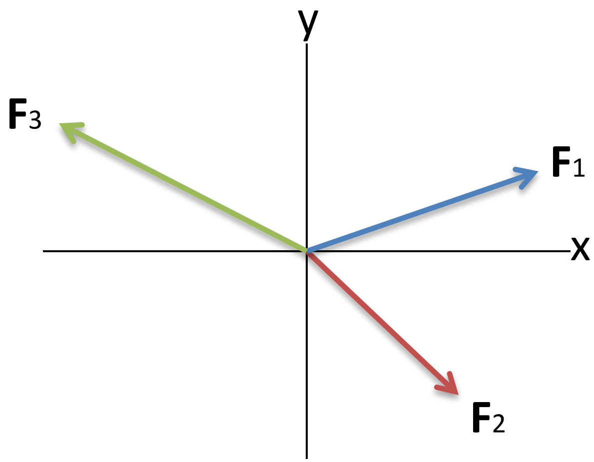
Step 1: Break up each force into its x and y elements.

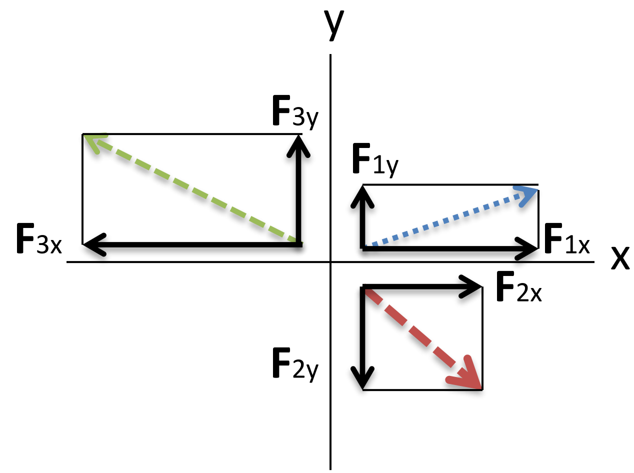
$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j} \quad ;$$

$$\mathbf{F}_2 = F_{2x}\mathbf{i} - F_{2y}\mathbf{j} \quad ;$$

$$\mathbf{F}_3 = -F_{3x}\mathbf{i} + F_{3y}\mathbf{j}$$

Addition of Coplanar Forces





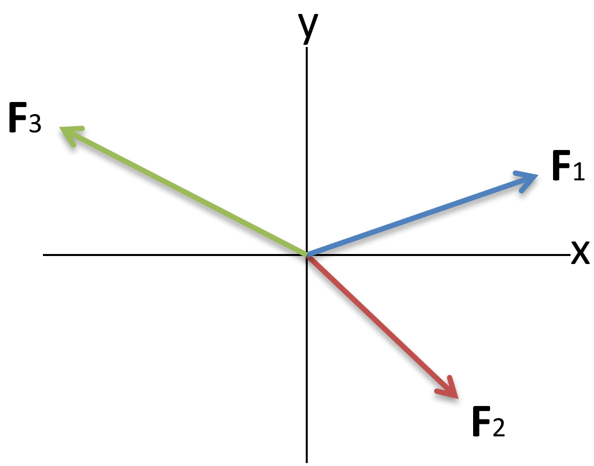
Step 2: Add all the x elements together and add all the y elements together.

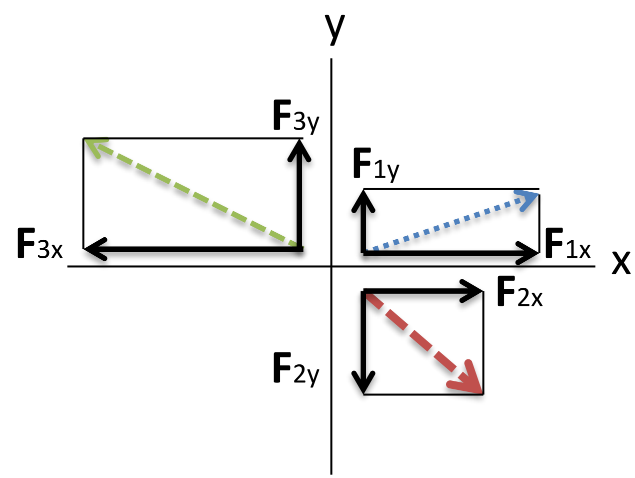
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (F_{1x} + F_{2x} - F_{3x})\mathbf{i} + (F_{1y} - F_{2y} + F_{3y})\mathbf{j}$$

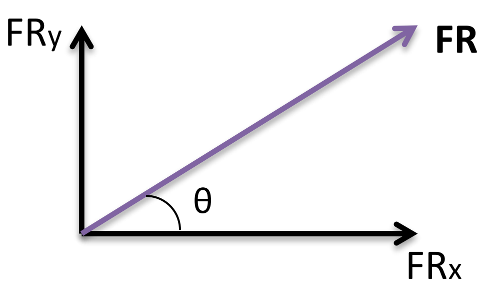
$$= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}$$

Addition of Coplanar Forces





Step 3: Find the magnitude and angle of the resultant vector using the total of x and y elements.

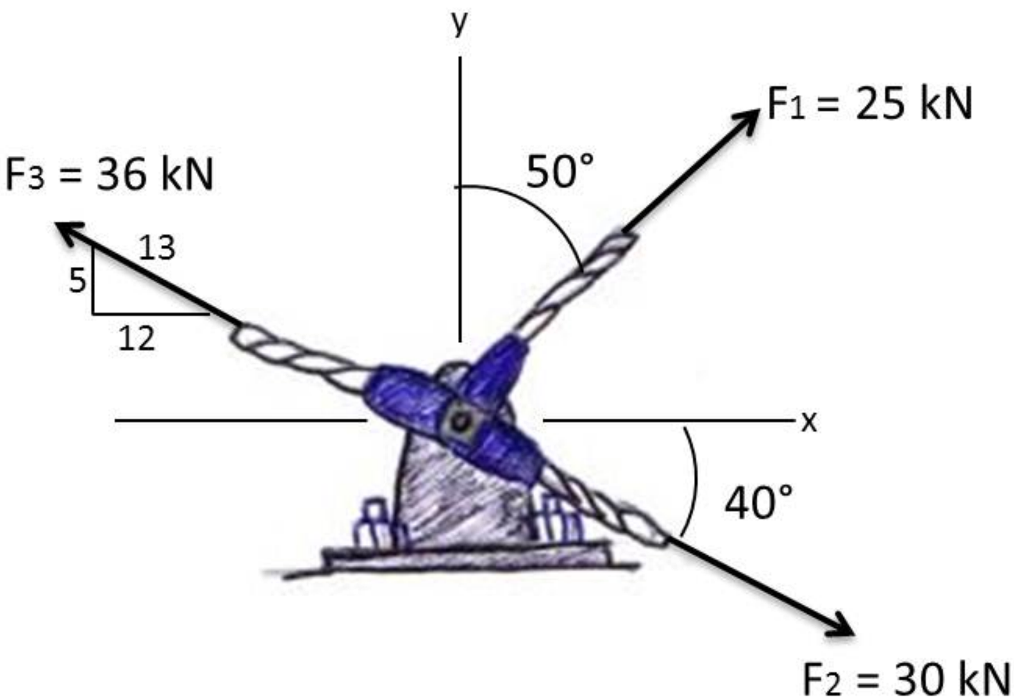


$$FR = \sqrt{FRx^2 + FRy^2}$$

$$\theta = \tan^{-1}\left|\frac{FRy}{FRx}\right|$$

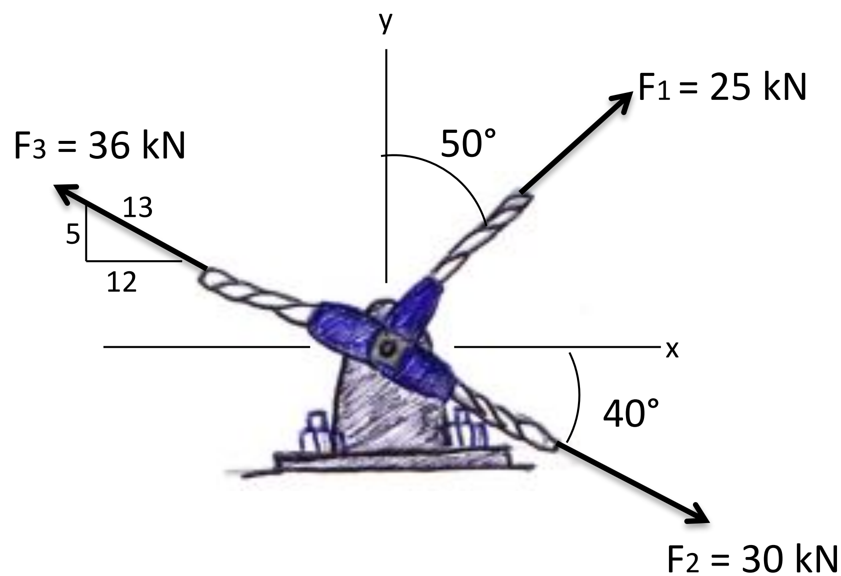
Example

A bracket is subjected to three simultaneous forces. Determine the magnitude and angle of the resultant force.



Addition of Coplanar Forces

Example (continued)



Step 1: Resolve the forces in their x-y elements.

$$F1 = \{ 25 \sin 50° \, i + 25 \cos 50° \, j \} \text{ kN}$$
$$= \{ 19.15 \, i + 16.07 \, j \} \text{ kN}$$

$$F2 = \{ 30 \cos 40° \, i - 30 \sin 40° \, j \} \text{ kN}$$
$$= \{ 22.98 \, i - 19.28 \, j \} \text{ kN}$$

$$F3 = \{ -(12/13)36 \, i + (5/13)36 \, j \} \text{ kN}$$
$$= \{ -33.23 \, i + 13.85 \, j \} \text{ kN}$$

Example (continued)

Step 2: Add the respective elements to get the resultant vector.

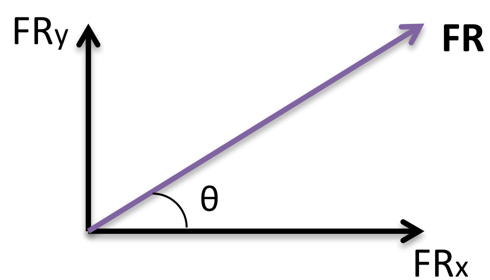
$$\mathbf{F}_R = \{ (19.15 + 22.98 - 33.23)\, \mathbf{i} + (16.07 - 19.28 + 13.85)\, \mathbf{j} \}\ \text{kN}$$

$$= \{ 8.9\, \mathbf{i} + 10.64\, \mathbf{j} \}\ \text{kN}$$

Step 3: Find magnitude and angle from the resultant elements.

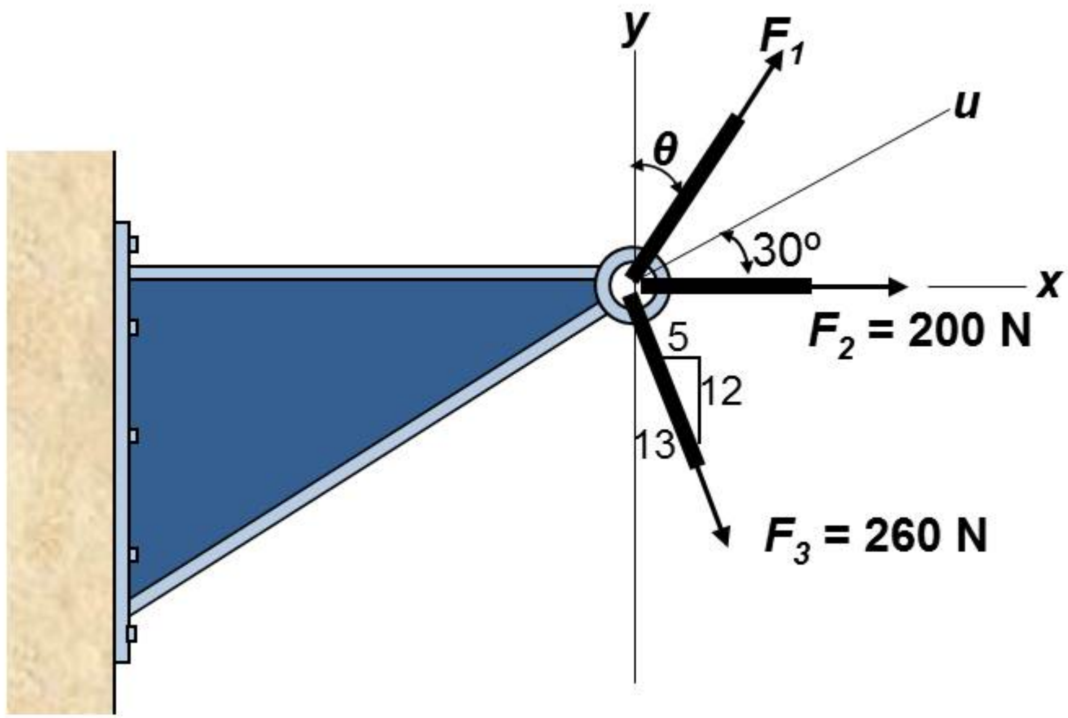
$$F_R = \sqrt{(8.9)^2 + (10.64)^2} = 13.87\ \text{kN}$$

$$\theta = \tan^{-1}\left|\frac{10.64}{8.9}\right| = 50.1°$$

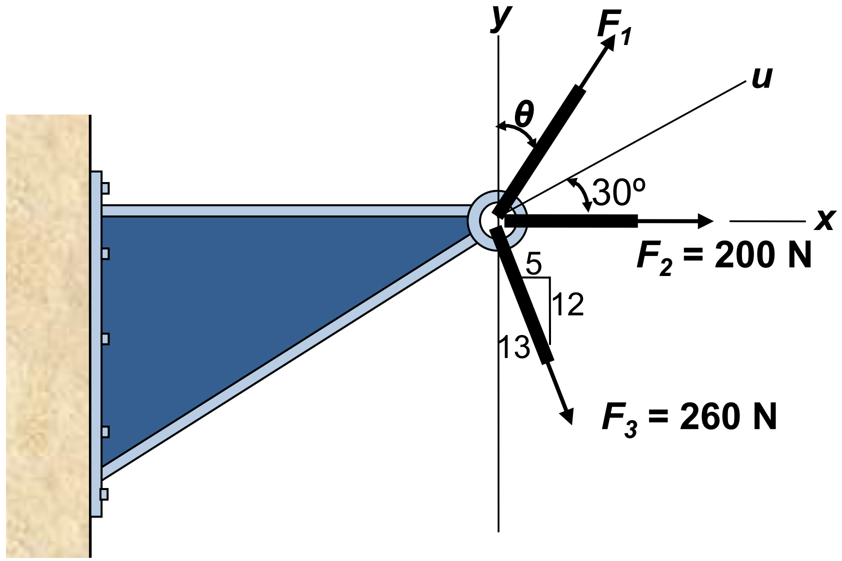


Example

Figure below shows a bracket subjected to three forces of F_1, F_2 and F_3. If the magnitude of the resultant force acting on the bracket is 450 N directed along the positive ***u*** axis, determine the magnitude of F_1 and its direction θ.



Example (continued)



Step 1: Resolve the forces in their x-y elements.

$$F1 = \{ F1 \sin\theta \, i + F1 \cos\theta \, j \} \text{ N}$$

$$F2 = \{ 200 \, i \} \text{ N}$$

$$F3 = \{ (5/13)260 \, i - (12/13)260 \, j \} \text{ kN}$$
$$= \{ 100 \, i - 240 \, j \} \text{ kN}$$

Addition of Coplanar Forces

Example (continued)

Step 2: Add the respective elements to get the resultant vector.

$$\mathbf{F}_R = \{ (F1 \sin\theta + 200 + 100)\, i + (F1 \cos\theta - 240)\, j \}\ N$$
$$= \{ (F1 \sin\theta + 300)\, i + (F1 \cos\theta - 240)\, j \}\ N$$

Step 3: Find magnitude and angle of F1 where FR=450 N and θR=30°.

$$\mathbf{F}_{Rx} = 450 \cos 30° = F1 \sin\theta + 300$$
$$\rightarrow \quad F1 \sin\theta = 89.71 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\mathbf{F}_{Ry} = 450 \sin 30° = F1 \cos\theta - 240$$
$$\rightarrow \quad F1 \cos\theta = 465 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Example (continued)

Solve equation: (1) ÷ (2)

$$\tan\theta = \frac{89.71}{465}$$

$$\theta = 10.9°$$

Substitute θ=10.9° into equation (2).

→ F1 cos 10.9° = 465

F1 = 473.5 N

End of Lesson

<u>Recall:</u>

- Can you differentiate between scalar and vector of force?
- What is parallelogram law?
- What is resolution of vector?
- How to draw the FBD?
- What are the coplanar forces?
- Can you do the addition of coplanar forces?

References

- Hibbeler, R.C. and Yap, K.B., 2013, **Mechanics for Engineers – Statics**, Thirteenth SI Edition, Pearson, Singapore.