

MECHANISM DESIGN

CHAPTER 4:

VELOCITY ANALYSIS

Shamsul Anuar Shamsudin

Mohd Nizam Sudin

LINEAR VELOCITY

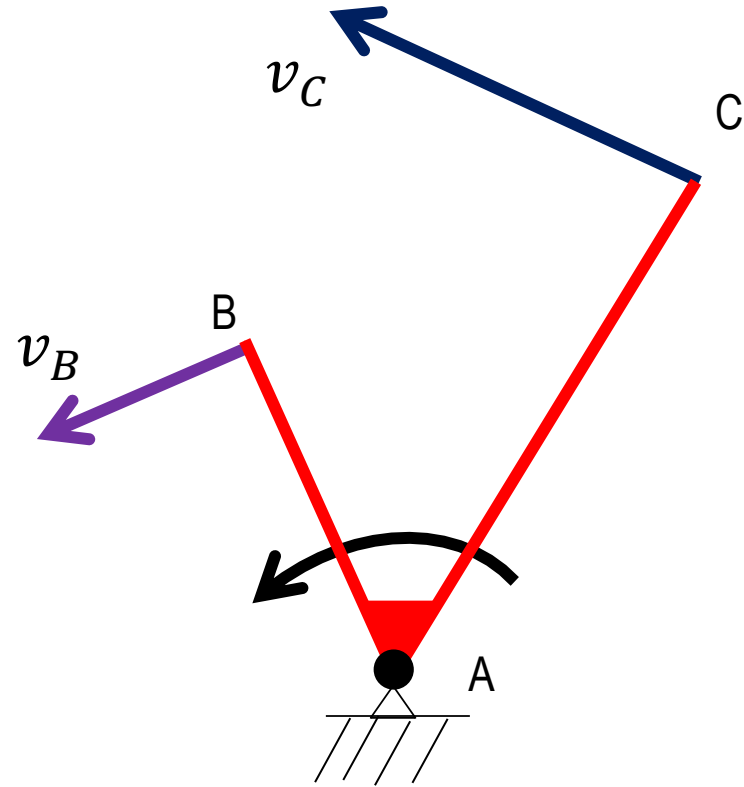
Speed is a measure of how fast an object moves. Basically that is distance covered in a time unit.

Velocity include the scalar speed magnitude and direction. Here v_B is different from v_C .

Common: m/s, ft/min (fpm), mile/h (mph), and km/h .

The linear velocity of a point is perpendicular to the radial arm of the point.

The sense or direction must agree with the rotation of the arm.



$$v = \frac{ds}{dt}$$

ANGULAR VELOCITY

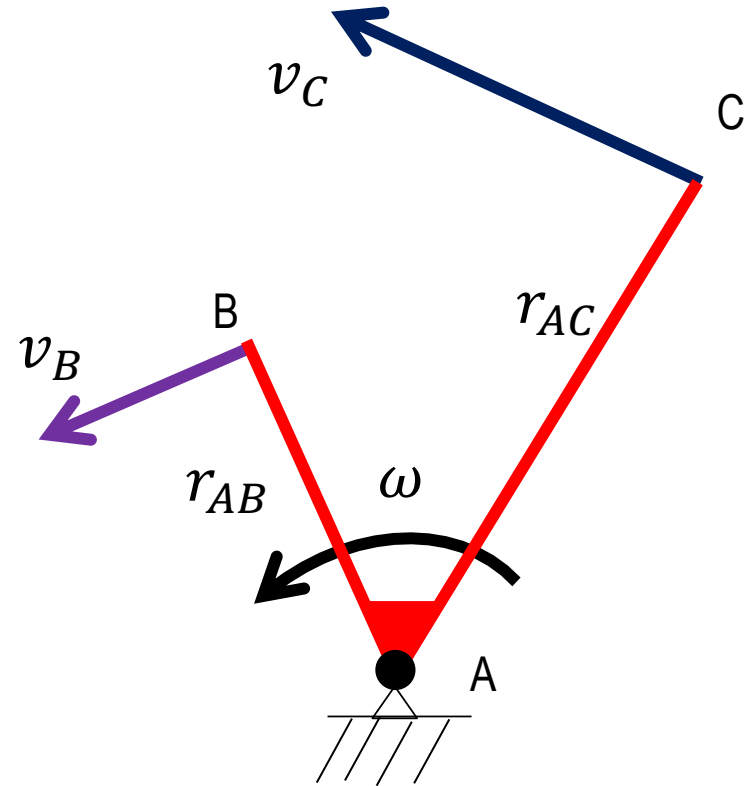
Angular speed is a measure of how fast an arm rotates. Basically that is the change in angle over a time period.

Common units are rev/min (rpm), rad/s (rps), and deg/s.

Angular velocity include the scalar angular speed magnitude and rotation direction.

Here v_B and v_C will result in or are the result of the same angular velocity.

Directions: clockwise (CW) or counterclockwise (CCW).



$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

MAIN RELATIONSHIP

The sense of v and ω must be consistent.

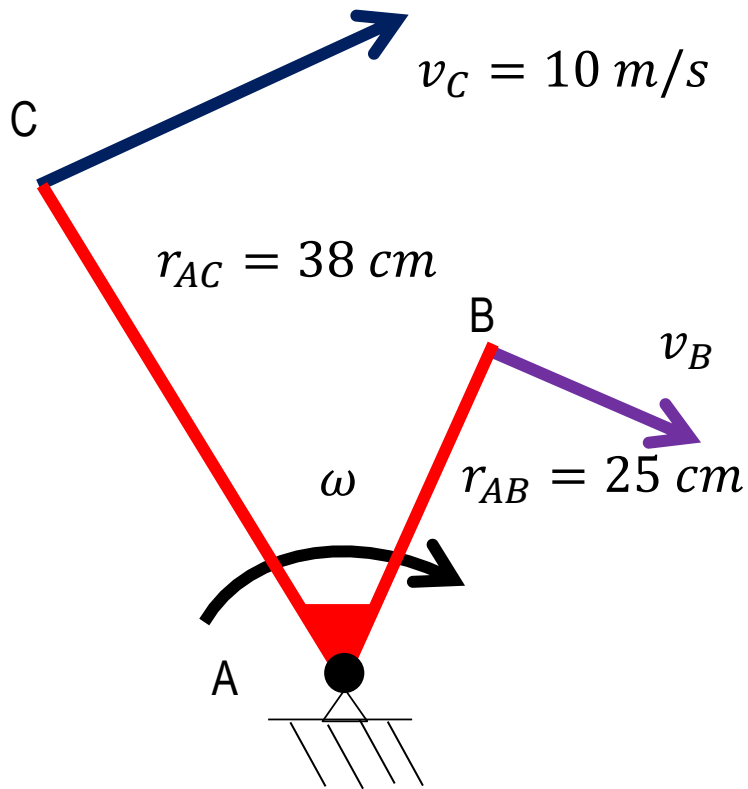
$$v = r\omega$$

The ω may be in **rad/s** if it needs changing to linear velocity.

EXAMPLE 1

The velocity of point C is 10 m/s, determine the velocity of point B if the link is rotating clockwise about pivot A.

Solution:

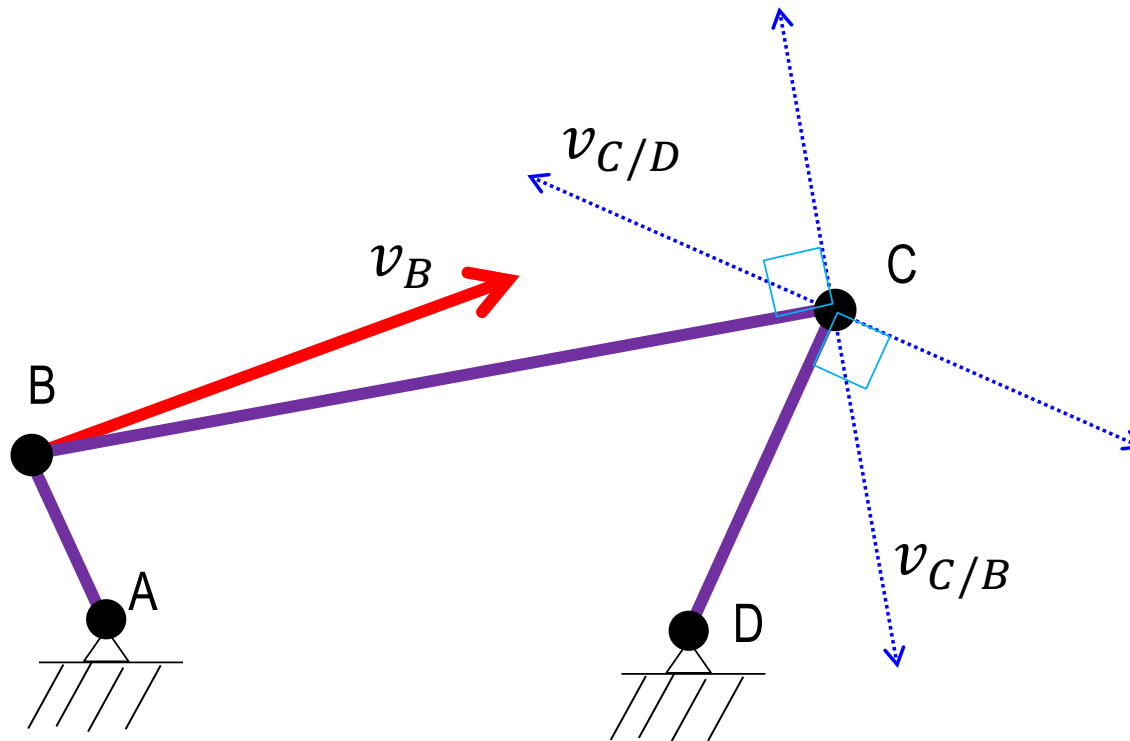


$$\begin{aligned}\omega &= \frac{v_C}{r_C} \\ &= \frac{10 \text{ m/s}}{0.38 \text{ m}} \\ &= 26.32 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}v_B &= r_B \omega \\ &= (0.25 \text{ m})(26.32 \text{ rad/s}) \\ &= 6.58 \text{ m/s}\end{aligned}$$

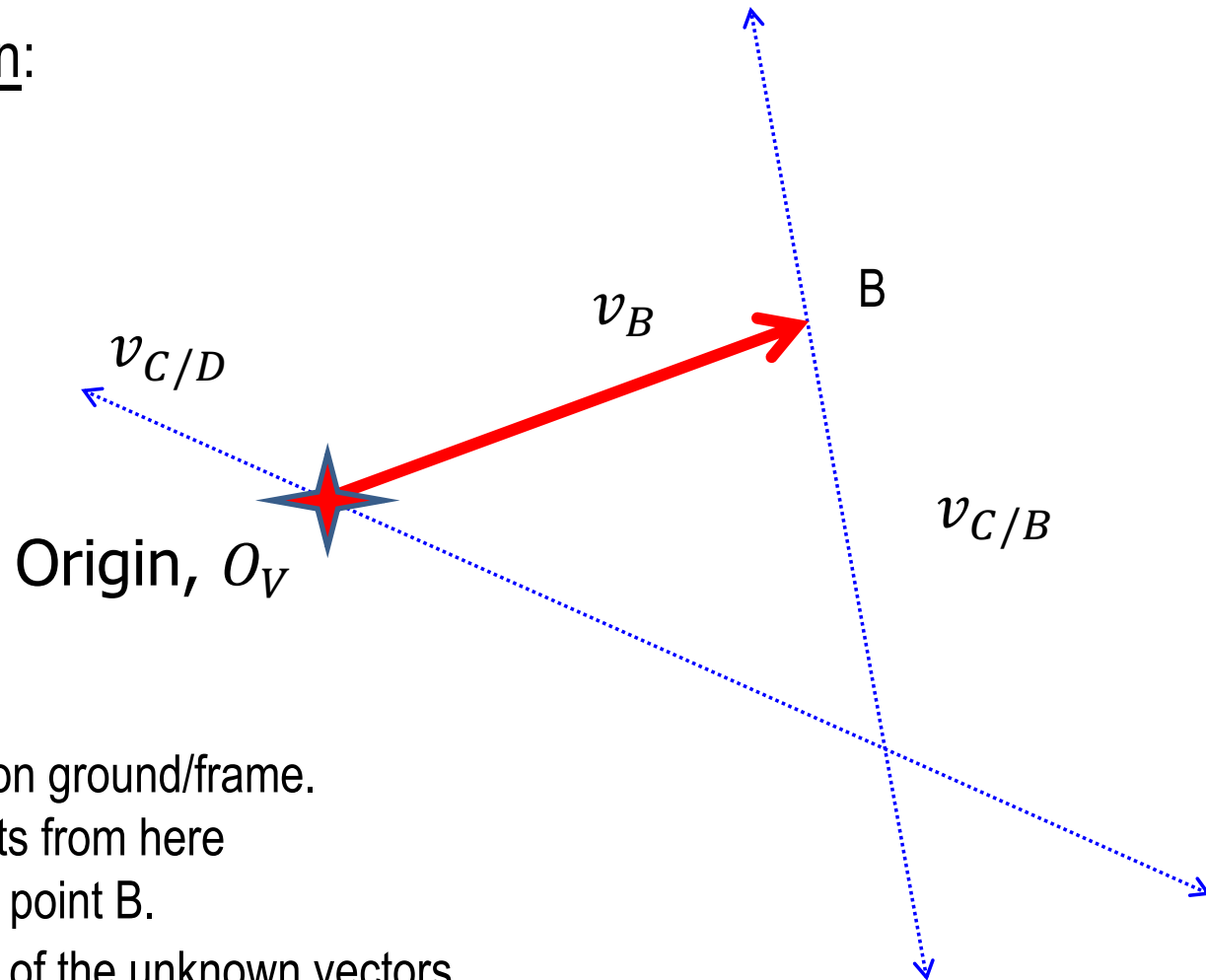
GRAPHICAL VELOCITY ANALYSIS

We may treat the linear velocity as vectors. Use the vector polygon to solve for unknowns. This can be done analytically or graphically.



GRAPHICAL VELOCITY ANALYSIS

Vector polygon:



A and D are pivots on ground/frame.

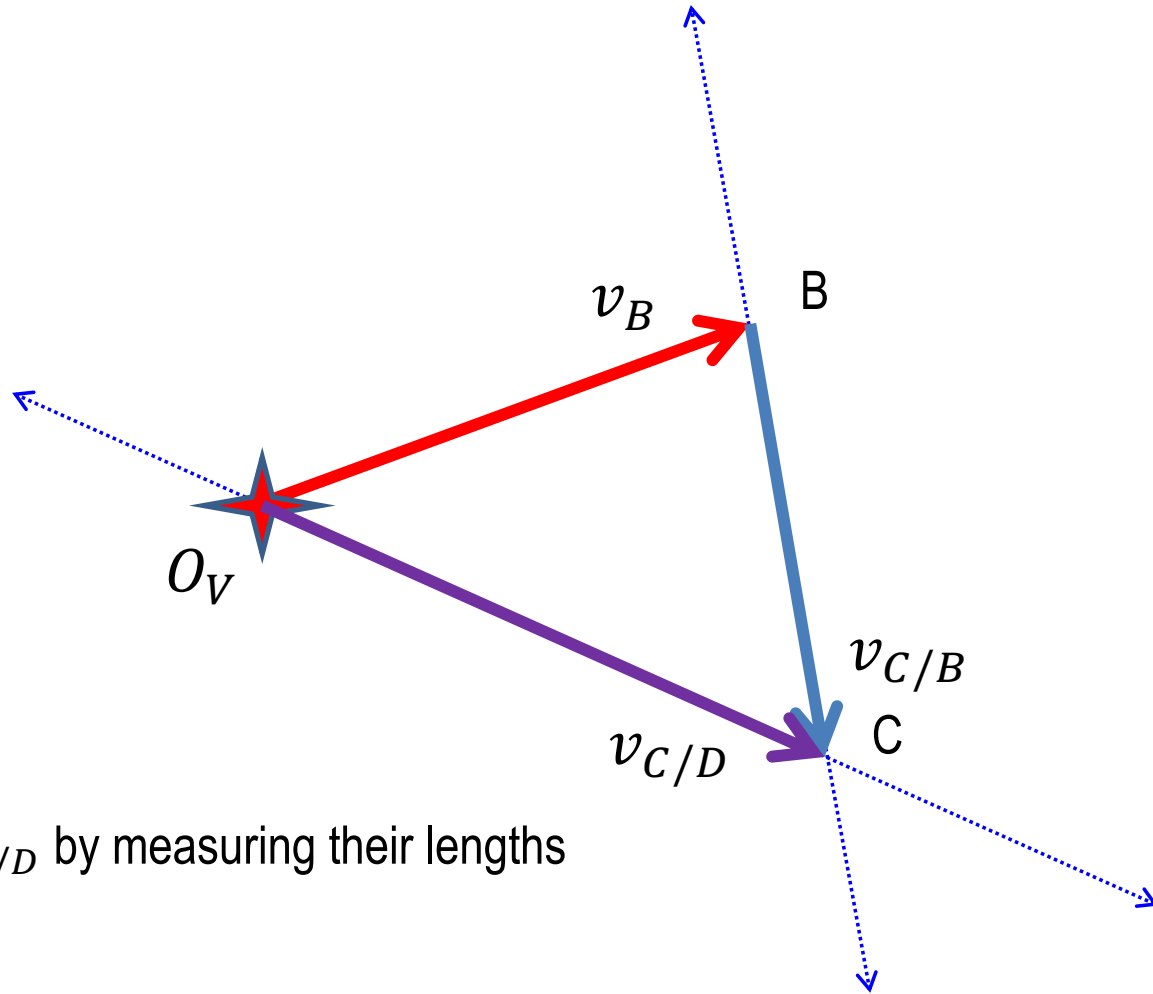
Their velocities start from here

$v_{C/B}$ must relate to point B.

Get the intersection of the unknown vectors

GRAPHICAL VELOCITY ANALYSIS

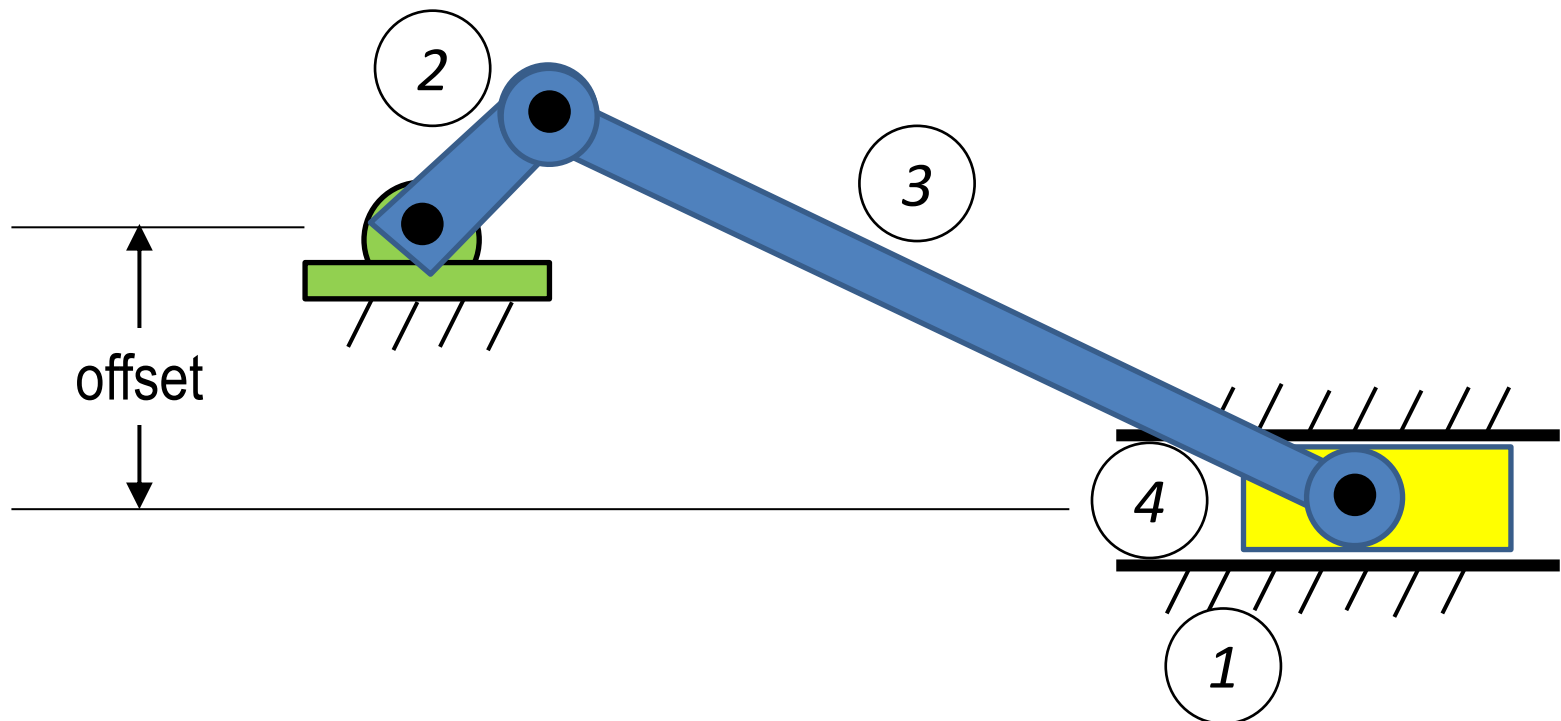
Vector polygon:



Solve for $v_{C/B}$ and $v_{C/D}$ by measuring their lengths and direction angles.

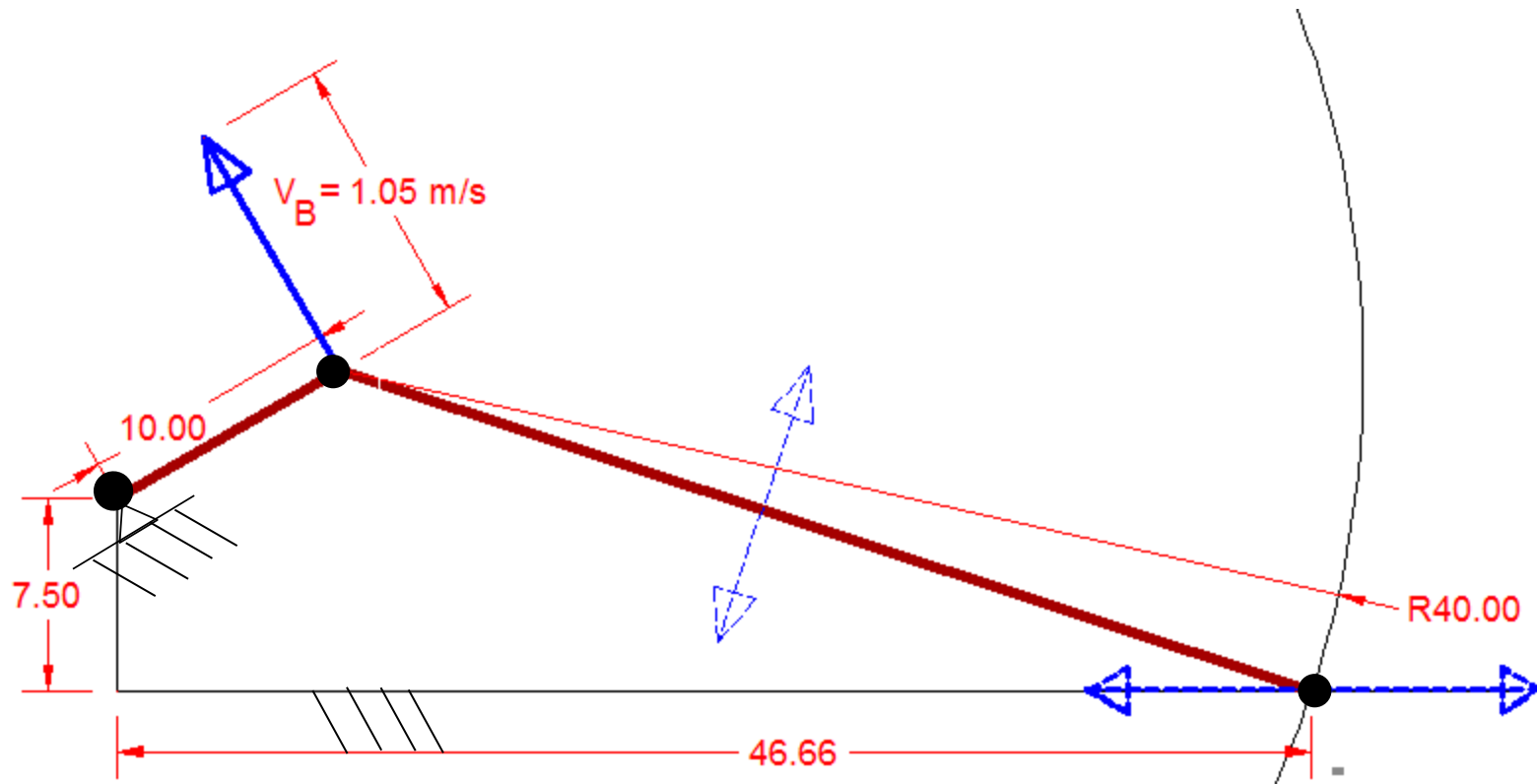
EXAMPLE 2

The 10 cm crank revolves at 100 rpm, counterclockwise. The connecting rod is 50 cm. The offset is 7.5 cm. Crank angle is 30° . Find the velocity of the piston.



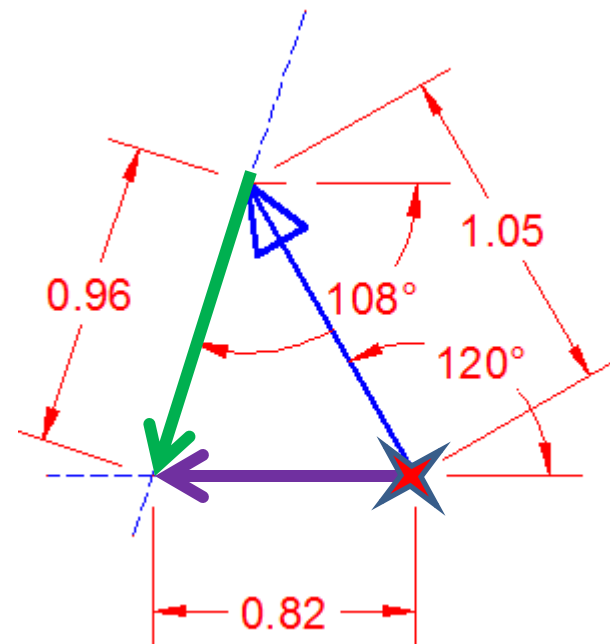
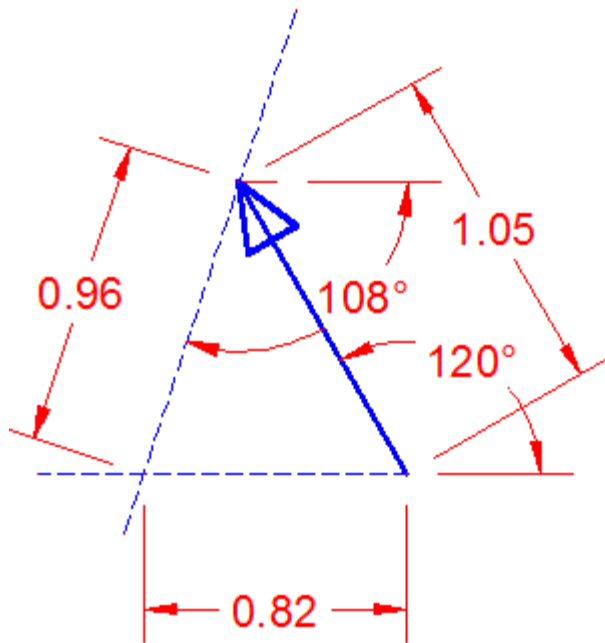
EXAMPLE 2

Do position analysis first using graphical method.



EXAMPLE 2

Next, solve the vector polygon for velocity.



$$\vec{V}_{C/B} = 0.96 \begin{Bmatrix} \cos(-108) \\ \sin(-108) \end{Bmatrix} = \begin{Bmatrix} -0.297 \\ -0.913 \end{Bmatrix} \text{ m/s}, \quad \vec{V}_C = \begin{Bmatrix} -0.82 \\ 0 \end{Bmatrix} \text{ m/s}$$

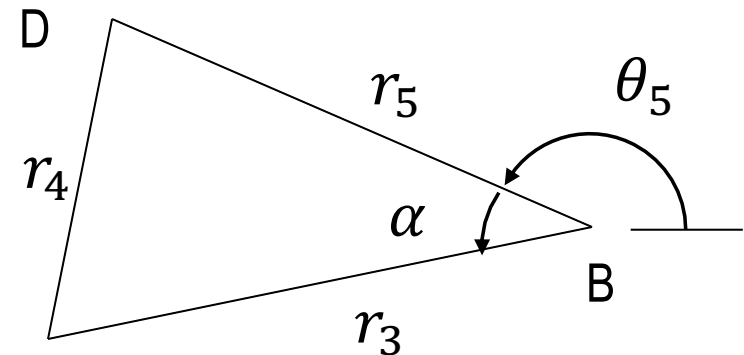
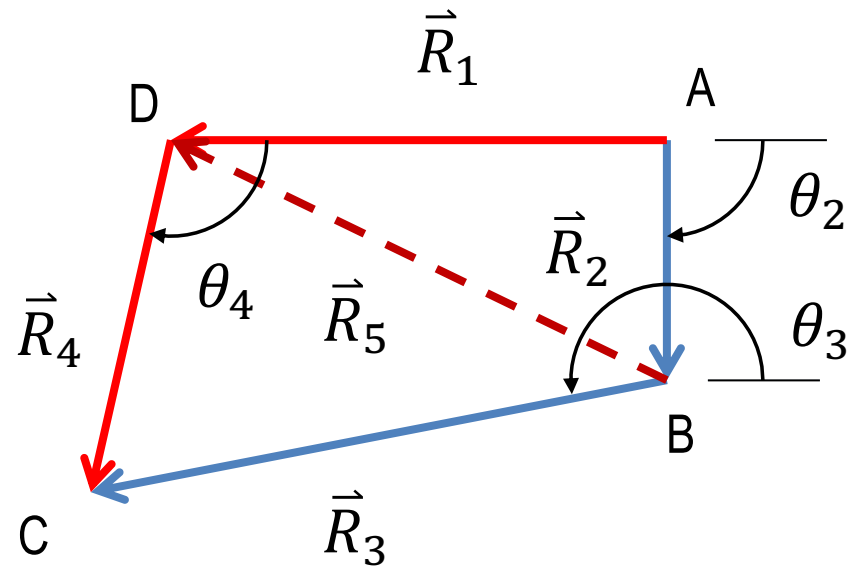
VECTOR ANALYTICAL METHOD

Loop equations:

1. $\vec{R}_2 + \vec{R}_3 = \vec{R}_1 + \vec{R}_4$
 (θ_3, θ_4) - unknowns
2. $\vec{R}_5 = \vec{R}_1 - \vec{R}_2$
3. $\vec{R}_5 = \vec{R}_3 - \vec{R}_4$
4. $\vec{R}_c = \vec{R}_2 + \vec{R}_3$

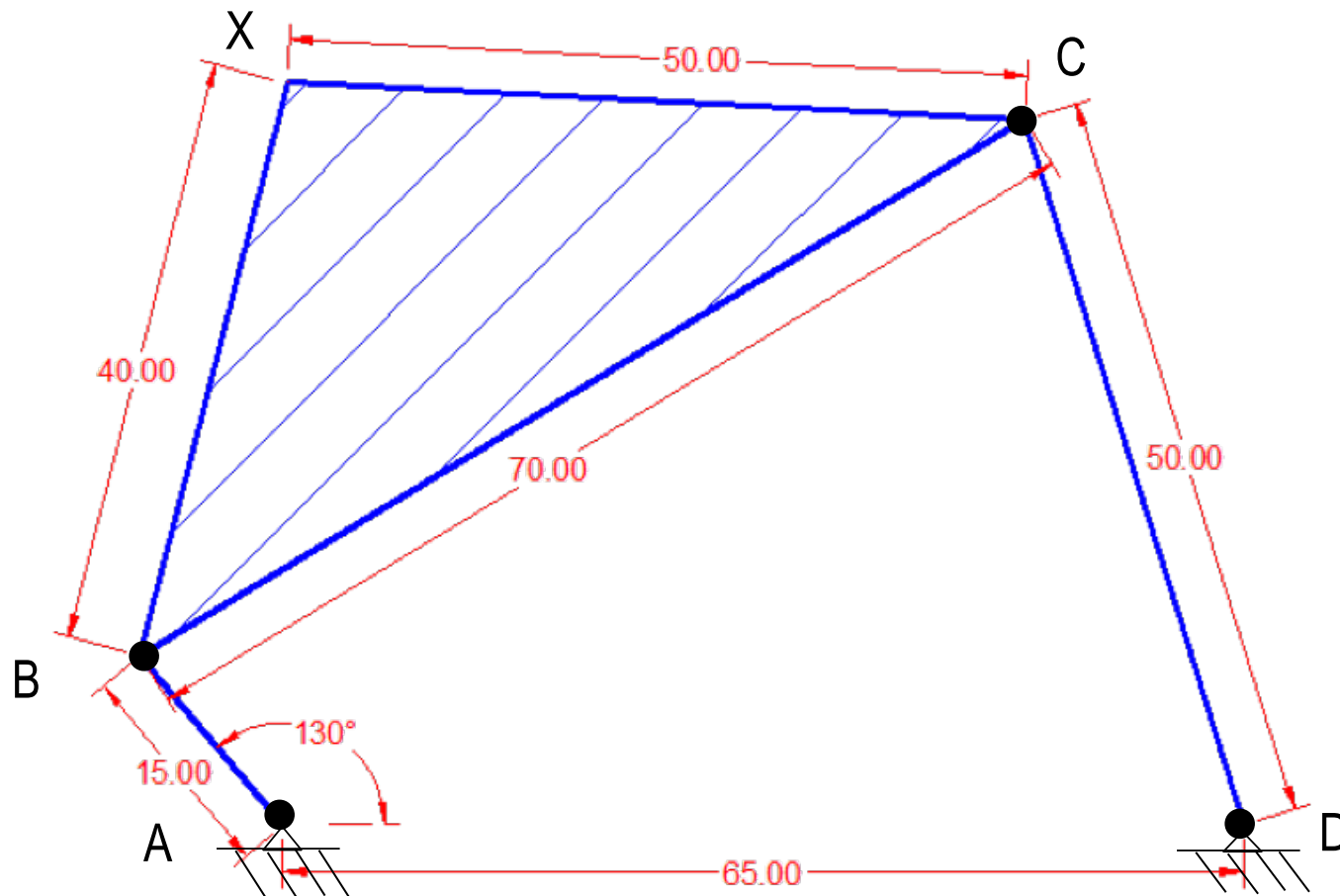
Differentiate w.r.t. time:

1. $\vec{V}_2 + \vec{V}_3 = \vec{V}_4$
 (ω_3, ω_4) - unknowns
2. $\vec{V}_c = \vec{V}_2 + \vec{V}_3 = \begin{Bmatrix} ? \\ ? \end{Bmatrix} \text{ mm/s}$



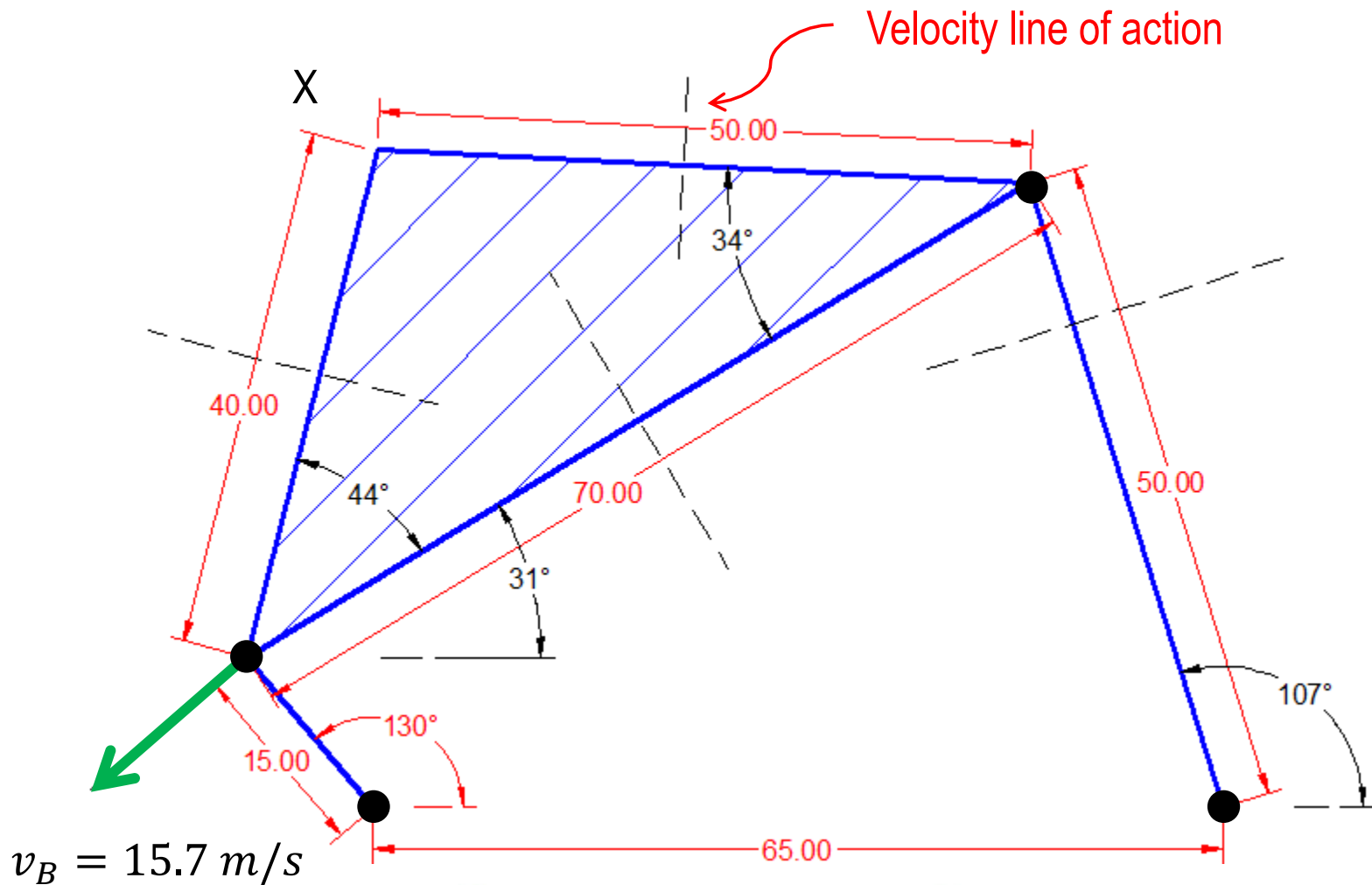
EXAMPLE 3

The 15 mm crank rotates at 1000 rpm, CCW. Find the linear velocity of the X.



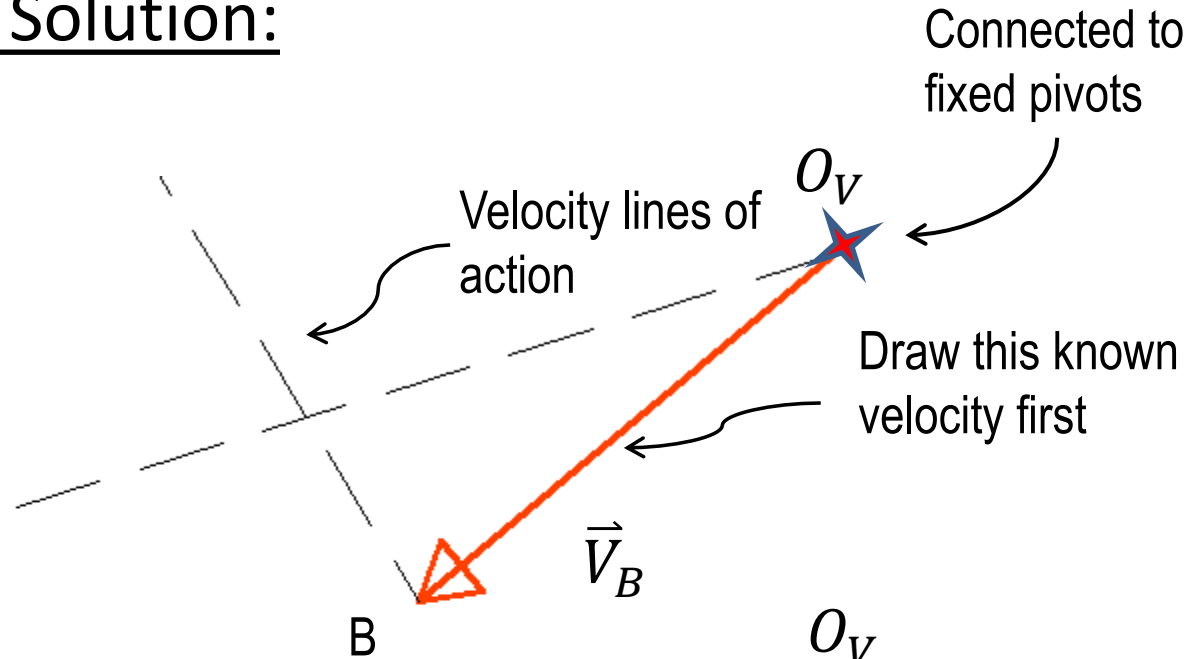
EXAMPLE 3

Do the position analysis and get the perpendicular lines of action for velocities.

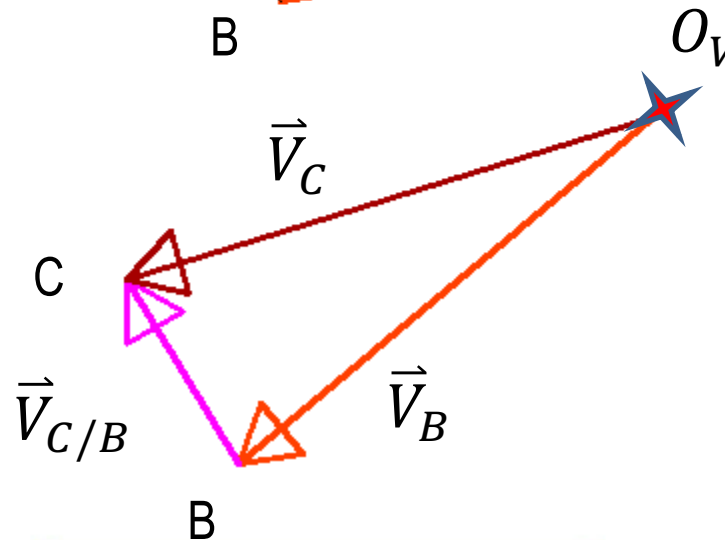


Graphical Solution:

1



2

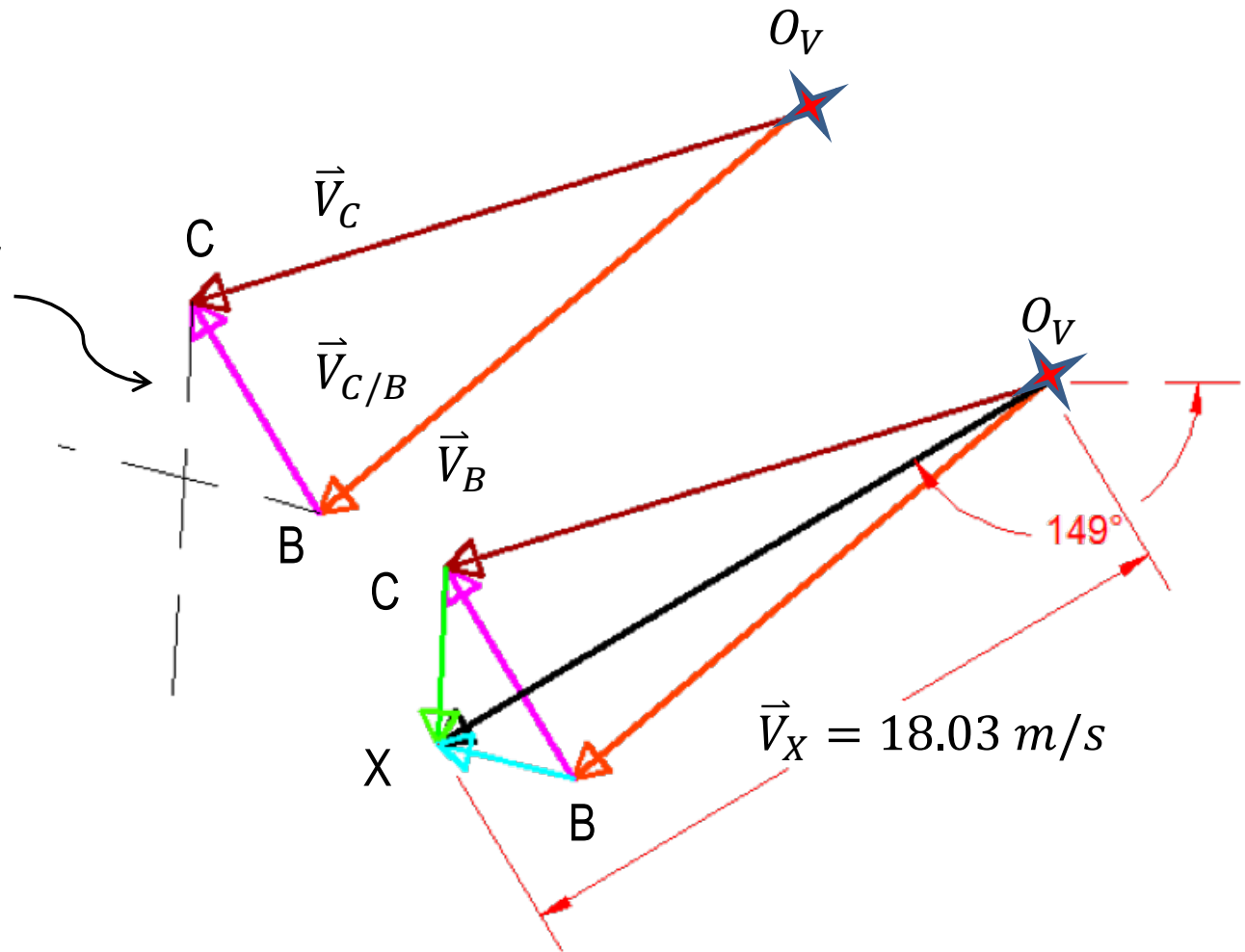


Graphical Solution:

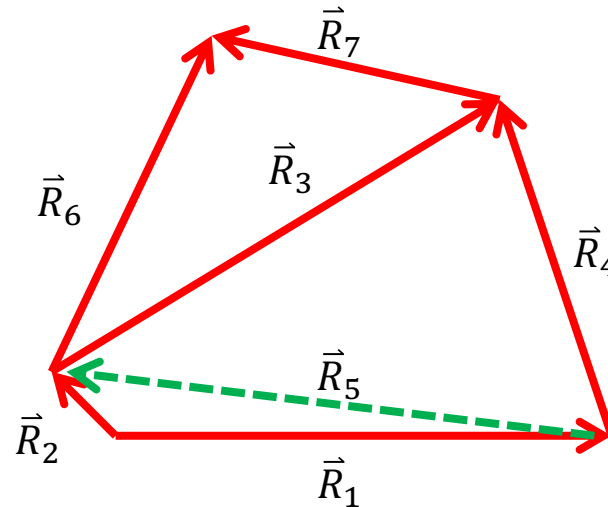
3

Unknown Velocity
lines of action

4



Analytical Solution:



1. $\vec{R}_2 + \vec{R}_3 = \vec{R}_1 + \vec{R}_4$ (θ_3, θ_4) - unknown
2. $\vec{R}_5 = -\vec{R}_1 + \vec{R}_2$
3. $\vec{R}_5 = -\vec{R}_3 + \vec{R}_4$
4. $\vec{R}_X = \vec{R}_2 + \vec{R}_6$

$$\theta_3 = 31.2139^\circ$$

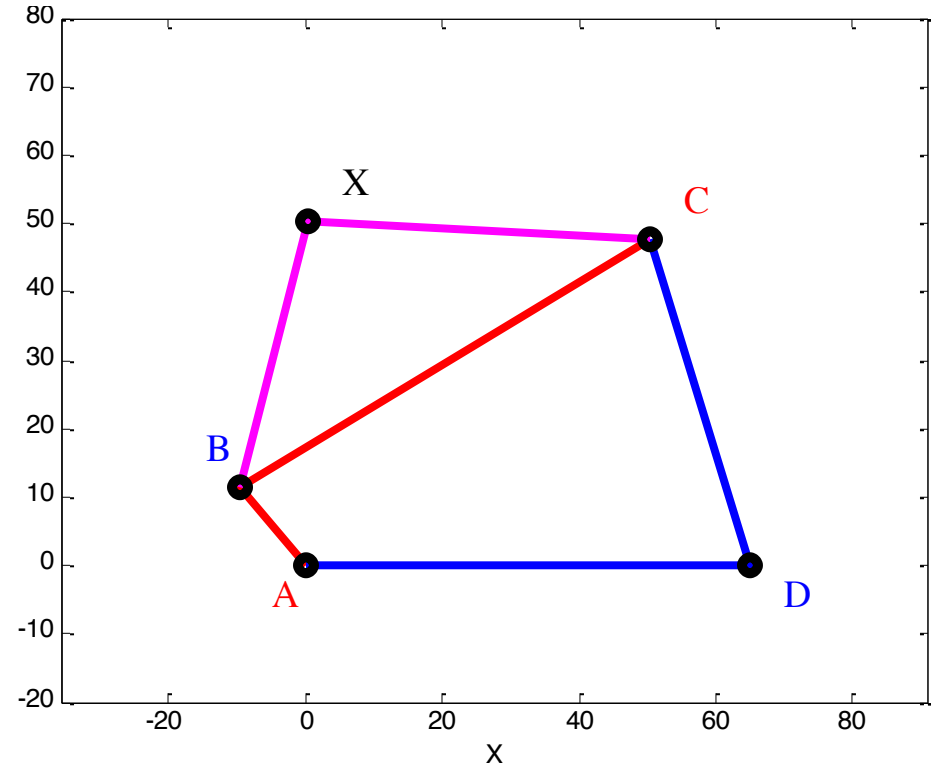
$$\theta_4 = 107.1877^\circ$$

Analytical Solution:

1. $\vec{V}_2 + \vec{V}_3 = \vec{V}_4$ (ω_3, ω_4) - unknown
2. $\omega_6 = \omega_3$
3. $\vec{V}_X = \vec{V}_2 + \vec{V}_6 = \begin{Bmatrix} -1550.8 \\ -920.7 \end{Bmatrix} \text{ mm/s}$
 $v_X = 18.035 \text{ m/s}, \theta_X = -149.3035^\circ$

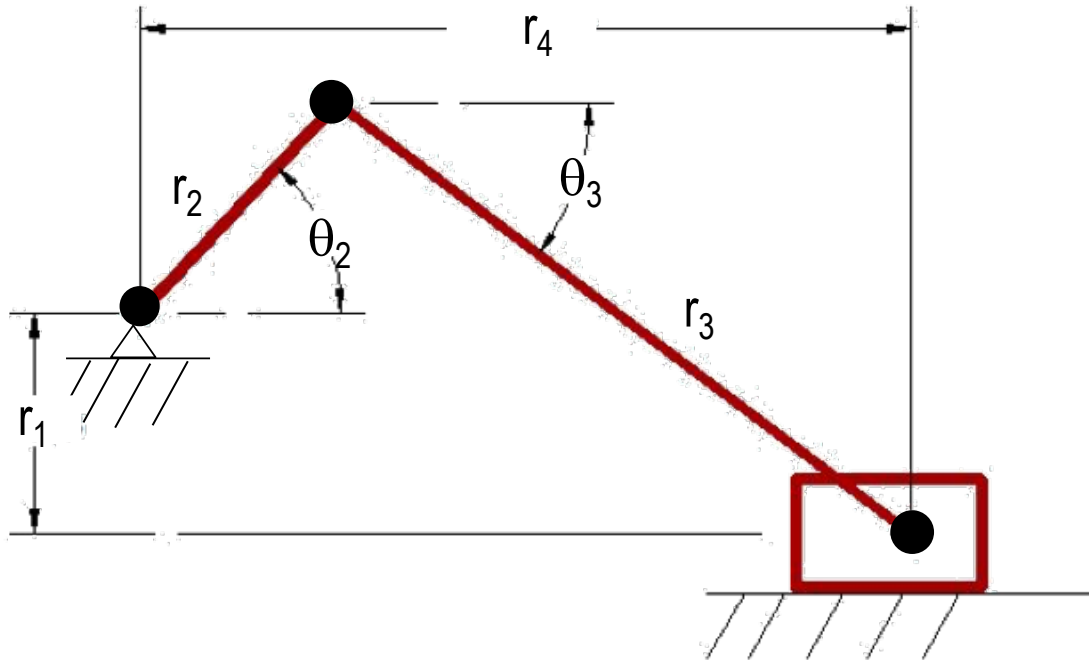
About the same as in graphical solution!

Sketch of kinematic diagram at $\theta_2 = 130$ degrees



$$\begin{bmatrix} -r_3 \sin \theta_3 & r_4 \sin \theta_4 \\ r_3 \cos \theta_3 & -r_4 \cos \theta_4 \end{bmatrix} \begin{Bmatrix} \omega_3 \\ \omega_4 \end{Bmatrix} = -r_2 \omega_2 \begin{Bmatrix} -\sin \theta_2 \\ \cos \theta_2 \end{Bmatrix}$$

FOUR-BAR VELOCITY



First do the position analysis to unknown angles or lengths.
Next, find all rotational and linear velocities.

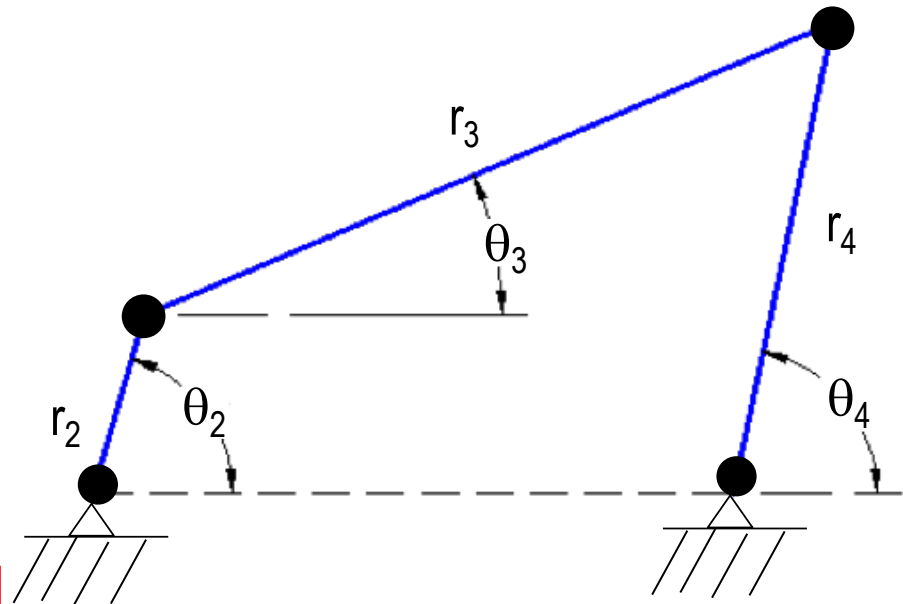
$$\omega_3 = \omega_2 \left[\frac{r_2 \cos \theta_2}{r_3 \cos \theta_3} \right]$$

$$v_4 = \omega_2 r_2 \sin \theta_2 - \omega_3 r_3 \sin \theta_3$$

FOUR-BAR VELOCITY

First do the position analysis to unknown angles or lengths.
Next, find all rotational and linear velocities.

$$v = r\omega$$



$$\omega_3 = -\omega_2 \left[\frac{r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin \gamma} \right]$$

$$\omega_4 = -\omega_2 \left[\frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin \gamma} \right]$$

Thank You

Main Reference:

Myszka, David H., 2012. Machines and mechanism: applied kinematic analysis, 4th ed., Prentice Hall, New York.



Source:

https://en.wikipedia.org/wiki/Speedometer#/media/File:Animated_Aston_Martin_Speedometer.gif