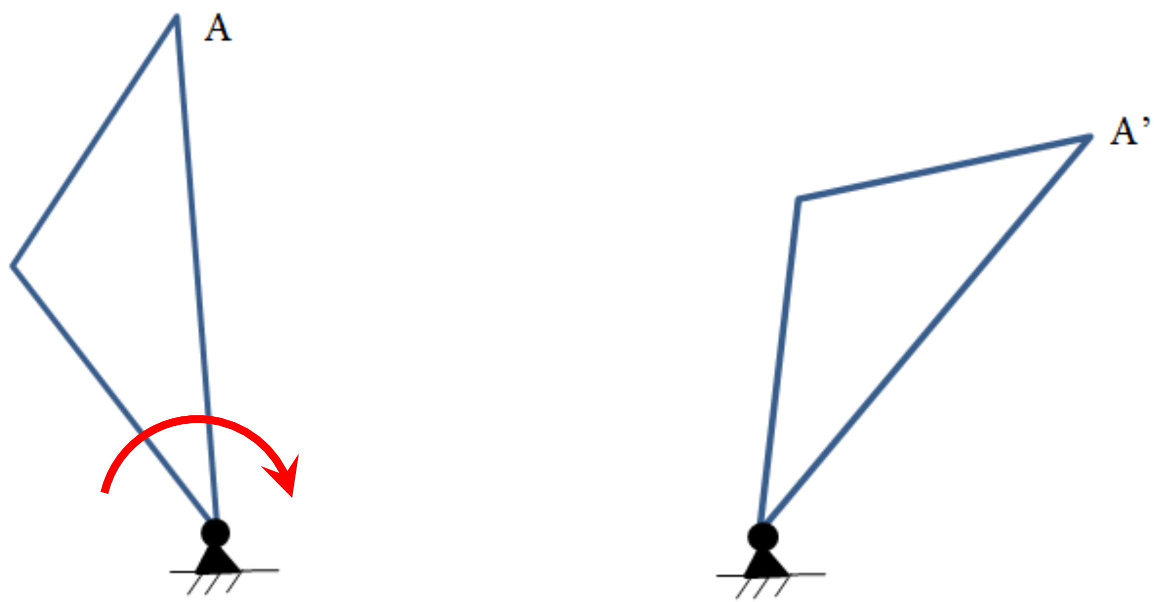
MECHANISM DESIGN

CHAPTER 2: POSITION ANALYSIS

Shamsul Anuar Shamsudin
Mohd Nizam Sudin

INTRODUCTION

- Position of particular points can be important at any time or input.
- As one point A changes to A’, other points will change accordingly.



VECTOR REVIEW

Scalar:

They just need magnitudes.

- Temperature
- Volume
- Time

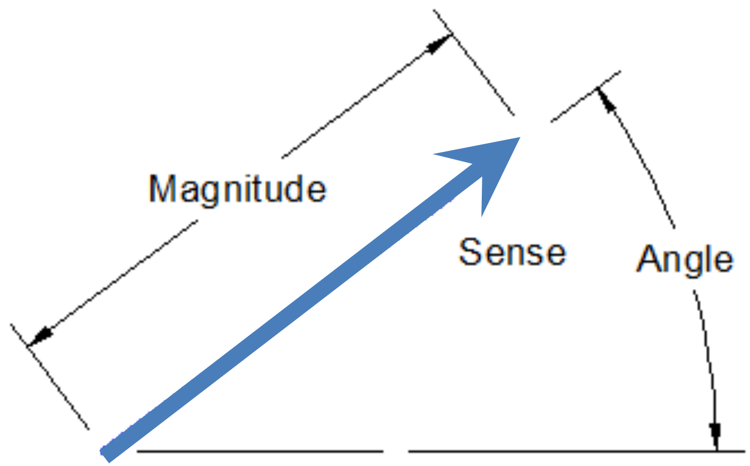
Vector:

They consist of scalar magnitudes and direction angles.

- Forces
- Displacement (Position)
- Velocity

VECTOR REVIEW

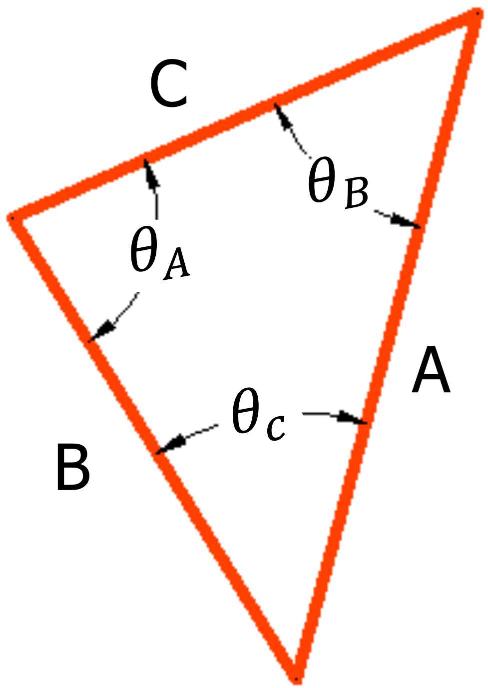
- Scalar Magnitude
- Sense or arrowhead
- Direction Angle at the tail



Take: angle 0°from positive x. Positive θ going counterclockwise. Clockwise θ is negative.

VECTOR REVIEW

Use triangles & trigonometry when working with two vectors.



Cosine Law.

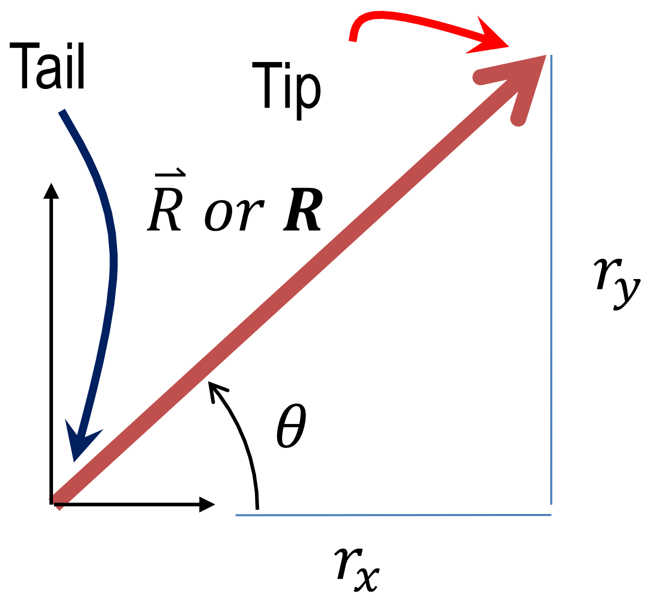
$$C^2 = A^2 + B^2 - 2AB\cos\theta_C$$

Sine Law.

$$\frac{A}{\sin\theta_A} = \frac{B}{\sin\theta_B} = \frac{C}{\sin\theta_c}$$

VECTOR REVIEW

- These operations give the **resultant** of several vectors.
- Place the vectors tail-to-tip, maintaining directions.
- The **resultant** shows the final destination.



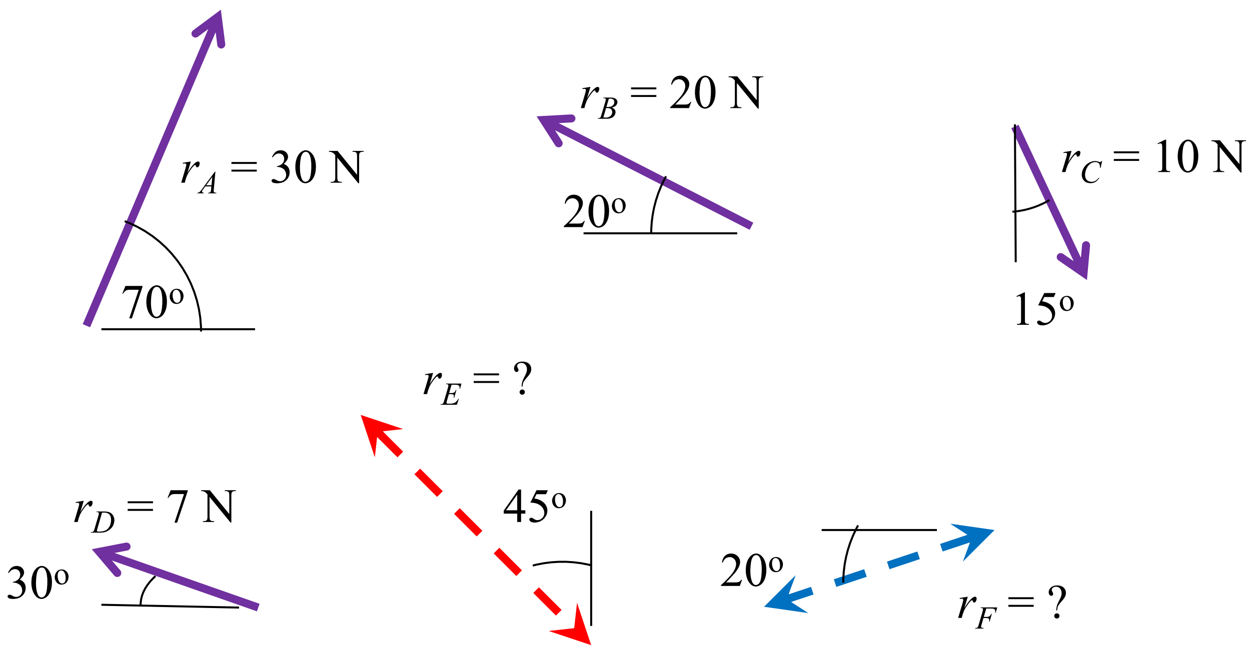
$$\vec{R} = \begin{Bmatrix} r_x \\ r_y \end{Bmatrix} = |\vec{R}| \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} = r \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix}$$

VECTOR EQUATIONS

We can solve for either:

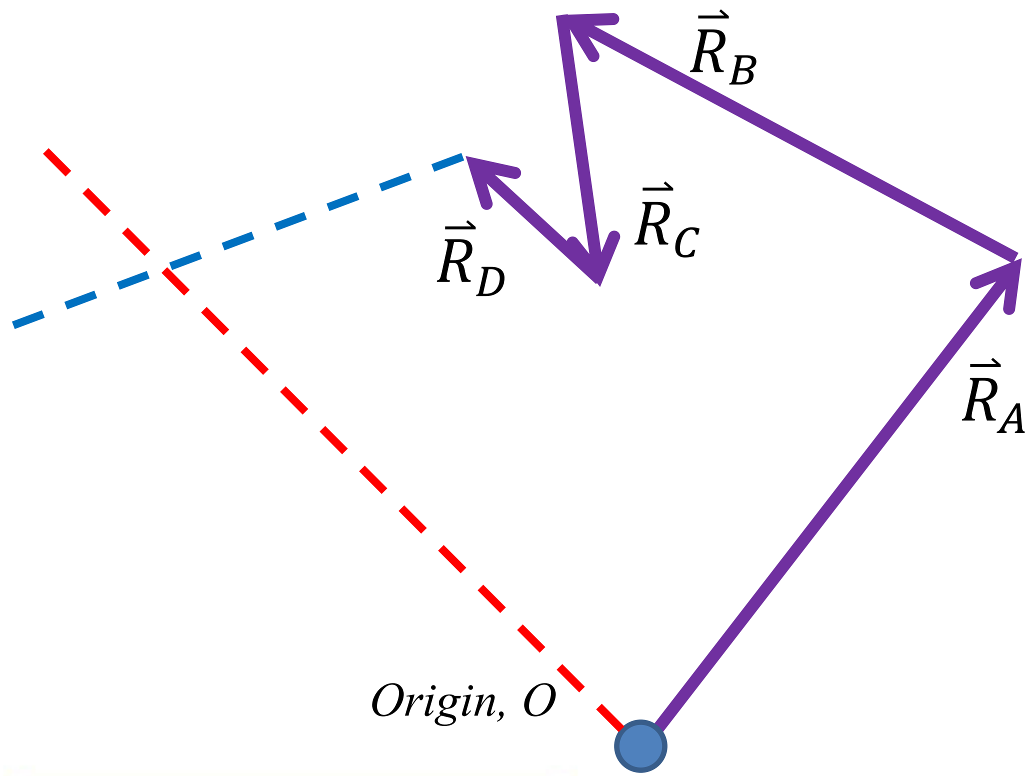
- The magnitude & direction of one vector
- The magnitude of two vectors.
- Two angles of the two vectors.

Only 2 unknowns to solve from 1 equation



VECTOR EQUATIONS

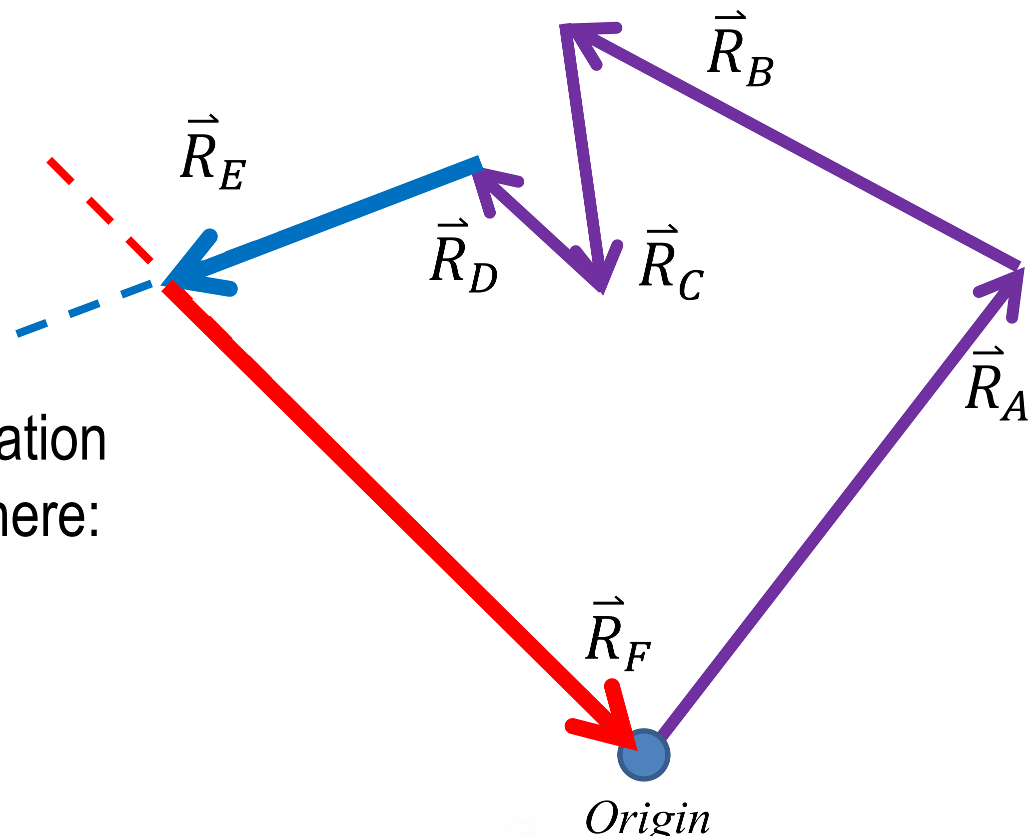
Based on known vectors, a vector polygon can be drawn to scale. Since the polygon is not closed yet, the two other vectors $\vec{R}_E$ and $\vec{R}_F$ are needed to return to the Origin, O.



VECTOR EQUATIONS

If done graphically, it is best to use full scale. Then the magnitude and angles of unknown vectors can be measured. Alternatively, scale down the polygon carefully.

$$\vec{R}_A + \vec{R}_B + \vec{R}_C + \vec{R}_D = -\vec{R}_E - \vec{R}_F$$

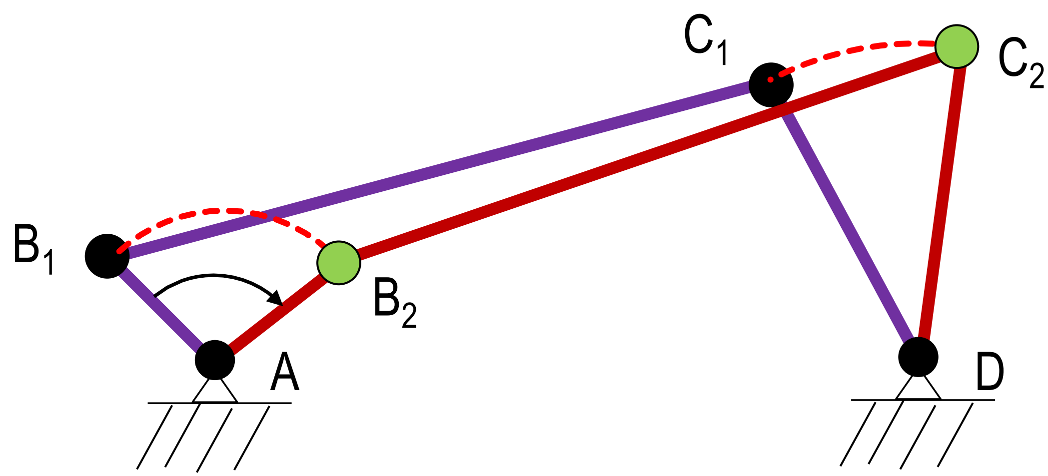


Analytically, this loop equation needs to be expanded where:

$$\vec{R}_i = r_i \begin{Bmatrix} \cos\theta_i \\ \sin\theta_i \end{Bmatrix}$$

GRAPHICAL METHOD

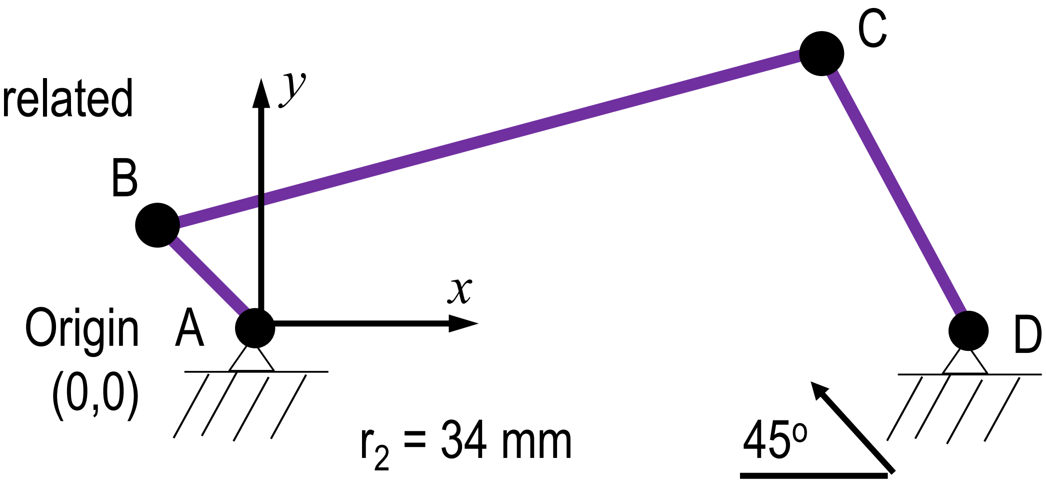
The aim is to find the position of all points and links, as the driver link 2 changes position.



Oftentimes, there is one point-of-interest (POI) that needs to be monitored.

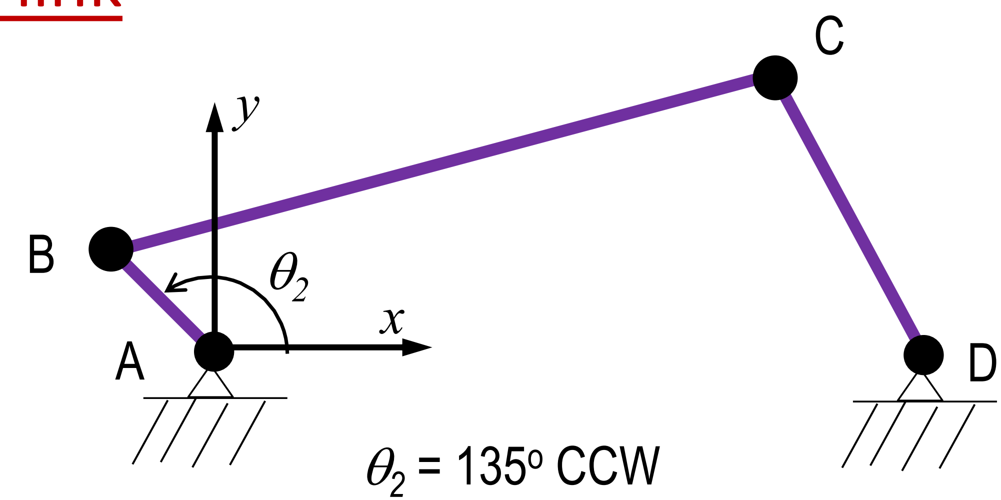
Location of a point

Usually the position must be related to the Origin.



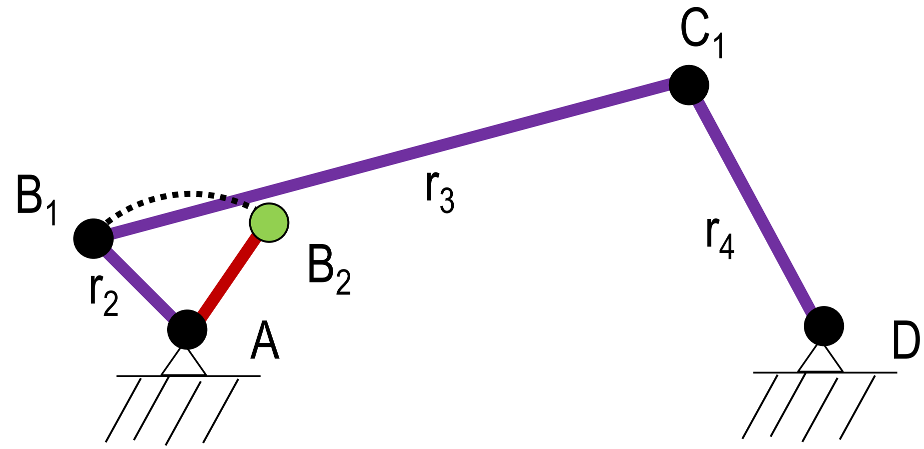
Angular position of a link

Angular direction of a link in a reference axis.

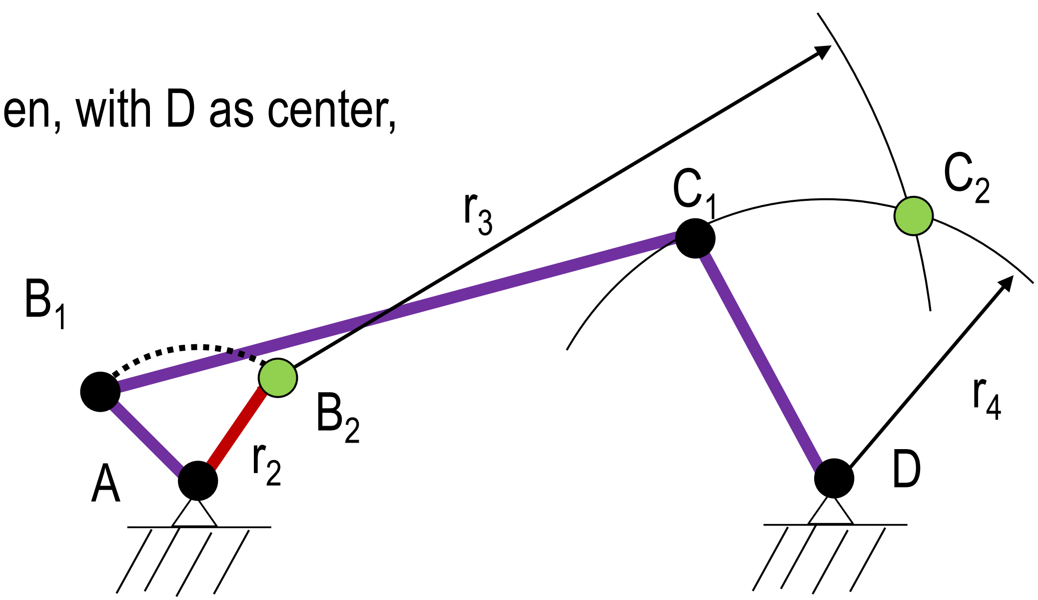


GRAPHICAL POSITION ANALYSIS

Identify the new position, B_2.

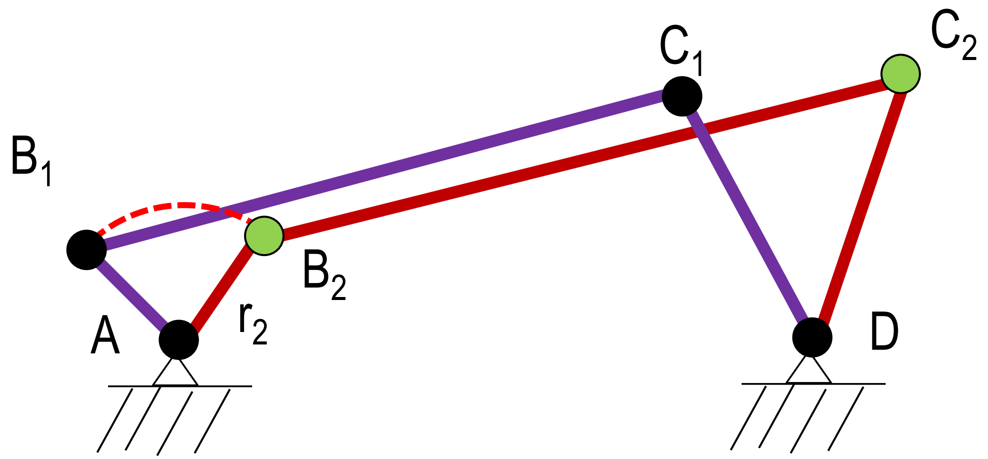


From B_2 make an arc with r_3. Then, with D as center, make another arc with r_4.



GRAPHICAL METHOD: COMPLETE THE DRAWING

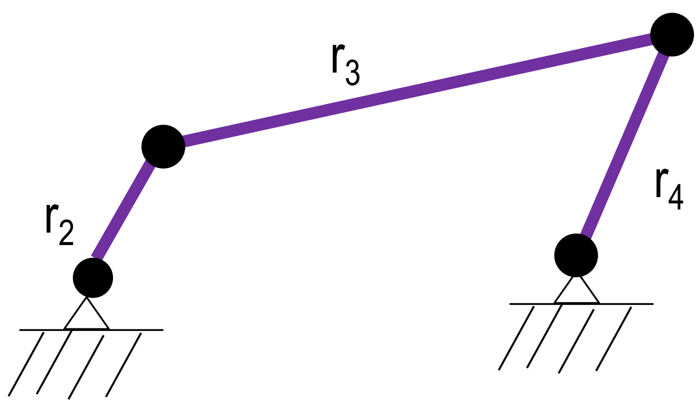
Lastly, complete the mechanism with links without altering the shape and size of any of them.



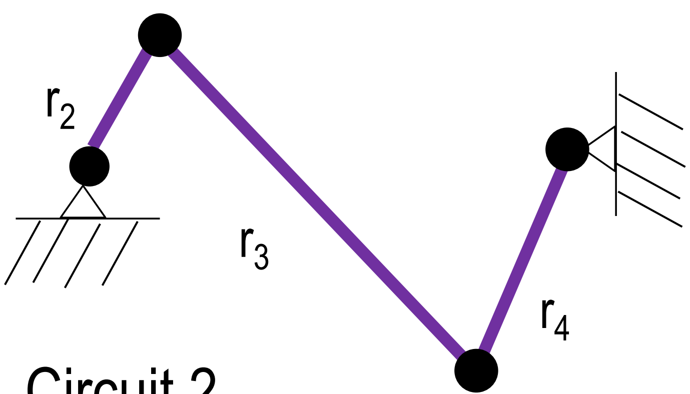
KINEMATIC CIRCUITS

This happens when the type of motion is crank-rocker or rocker-crank. The mechanism cannot cross circuit unless force is high or the pin at C is taken out and reassembled in Circuit 2.

There are two possible circuits:



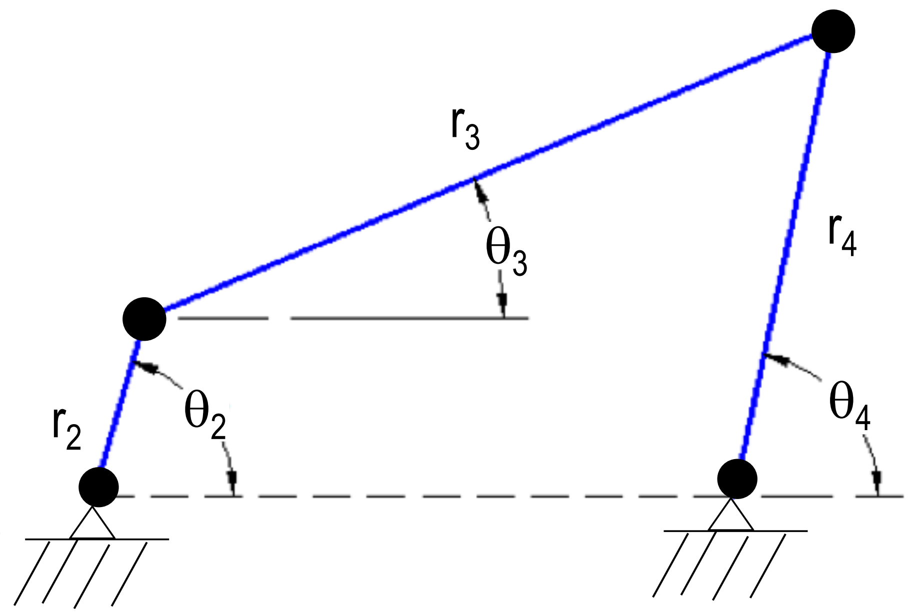
Circuit 1



Circuit 2

ANALYTICAL METHOD

CIRCUIT 1



$$BD = \sqrt{{r_1}^2 + {r_2}^2 - 2r_1r_2\cos\theta_2}$$

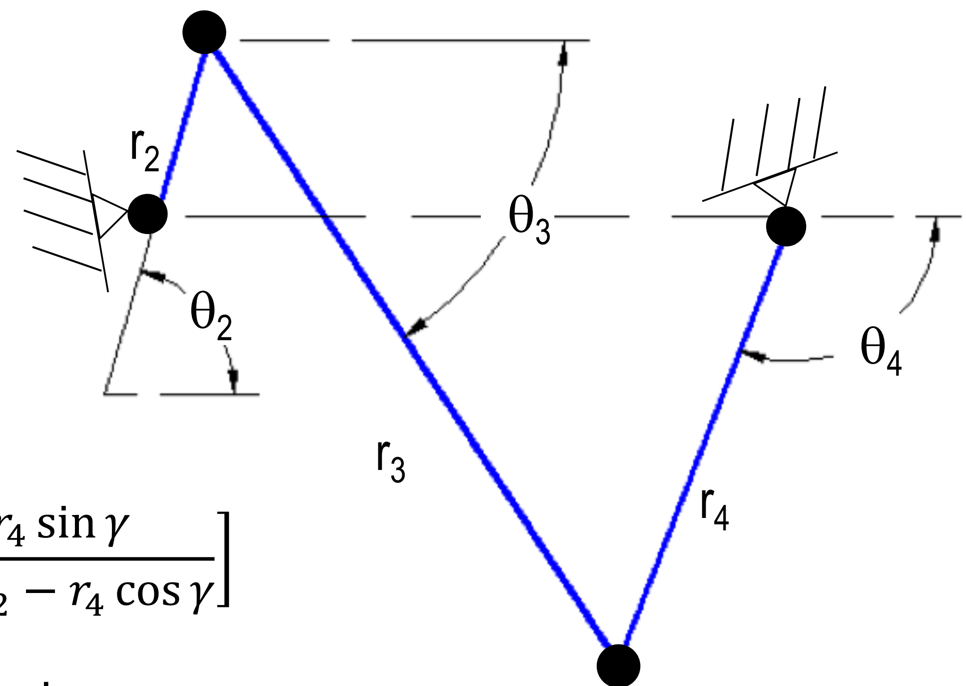
$$\gamma = \cos^{-1}\left[\frac{{r_3}^2+{r_4}^2-BD^2}{2r_3r_4}\right]$$

$$\theta_3 = 2\tan^{-1}\left[\frac{-r_2\sin\theta_2 + r_4\sin\gamma}{r_1 + r_3 - r_2\cos\theta_2 - r_4\cos\gamma}\right]$$

$$\theta_4 = 2\tan^{-1}\left[\frac{r_2\sin\theta_2 - r_3\sin\gamma}{r_4 - r_1 + r_2\cos\theta_2 - r_3\cos\gamma}\right]$$

ANALYTICAL METHOD

CIRCUIT 2



$$\theta_3 = 2\tan^{-1}\left[\frac{-r_2 \sin\theta_2 - r_4 \sin\gamma}{r_1 + r_3 - r_2 \cos\theta_2 - r_4 \cos\gamma}\right]$$

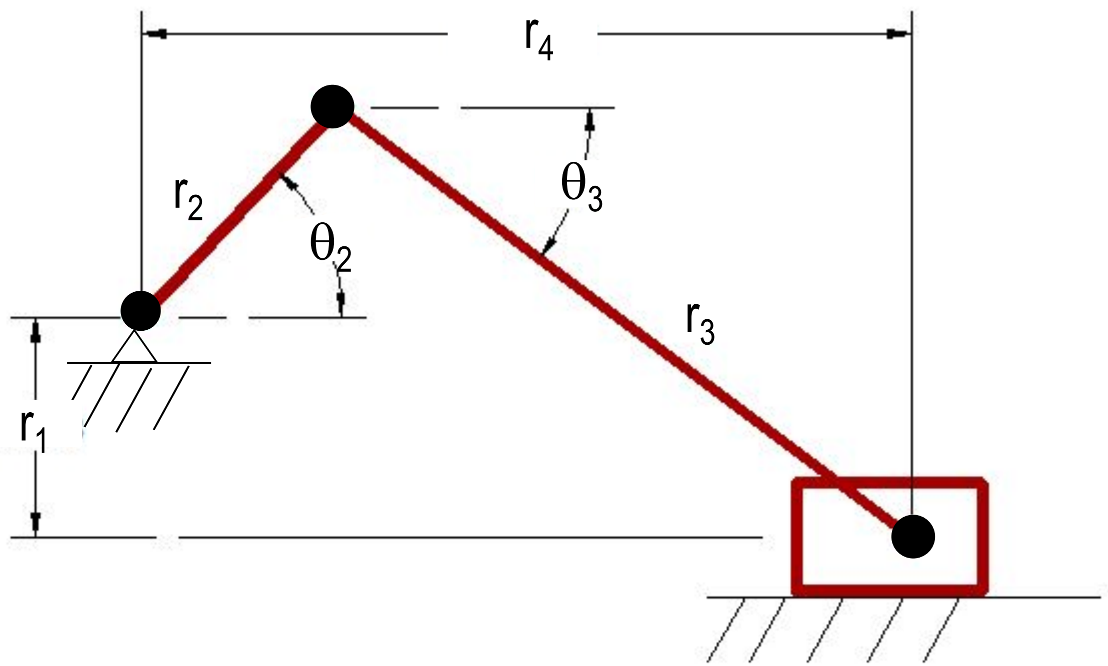
$$\theta_4 = 2\tan^{-1}\left[\frac{r_2 \sin\theta_2 + r_3 \sin\gamma}{r_4 - r_1 + r_2 \cos\theta_2 - r_3 \cos\gamma}\right]$$

GENERAL SLIDER-CRANK

This offset type. For inline type, consider $r_1 = 0$.

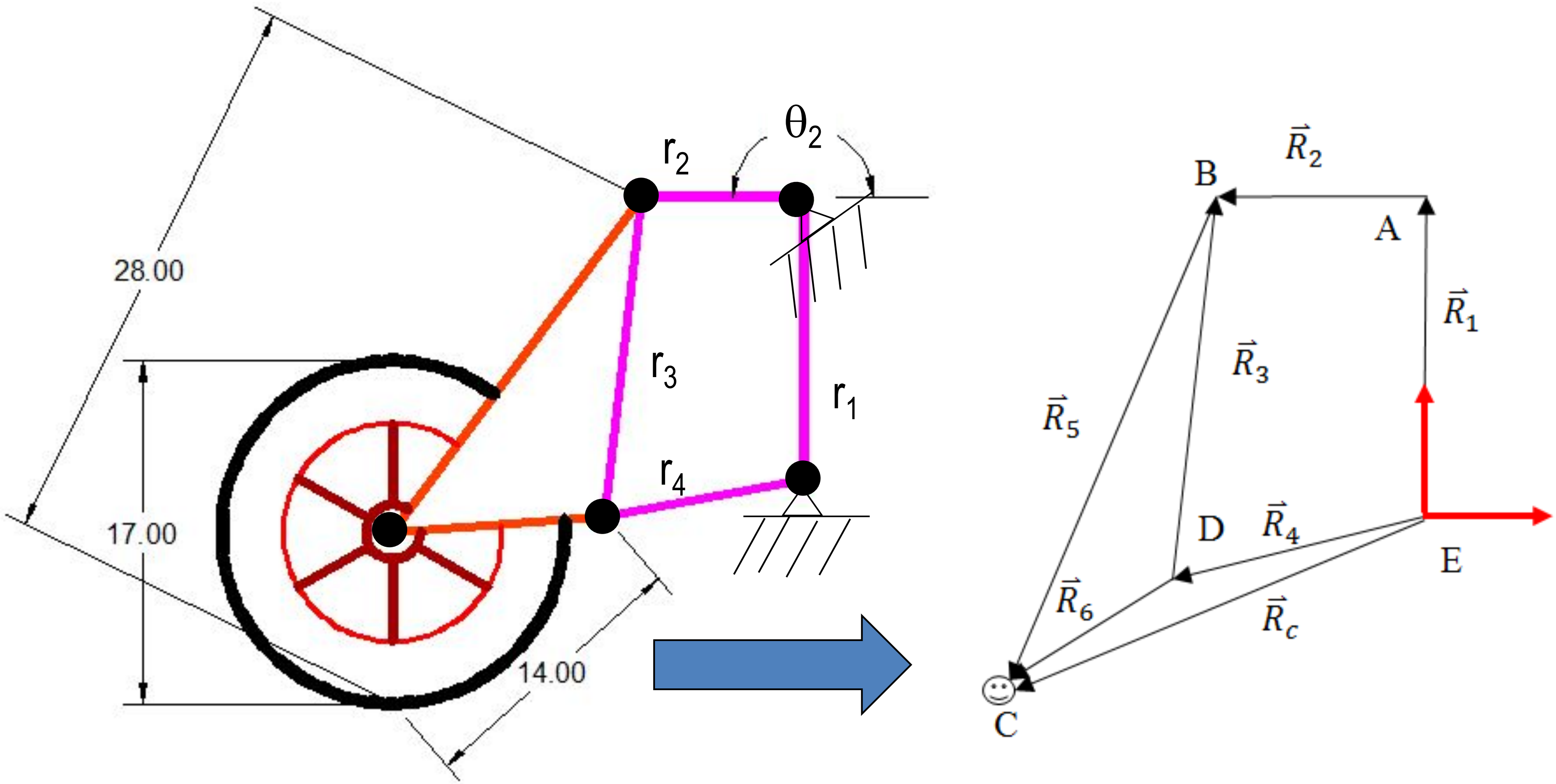
$$\theta_3 = \sin^{-1}\left[\frac{r_1 + r_2 \sin\theta_2}{r_3}\right]$$

$$r_4 = r_2 \cos\theta_2 + r_3 \cos\theta_3$$



VECTOR ANALYSIS - BRIEF

It is a more general type of solution. Identify vector loop equations.



EXAMPLE USING VECTOR APPROACH

Find the location of the bottom of the wheel when the upper control arm turns 15° CCW.

Solution

Fixed variables:

$\alpha = 137.82°,\ \theta_1 = 90°,\ r_1,\ r_2,\ r_3,\ r_4,\ r_5,\ r_6$

Varying variables:

$\theta_2 = 195°,\ \theta_3,\ \theta_4,\ \vec{R}_5,\ \theta_6, \vec{R}_c$

Loops:

$\vec{R}_4 + \vec{R}_3 = \vec{R}_1 + \vec{R}_2$ (θ_3, θ_4) - unknown

$$\vec{R}_5 = -\vec{R}_3 + \vec{R}_6$$

$$\vec{R}_c = \vec{R}_4 + \vec{R}_6$$

Known:

$\vec{R}_0 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \vec{R}_1 = \begin{Bmatrix} 0 \\ 14 \end{Bmatrix}$, $\theta_2 = 195°$, $\theta_1 = 90°$ $r_2 = 8$, $r_3 = 16$, and $r_4 = 10$.

Expansion:

$$10\begin{Bmatrix} \cos(\theta_4) \\ \sin(\theta_4) \end{Bmatrix} + 16\begin{Bmatrix} \cos(\theta_3) \\ \sin(\theta_3) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 14 \end{Bmatrix} + 8\begin{Bmatrix} \cos(195°) \\ \sin(195°) \end{Bmatrix}$$

$$= \begin{Bmatrix} -7.73 \\ 11.93 \end{Bmatrix}$$

$$\vec{R}_H = \vec{R}_1 + \vec{R}_2 = \begin{Bmatrix} -7.73 \\ 11.93 \end{Bmatrix}$$

$$\theta_H = \tan^{-1}\left(\frac{11.93}{-7.73}\right) + 180$$

$$= 123°$$

$$r_H = \sqrt{7.73^2 + 11.93^2} = 14.22$$

$$r_3{}^2 = r_H{}^2 + r_4{}^2 - 2r_4 r_H \cos\varphi$$

$$\varphi = \cos^{-1}\left(\frac{r_H{}^2 + r_4{}^2 - r_3{}^2}{2r_4 r_H}\right)$$

$$= \cos^{-1}\left(\frac{14.22^2 + 10^2 - 16^2}{2(10)(14.22)}\right)$$

$$= 80.65°$$

$$10\begin{Bmatrix} c(203.65) \\ s(203.65) \end{Bmatrix} + 16\begin{Bmatrix} c\theta_3 \\ s\theta_3 \end{Bmatrix} = \begin{Bmatrix} -7.73 \\ 11.93 \end{Bmatrix}$$

$$\begin{Bmatrix} c\theta_3 \\ s\theta_3 \end{Bmatrix} = \begin{Bmatrix} 0.0894 \\ 0.9963 \end{Bmatrix}$$

$$\theta_3 = 84.87°$$

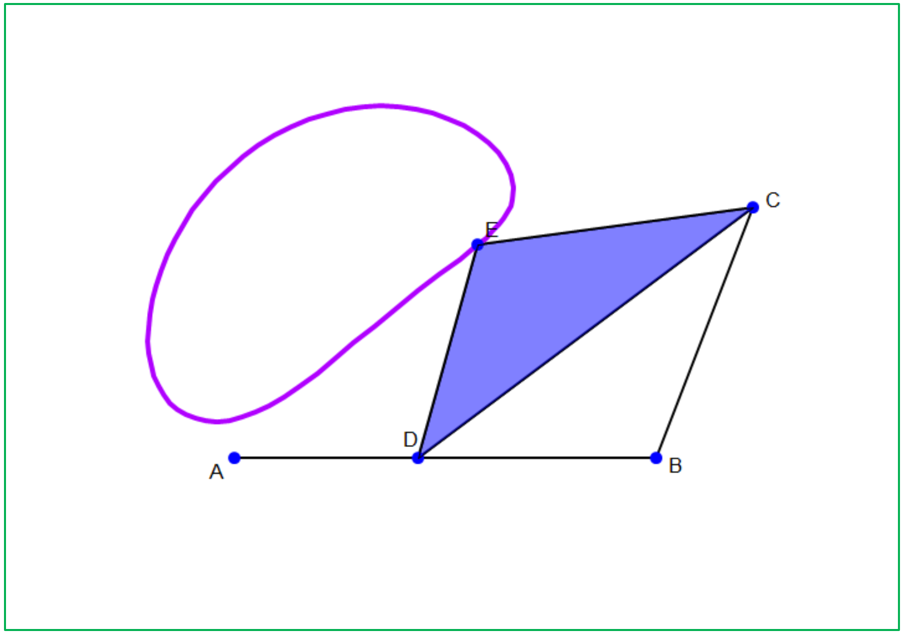
$$\theta_6 = 84.87° + 137.82° = 222.69°$$

$$\vec{R}_c = 10\begin{Bmatrix} c(203.65) \\ s(203.65) \end{Bmatrix} + 14\begin{Bmatrix} c(222.69) \\ s(222.69) \end{Bmatrix}$$
$$= \begin{Bmatrix} -19.45 \\ -13.50 \end{Bmatrix}$$

THANK YOU

Main Reference:

Myszka, David H., 2012. Machines and mechanism: applied kinematic analysis, 4th ed., Prentice Hall, New York.



Source:
https://commons.wikimedia.org/wiki/File:4_bar_linkage_animated.gif