ADVANCED ELECTRICAL CIRCUIT
BETI 1333
SECOND ORDER SOURCE-FREE SERIES RLC CIRCUIT

Halyani binti Mohd Yassim
halyani@utem.edu.my

LESSON OUTCOME

At the end of this chapter, students are able:

- to describe second order source-free series RLC circuit

SUBTOPICS

- Introduction of Second Order Circuit
- Source-free Series RLC Circuit

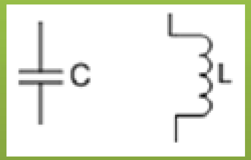
INTRODUCTION

$\frac{d^2v}{dt^2}$ or $\frac{d^2i}{dt^2}$

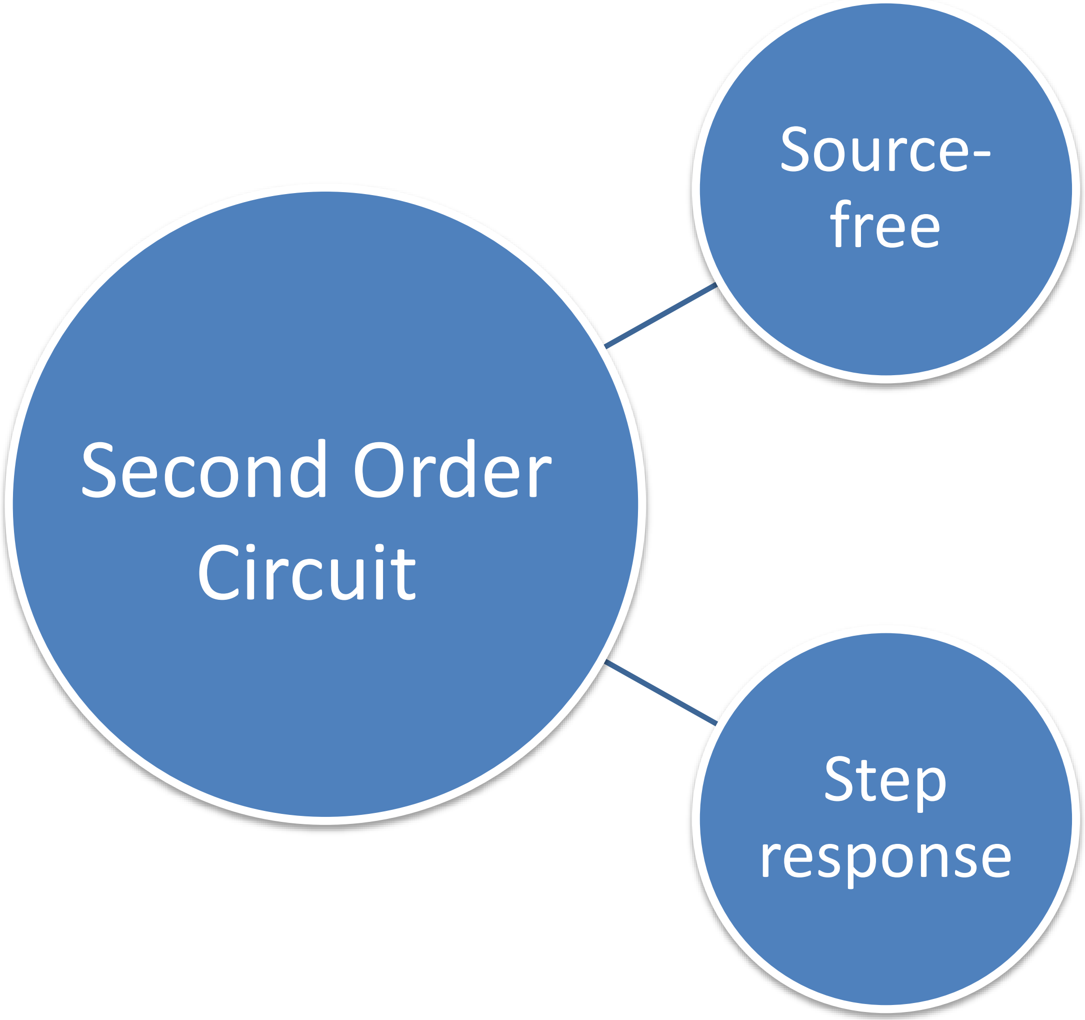
1. Using second order differential equation

Second Order Circuit

2. contains two storage elements



INTRODUCTION



Source-free:
- Series
- Parallel

Step response:
- Series
- Parallel

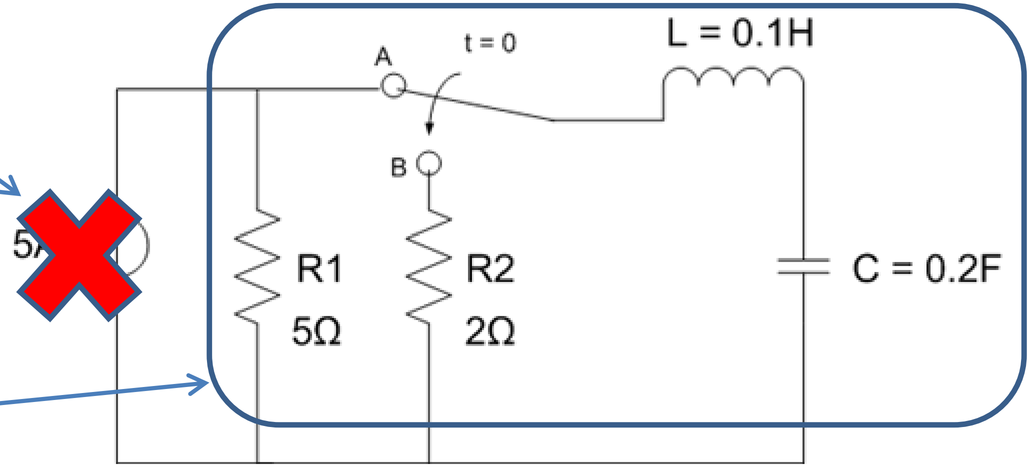
SOURCE-FREE SERIES RLC CIRCUIT

What?

1. No external sources of excitation

2. Contains several resistors, a capacitor and an inductor

3. Output response: Transient/natural response



SOURCE-FREE SERIES RLC CIRCUIT

Source-free Series RLC Circuit

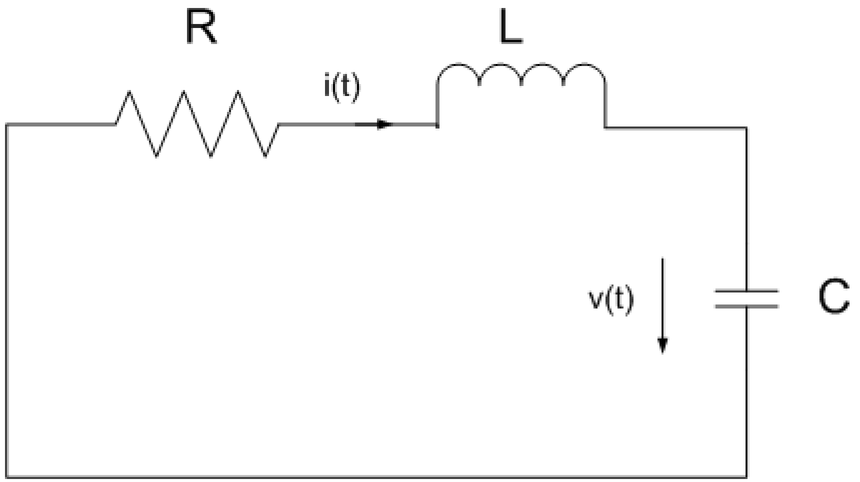


Figure 1

Assumption:

$i(0) = I_0$ and $v(0) = \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt = V_0$

Initial current through inductor

Initial voltage across capacitor

By Applying Kirchhoff's Voltage Law:

$$V_R + V_L + V_C = 0$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt = 0$$

Second order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

SOURCE-FREE SERIES RLC CIRCUIT

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Roots of the characteristic equation

$$\alpha = \frac{R}{2L},$$

Damping factor

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Undamped natural frequency

Types of natural response of source-free series RLC circuit:

1. Overdamped response ($\alpha > \omega_0$)

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

2. Critically damped response ($\alpha = \omega_0$)

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$

3. Underdamped response ($\alpha < \omega_0$)

$$i(t) = e^{-\alpha t}(B_1 cos\ \omega_d t + B_2\ sin\ \omega_d t)$$

Note: A_1 and A_2 can be determined from the initial conditions namely $i(0)$ and $\frac{di(0)}{dt}$.

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$

EXAMPLE 1

The switch in Figure 2 moves from position A to B at t = 0. Find i(t) for t > 0.

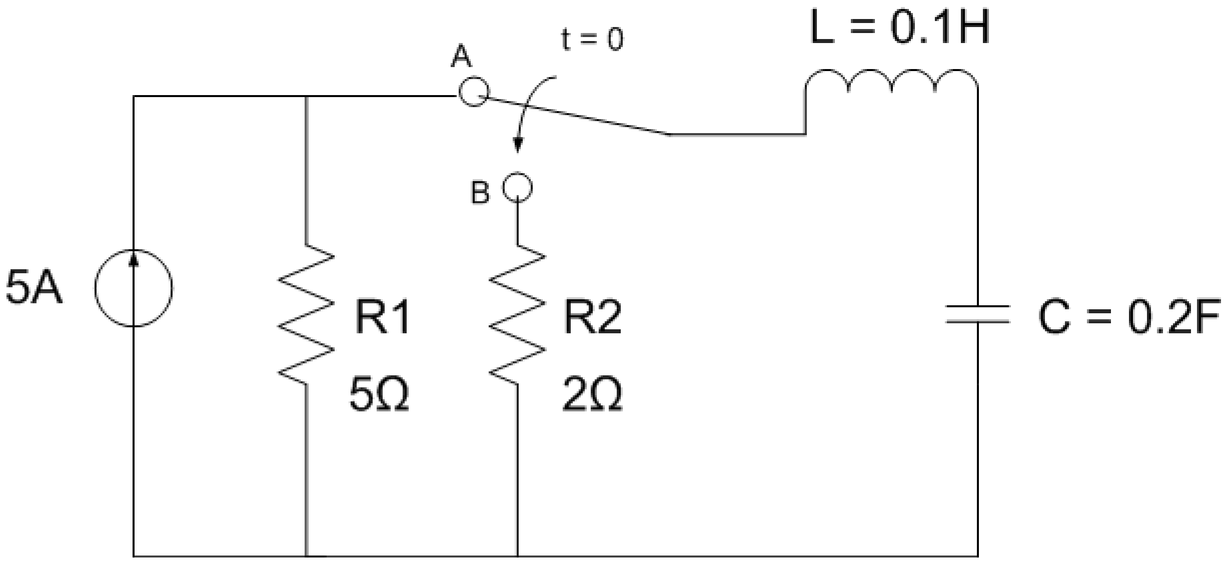


Figure 2

SOLUTION 1

Step 1: Find initial voltage across capacitor, V_0 and initial current through inductor, I_0 when t < 0.

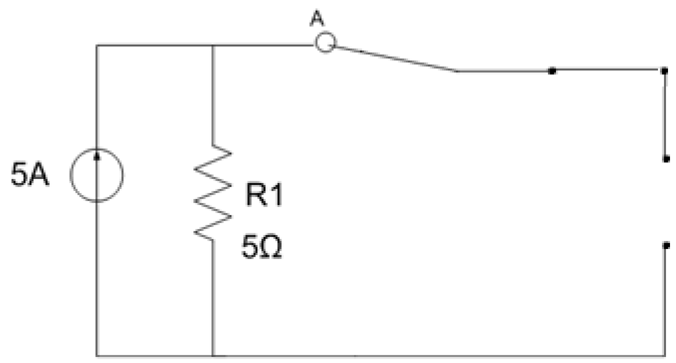


Figure 3

$i(0) = 0A$
$v(0) = iR = 5A * 5\Omega = 25V$

Tips 1:

When t < 0, capacitor acts like an open circuit and inductor acts like a short circuit.

Tips 2:

There is no current flow through inductor due to open circuit.

SOLUTION 1

Step 2: Determine type of natural response of this circuit when t > 0.

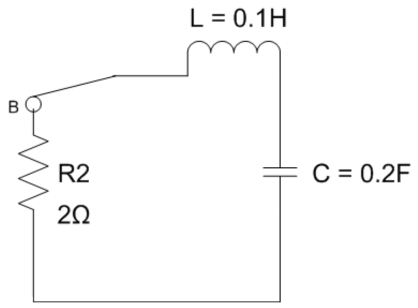


Figure 4

$$\alpha = \frac{R}{2L} = \frac{2}{2 * 0.1} = 10, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 * 0.2}} = 7.071$$

$\alpha > \omega_0 \rightarrow$ overdamped response

Current response for overdamped case:
$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

SOLUTION 1

Step 3: Find roots of characteristic equation for this circuit.

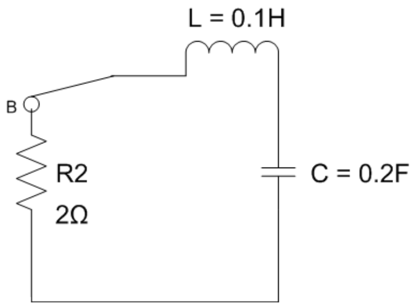


Figure 5

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\frac{2}{2 * 0.1} \pm \sqrt{\left(\frac{2}{2 * 0.1}\right)^2 - \frac{1}{0.1 * 0.2}}$$

$$s_{1,2} = -0.36, \qquad -19.64$$

SOLUTION 1

Step 4: Determine A_1 and A_2 from initial conditions

$i(0)$ and $\frac{di(0)}{dt}$, when t > 0.

$$i(0) = A_1 e^{-0.36(0)} + A_2 e^{-19.64(0)} = 0A$$

$$A_1 + A_2 = 0 \rightarrow A_1 = -A_2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -\frac{1}{0.1}(2(0) + 25) = -250\frac{A}{s}$$

$$\frac{di}{dt} = -0.36 * A_1\, e^{-0.36t} - 19.64 * A_2 e^{-19.64t}$$

$$\frac{di(0)}{dt} = -0.36 * A_1 e^{-0.36(0)} - 19.64 * A_2 e^{-19.64(0)} = -250$$

$$0.36 * A_2 - 19.64 * A_2 = -250 \rightarrow A_2 = 13$$

$$\rightarrow A_1 = -13$$

Current response:

$$i(t) = -13e^{-0.36t} + 13e^{-19.64}A$$

EXAMPLE 2

Calculate i(t) for t > 0 in the circuit of Figure 6.

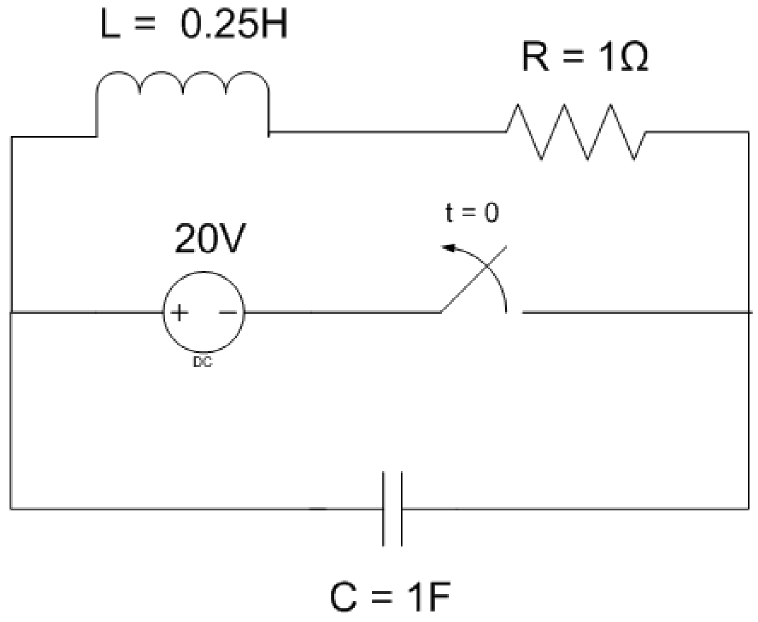


Figure 6

SOLUTION 2

Step 1: Find initial voltage across capacitor, V_0 and initial current through inductor, I_0 when t < 0.

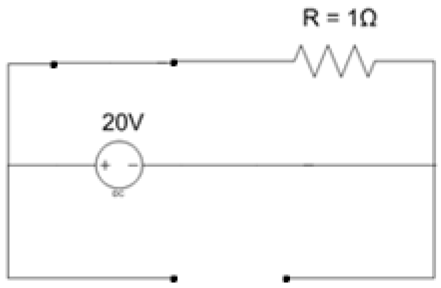


Figure 7

$$V_0 = 20V$$
$$I_0 = \frac{V}{R} = \frac{20V}{1\Omega} = 20A$$

Tips 1:

When t < 0, capacitor acts like an open circuit and inductor acts like a short circuit.

Tips 2:

Current flows through inductor, which is less resistance than capacitor.

SOLUTION 2

Step 2: Determine type of natural response of this circuit when t > 0.

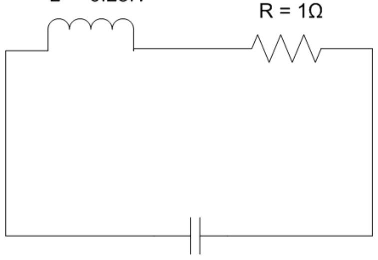


Figure 8

$$\alpha = \frac{R}{2L} = \frac{1}{2 * 0.25} = 2, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25 * 1}} = 2$$

$\alpha = \omega_0 \rightarrow$ critically damped response

Current response for critically damped case:

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$

SOLUTION 2

Step 3: Determine A_1 and A_2 from initial conditions

$i(0)$ and $\frac{di(0)}{dt}$, when t > 0.

$$i(0) = (A_2 + A_1(0))e^{-2(0)} = 20A, \qquad \rightarrow A_2 = 2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -\frac{1}{0.25}(1(20) + 20) = -160\frac{A}{s}$$

$$\frac{di}{dt} = -2(A_2 + A_1 t)e^{-2t} + A_1 e^{-2t}$$

$$\frac{di(0)}{dt} = -2(2 + A_1(0))e^{-2(0)} + A_1 e^{-2(0)} = -160$$

$$-4 + A_1 = -160 \quad \rightarrow A_1 = -156$$

Current response:

$$i(t) = (2 - 156t)e^{-2t}A$$

SELF REVIEW QUESTIONS

1. A source-free RL circuit is an example of second order circuit.

   TRUE FALSE

2. Name two storage elements in a second order source-free RLC circuit.

   Answer: _______________

3. Given R = 10Ω and C = 0.5F. Determine the value of L so that the natural response of source-free series RLC circuit is critically damped response.

   a) 1.25H b) 12.5H

   c) 125H d) 0.125H

4. Following is TRUE about a second order circuit **except**

a) consists of a resistor and two capacitors in a circuit

b) is described using second order differential equation

c) consists of one storage elements

d) has three types of natural response

5. Given L = 2H and C = 4F. Interpret the value of R when $\alpha > \omega_0$.

   a) R > 2 b) R < 0.5

   c) R > 0.2 d) R < 5

ANSWERS

1. FALSE
2. Inductor and capacitor
3. b
4. c
5. a