UNIVERSITI TEKNIKAL MALAYSIA MELAKA

OPENCOURSEWARE

ADVANCED ELECTRICAL CIRCUIT

BETI 1333

FIRST ORDER STEP RESPONSE RC CIRCUIT

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LESSON OUTCOMES

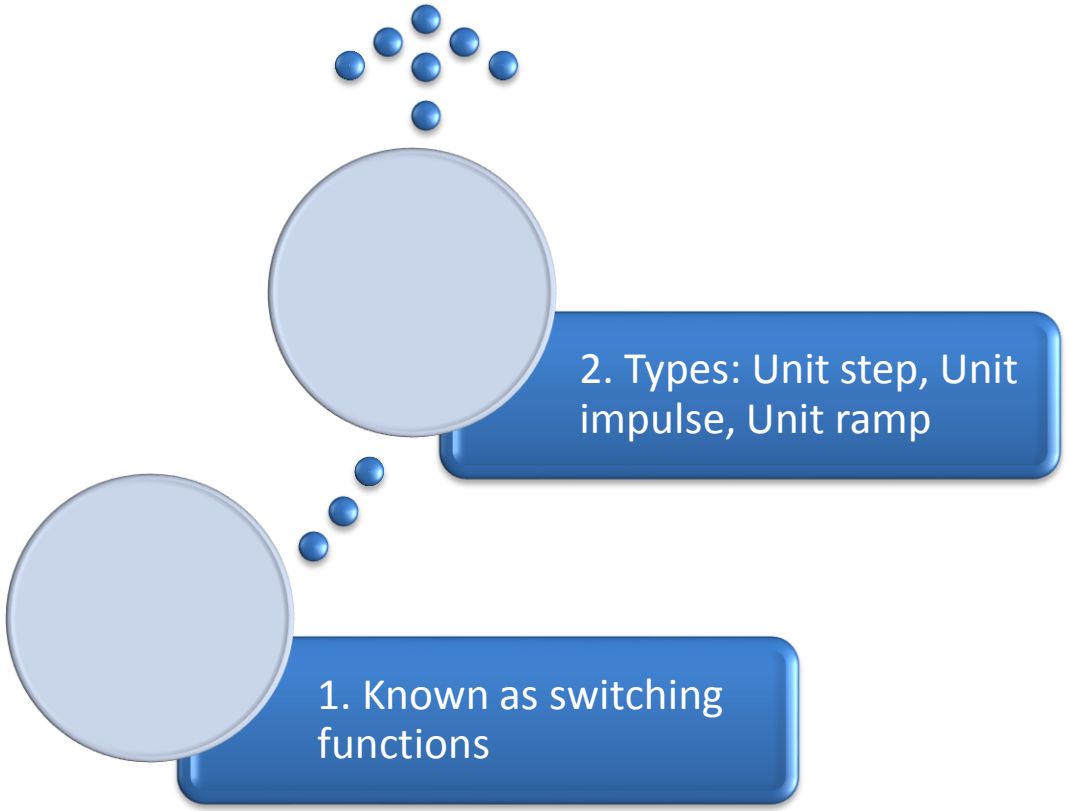
At the end of this chapter, students are able:

- to describe first order step response RC circuit
- to illustrate output response of first order step response RC circuit

SUBTOPICS

- Singularity Functions
- Step response RC Circuit

SINGULARITY FUNCTIONS



SINGULARITY FUNCTIONS

Voltage Source

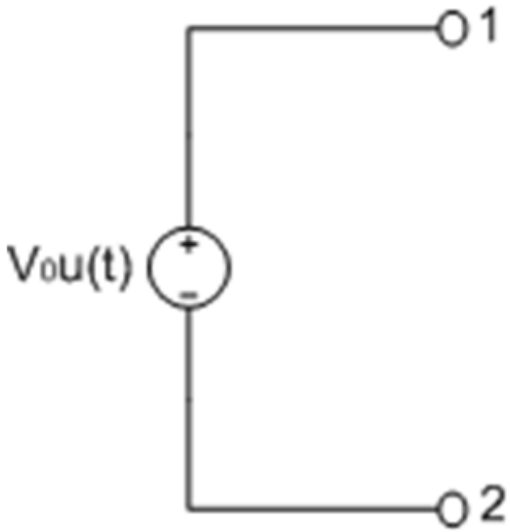


Figure 1

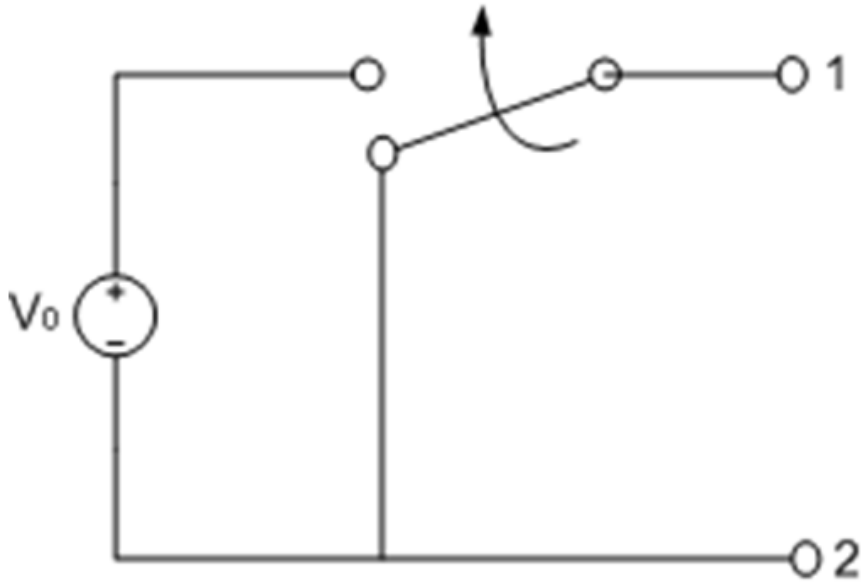


Figure 2

SINGULARITY FUNCTIONS

Current Source

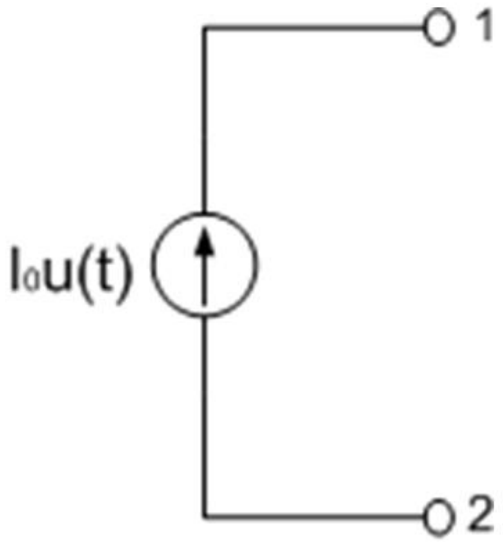


Figure 3

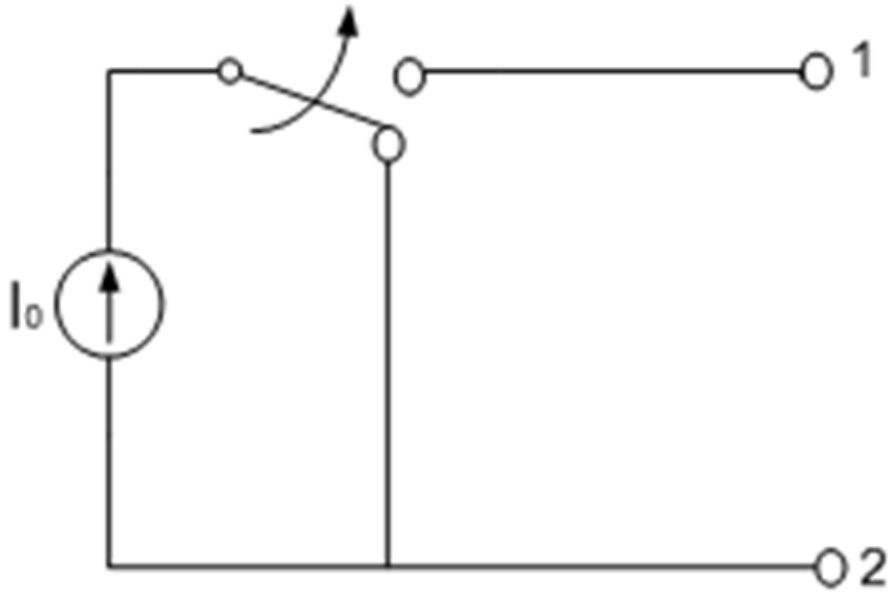


Figure 4

SINGULARITY FUNCTIONS

Unit Step Function:

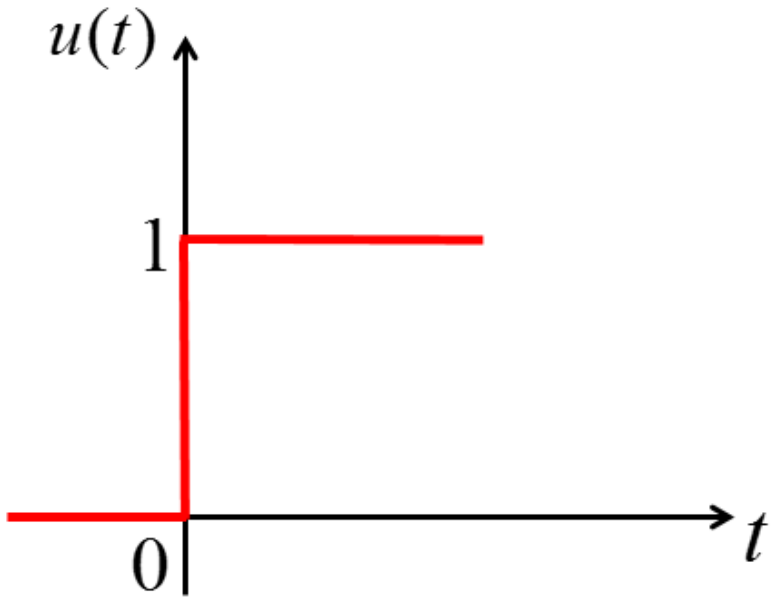


Figure 5

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

SINGULARITY FUNCTIONS

Unit Impulse Function:

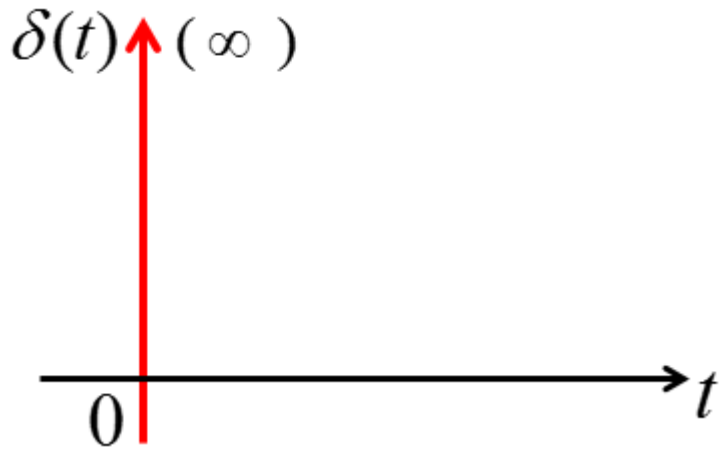


Figure 6

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ Undefined, & t = 0 \\ 0, & t > 0 \end{cases}$$

SINGULARITY FUNCTIONS

Unit Ramp Function:

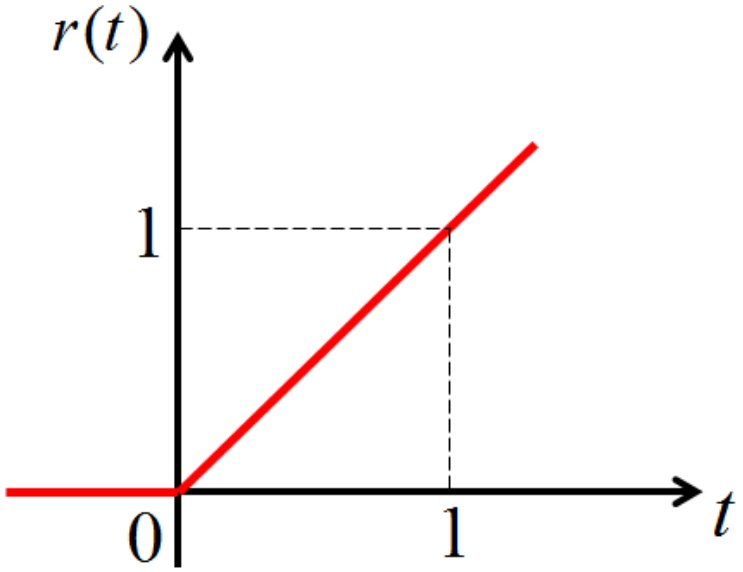
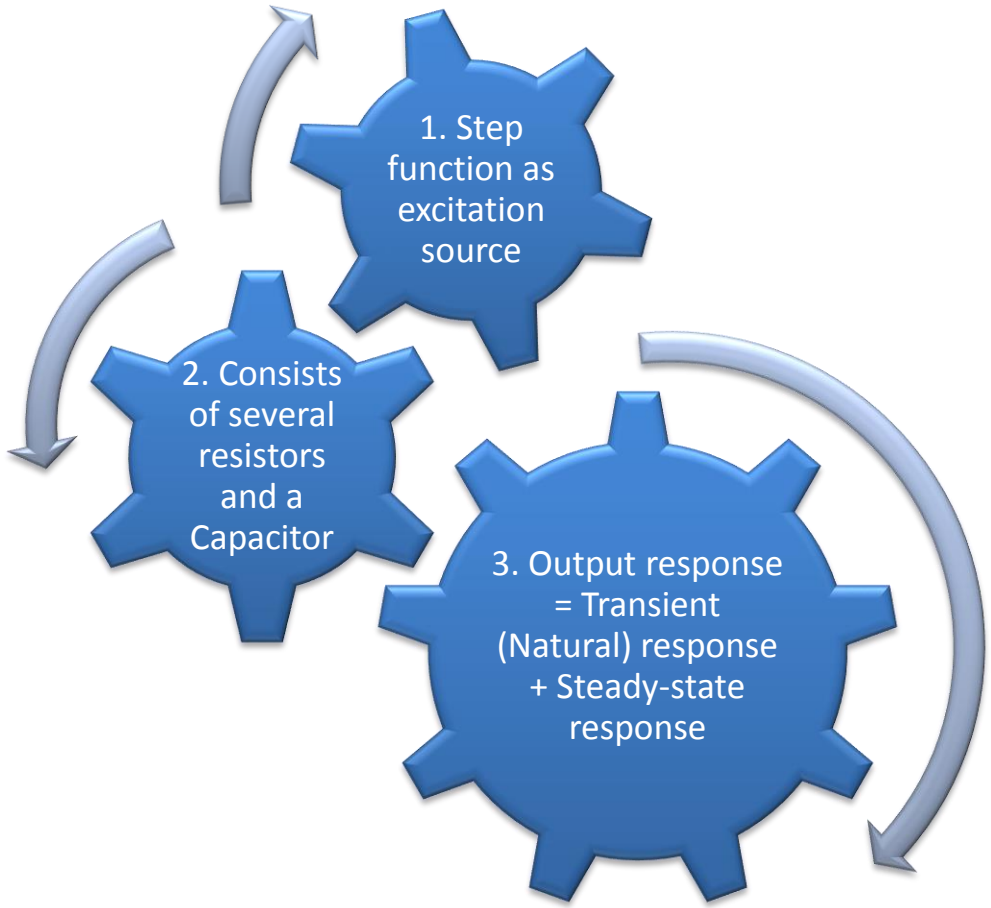


Figure 7

$$r(t) = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$$

STEP RESPONSE RC CIRCUIT

W H A T ?



STEP RESPONSE RC CIRCUIT

SERIES RC CIRCUIT

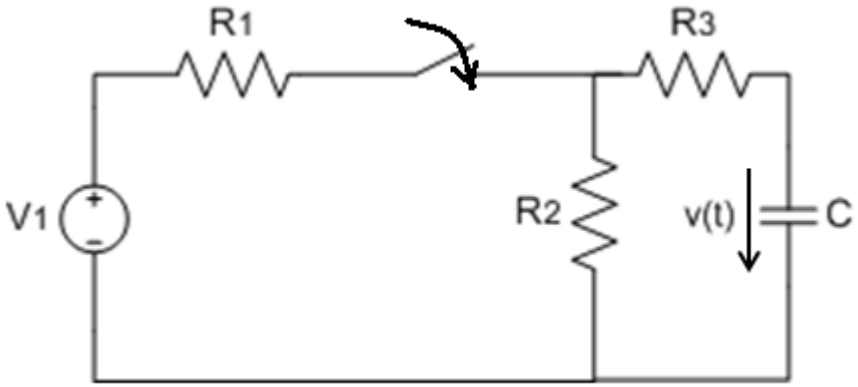


Figure 8

PARALLEL RC CIRCUIT

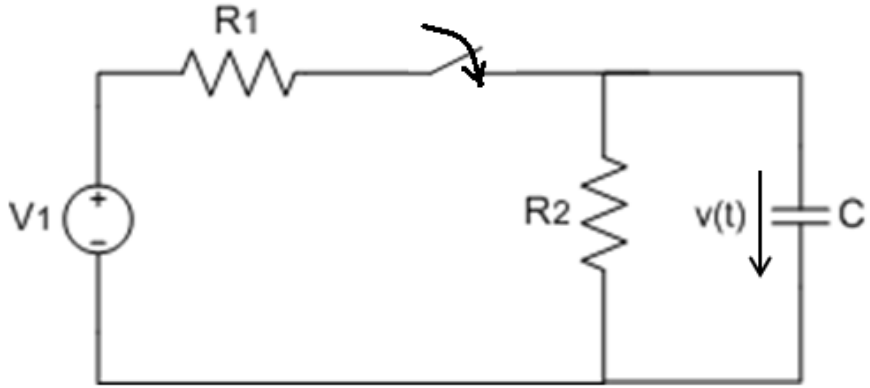


Figure 9

STEP RESPONSE RC CIRCUIT

RC Circuit:

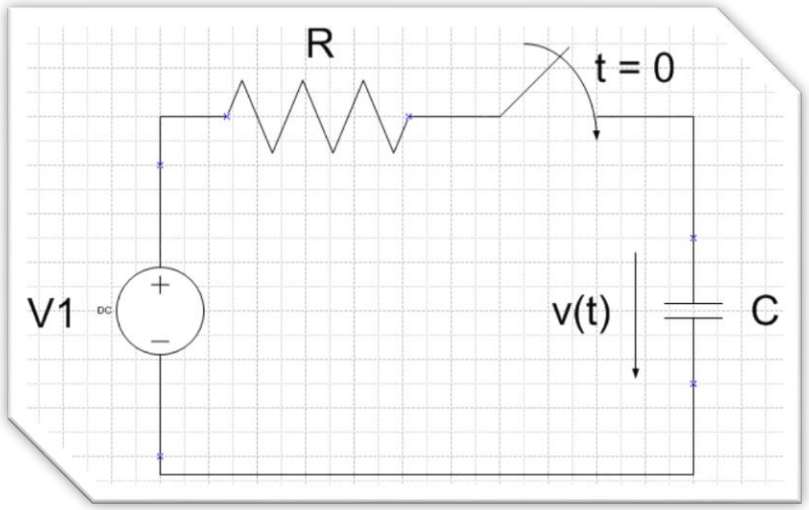


Figure 10

Output response:

$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$

- $v(\infty)$: Final voltage across capacitor, when $t = \infty$
- $v(0)$: Initial voltage across capacitor, when $t = 0$
- τ: Time constant ($\tau = RC$)

EXAMPLE 1

The switch in Figure 11 is closed at t = 0.5 s. Find $v(t)$ for t > 0.5 s.

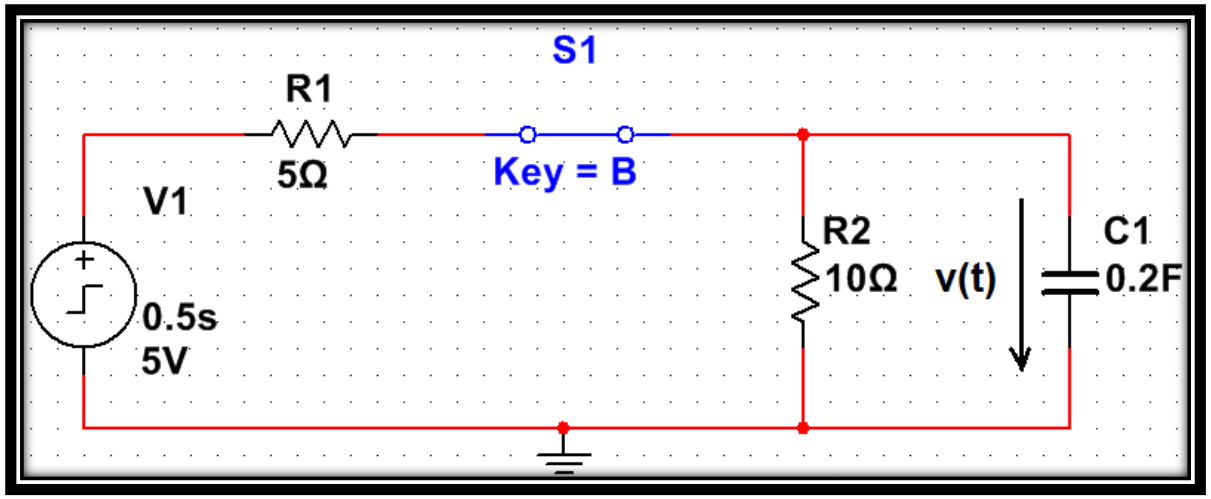


Figure 11

SOLUTION 1

Step 1: Find initial voltage across capacitor when t< 0.5 s. Below is the circuit when switch is opened for t < 0.5 s.

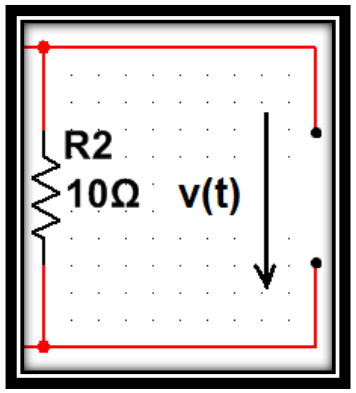


Figure 12

$$v(t) = v(0.5) = 0\ V$$

Tips 1:

When t < 0.5 s, capacitor acts like an open circuit.

SOLUTION 1

Step 2: Find time constant, τ for t > 0.5 s.

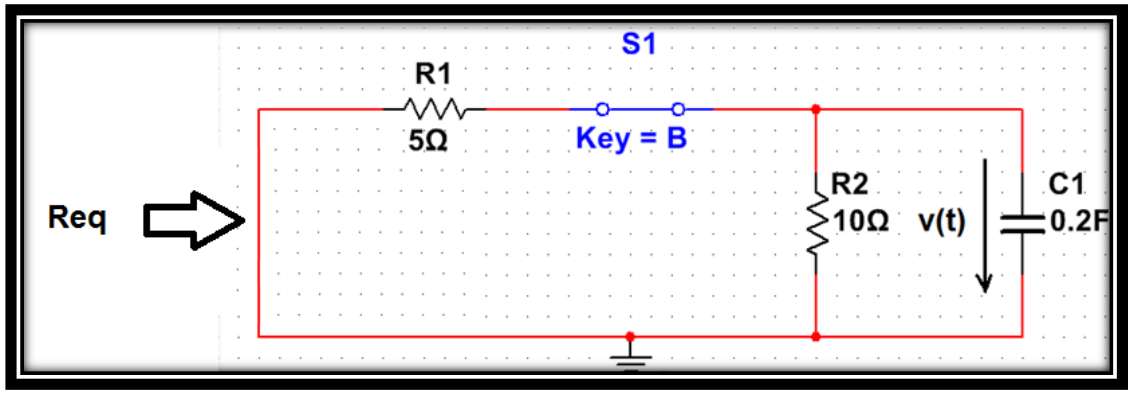


Figure 13

$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2} = \frac{5\Omega * 10\Omega}{(5+10)\Omega} = 3.33\Omega$$

$$\tau = R_{eq}C = 3.33\Omega * 0.2F = 0.67s$$

Tips 2:

Remove voltage source to find equivalent resistance, R_{eq}.

SOLUTION 1

Step 3: Find final voltage across capacitor when t = ∞.
Below is the circuit when switch is closed for t > 0.5 s.

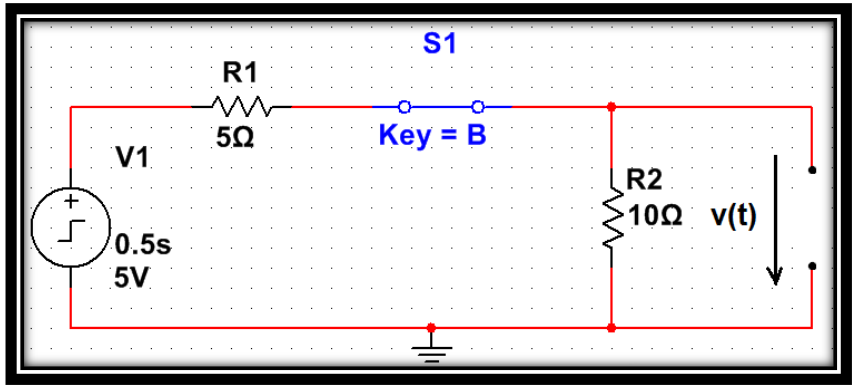


Figure 14

$$v(\infty) = \frac{R_2}{R_2 + R_1} V_1 = \frac{10\Omega}{(10+5)\Omega} 5V = 3.33V$$

Tips 2:
At dc steady state, capacitor is an open circuit.

SOLUTION 1

Step 4: Find voltage across capacitor when
t > 0.5 s.

$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$

$$v(t) = (3.33 - 3.33e^{-1.5t})V$$

OUTPUT RESPONSE 1

Voltage response of an RC circuit in Figure 15:

V(1) : Step function as voltage source

V(5) : Voltage response across capacitor

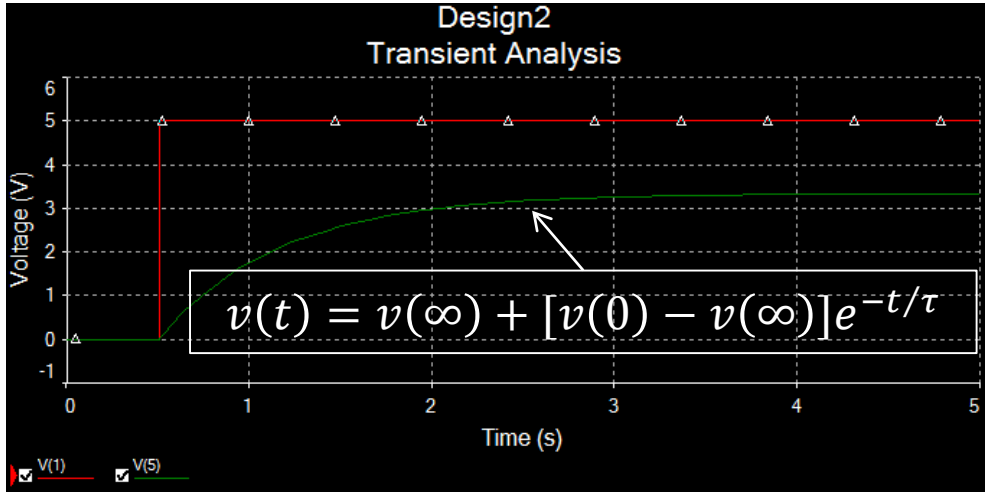


Figure 15

SELF REVIEW QUESTIONS

1. Switch Function is also known as _____
   a) Unit Step Function
   b) Singularity Function
   c) Unit Ramp Function
   d) Unit Impulse Function

2. State the behavior of step response of RC circuit.
   Answer: _____

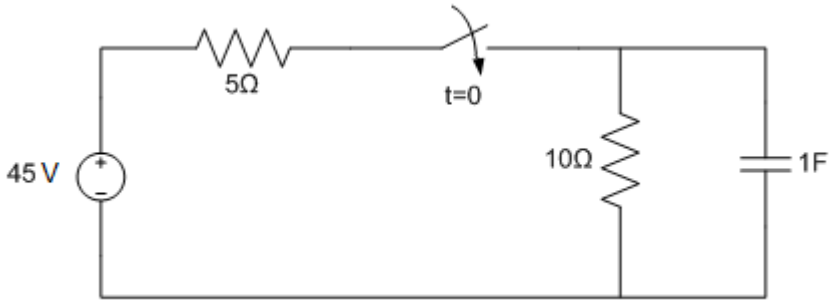


Figure 16

3. Find v(∞) for circuit in Figure 16.
   a) 30 V b) 35 V
   c) 60 V d) 65 V

4. Find the time constant, τ in Figure 16.
   a) 4.44 s b) 3.33s
   c) 5.55 s d) 2.22 s

5. Based on circuit in Figure 16, calculate the time required to reach half of its steady state value?
   a) 1.23 s b) 2.31 s
   c) 3.12 s d) 1.32 s

ANSWERS

1. b
2. Natural response and steady state response
3. a
4. b
5. b