UNIVERSITI TEKNIKAL MALAYSIA MELAKA

OPENCOURSEWARE

ADVANCED ELECTRICAL CIRCUIT
BETI 1333
FIRST ORDER SOURCE-FREE RC AND RL CIRCUIT

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LESSON OUTCOMES

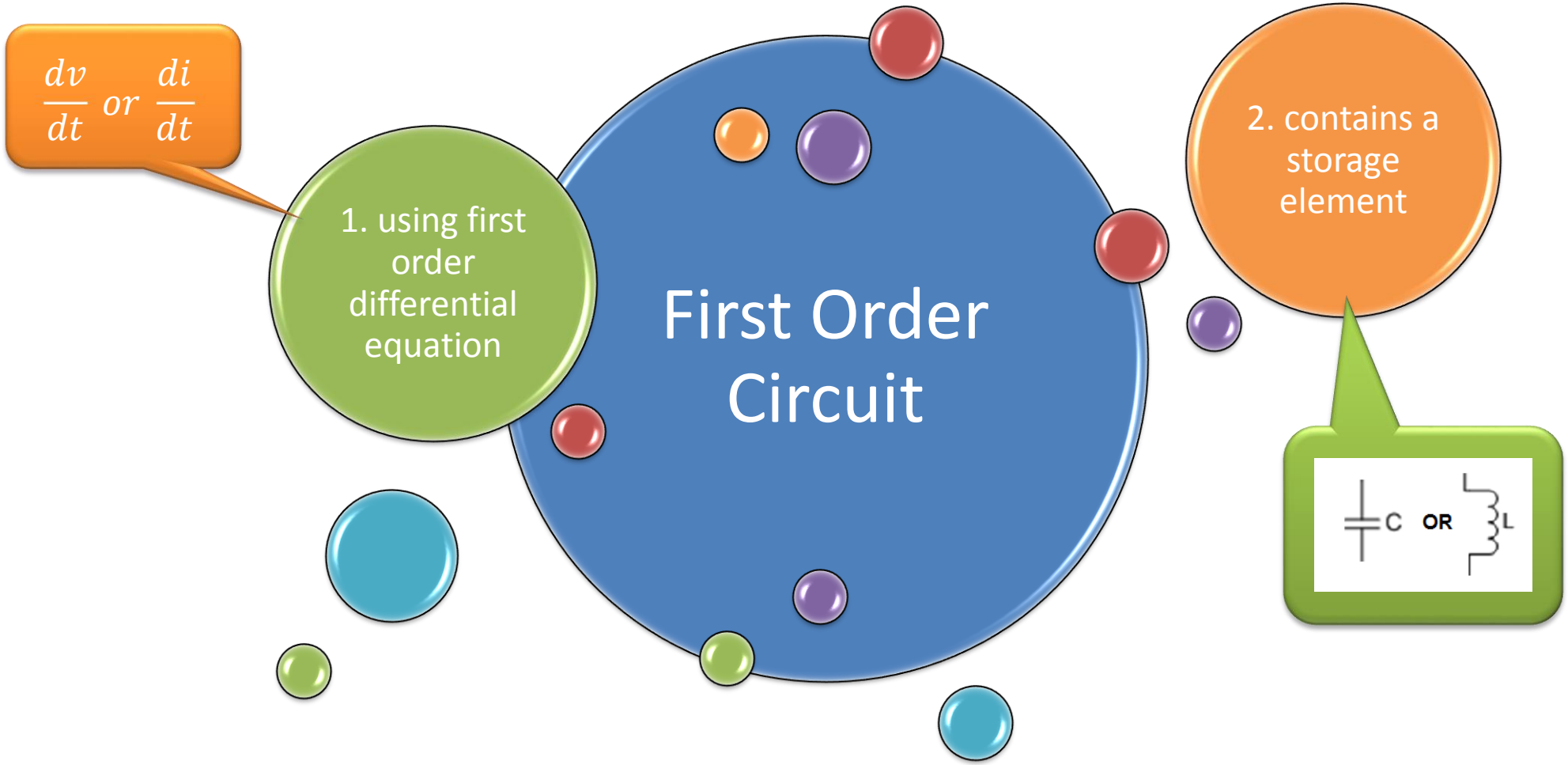
At the end of this chapter, students are able:

- to describe first order source-free RC and RL circuit
- to illustrate output response of first order source-free RC and RL circuit

SUBTOPICS

- Source-free RC Circuit
- Source-free RL Circuit

INTRODUCTION



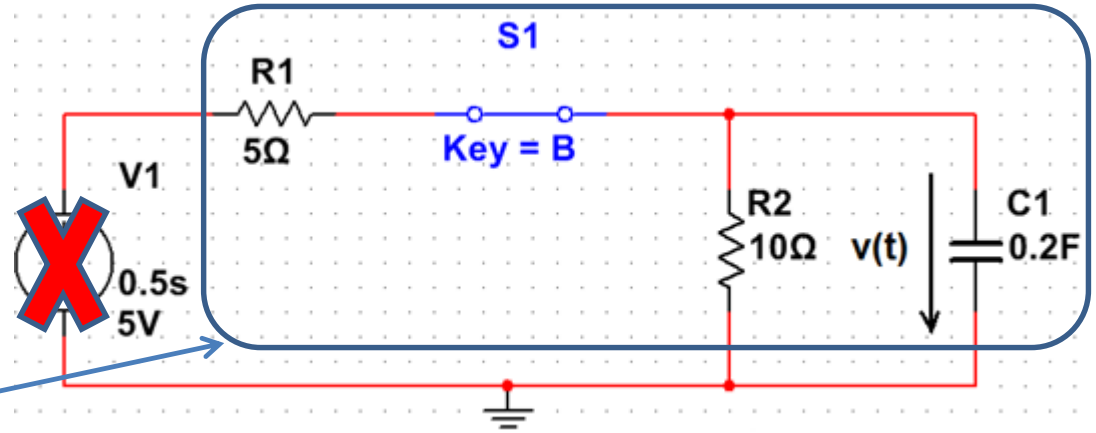
SOURCE-FREE RC CIRCUIT

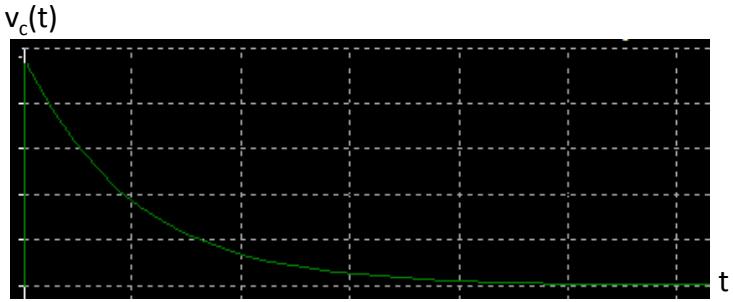
What?

1. No external sources of excitation

2. Contains several resistors and **a capacitor**

3. Output response = Natural response





SOURCE-FREE RC CIRCUIT

SERIES RC CIRCUIT

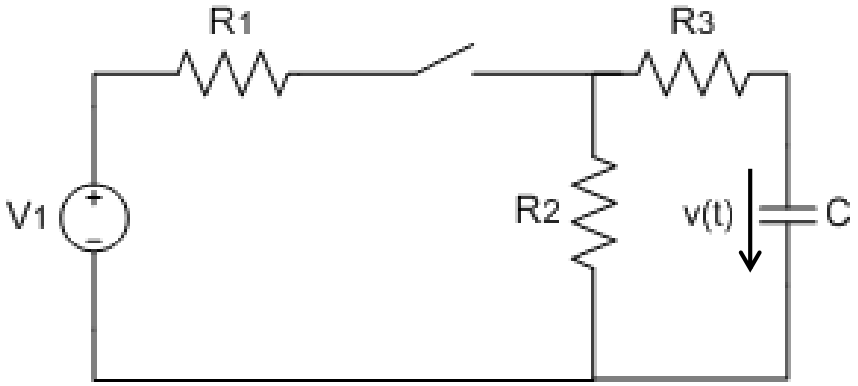


Figure 1

PARALLEL RC CIRCUIT

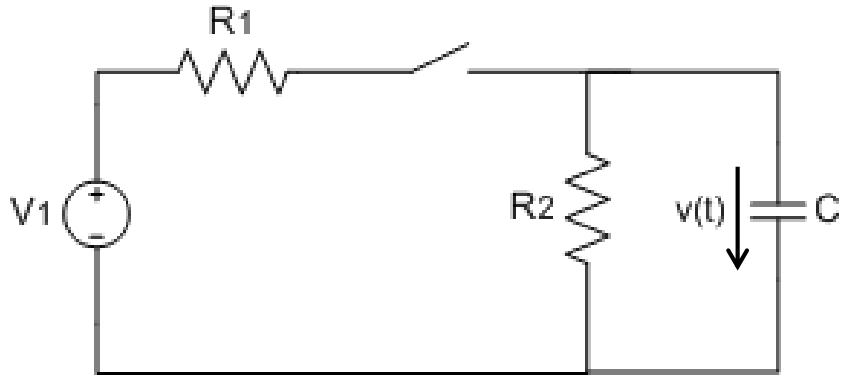


Figure 2

SOURCE-FREE RC CIRCUIT

Source-free RC Circuit:

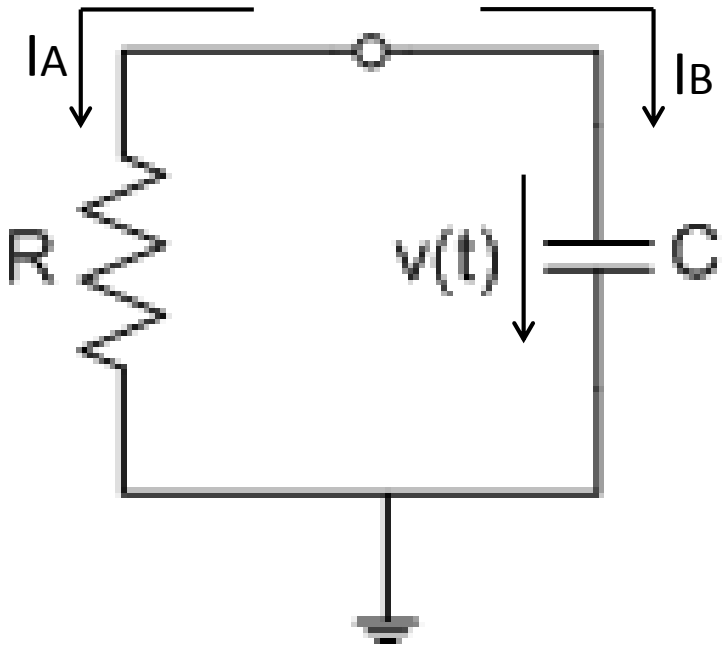


Figure 3

By applying Kirchhoff's Current Law:

$$I_A + I_B = 0$$

$$\frac{v}{R} + C\frac{dv}{dt} = 0$$

Ohm's Law

Capacitor Law

Output response:

$$v(t) = V_0 e^{-t/\tau}$$

Time constant $\tau = RC$

Initial voltage across Capacitor

EXAMPLE 1

The switch in Figure 1 is opened at t = 0. Find v(t) for t ≥ 0.

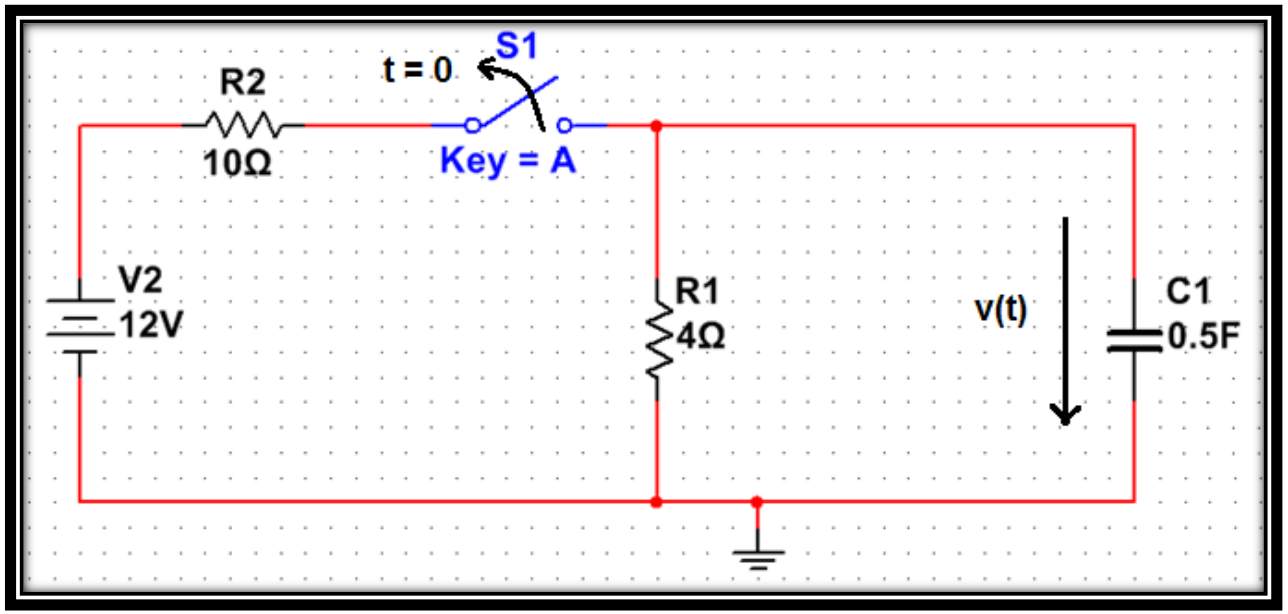


Figure 4

SOLUTION 1

Step 1: Find initial voltage across capacitor, V_0 when t < 0.

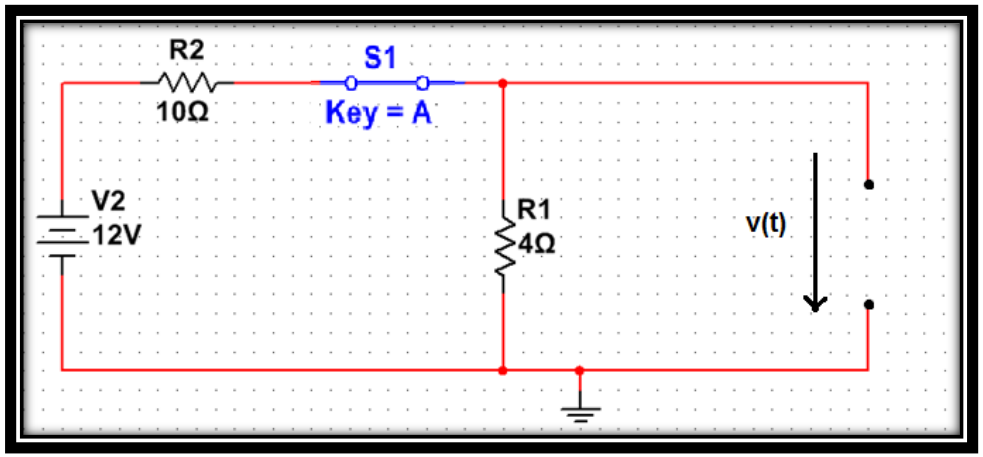


Figure 5

$$v(t) = V_0 = \frac{R_1}{R_1 + R_2} V_2 = \frac{4\Omega}{(4+10)\Omega} = 2V$$

Tips 1:

Initially, capacitor is not charged. When t < 0, it acts like an open circuit.

Tips 2:

$v(0^-) = v(0)$

SOLUTION 1

Step 2: Find time constant, τ for t $\geq$ 0.

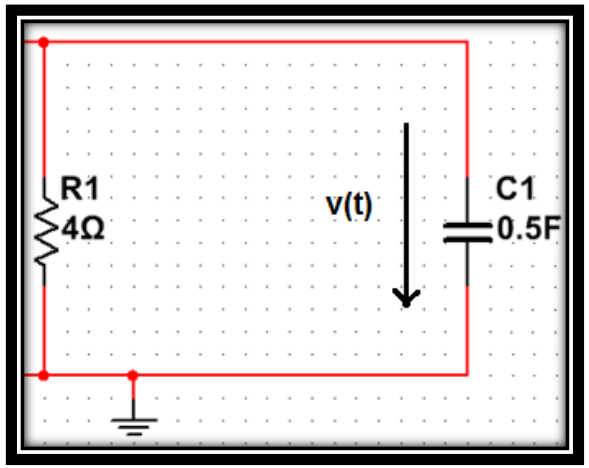


Figure 6

$$\tau = R_1 C = 4\Omega * 0.5F = 2\text{s}$$

Tips 3:

When t $\geq$ 0, circuit in Figure 4 is reduced to Figure 6.

SOLUTION 1

Step 3: Find voltage across capacitor, v(t) for t ≥ 0.

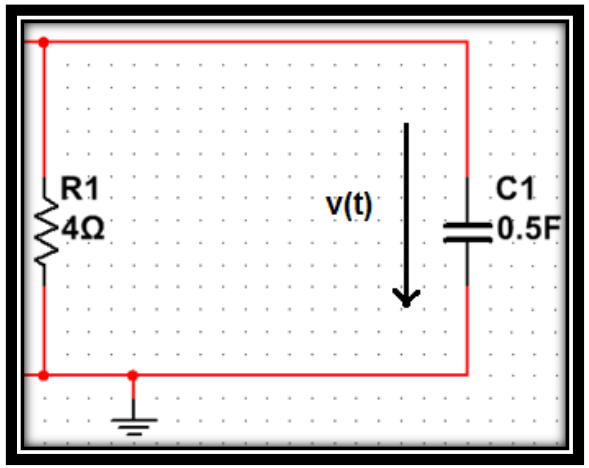


Figure 7

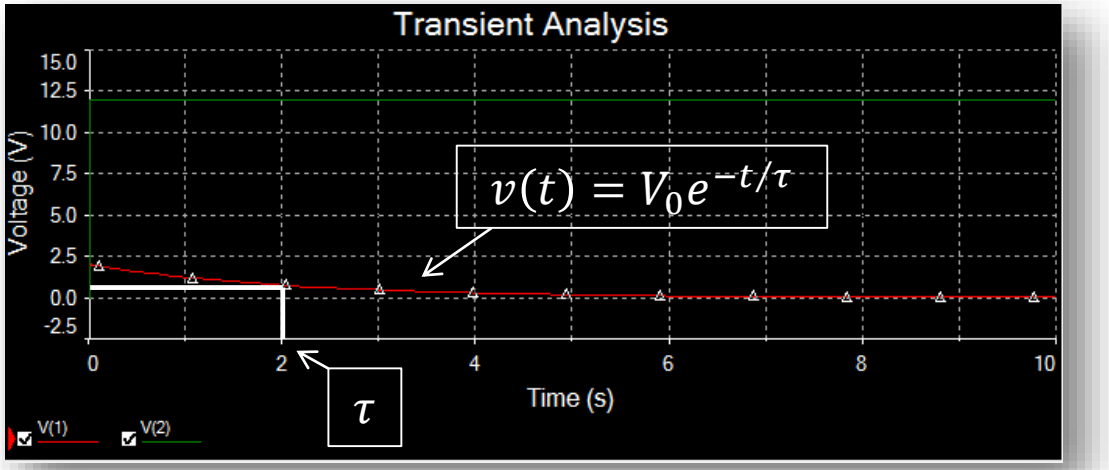
$$v(t) = V_0 e^{-t/\tau} = 2e^{-0.5t}\text{V}$$

OUTPUT RESPONSE 1

Natural response of an RC circuit in Figure 4:

V(1): Voltage across capacitor

V(2): Voltage source



Graph 1

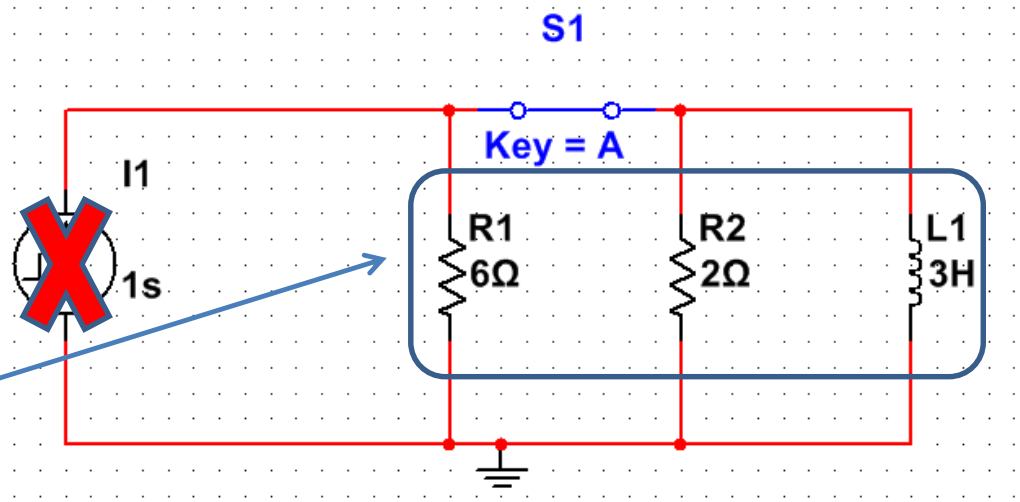
SOURCE-FREE RL CIRCUIT

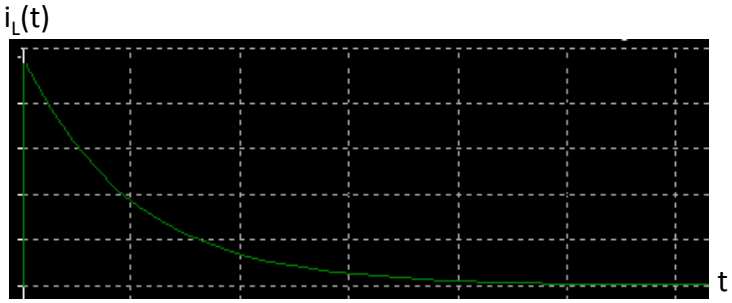
What?

1. No external sources of excitation

2. Contains several resistors and **an inductor**

3. Output response = Natural response





SOURCE-FREE RL CIRCUIT

SERIES RL CIRCUIT

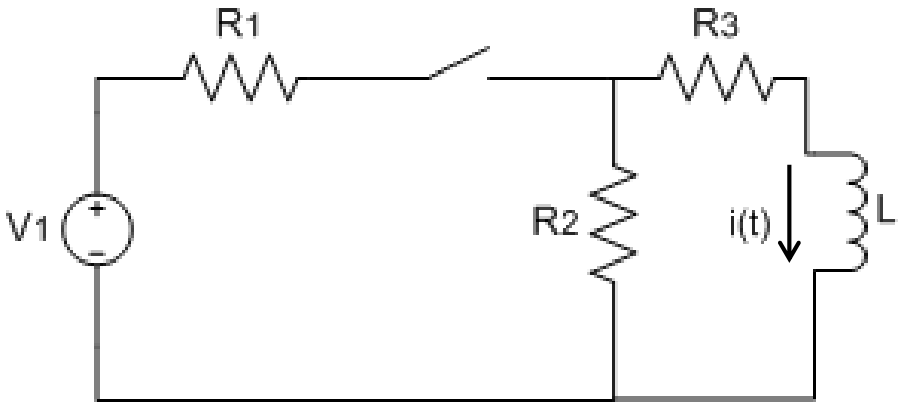


Figure 8

PARALLEL RL CIRCUIT

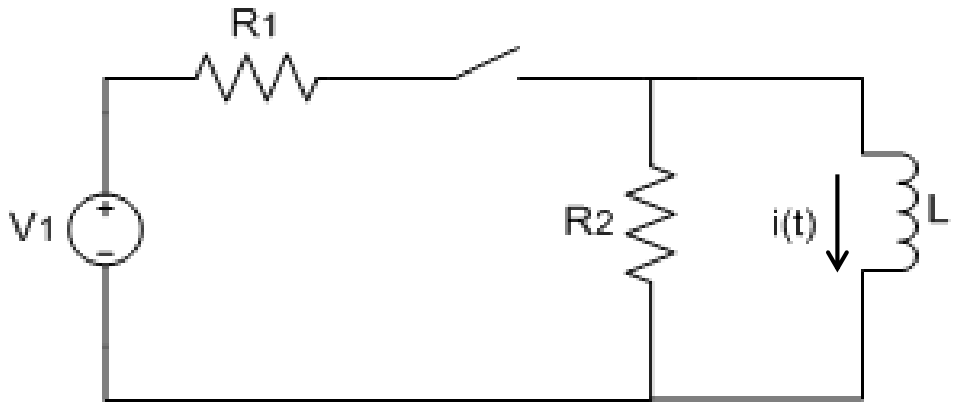


Figure 9

SOURCE-FREE RL CIRCUIT

Source-free RL Circuit:

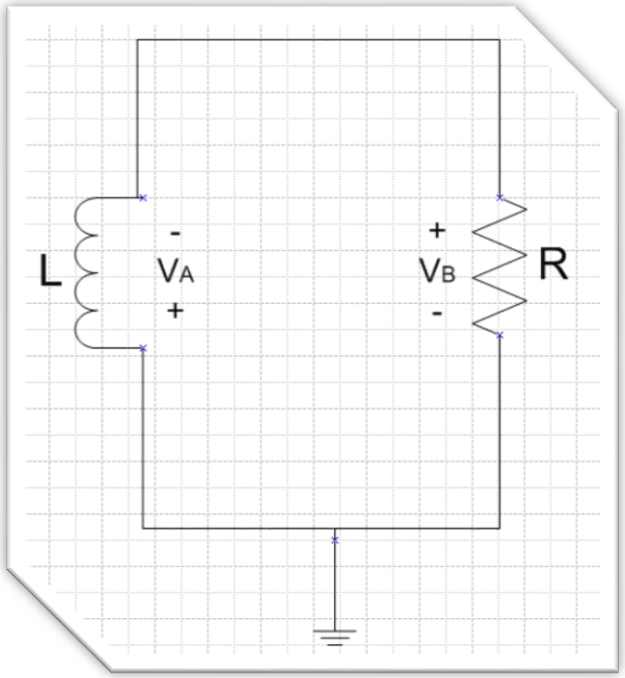


Figure 10

By applying Kirchhoff's Voltage Law:

$$V_A + V_B = 0$$

$$L\frac{di}{dt} + iR = 0$$

Ohm's Law

Inductor Law

Output response:

$$i(t) = I_0 e^{-t/\tau}$$

Time constant $\tau = \frac{L}{R}$

Initial current through inductor

EXAMPLE 2

The switch in Figure 11 is opened for t = 0. Find i(t) for t > 0.

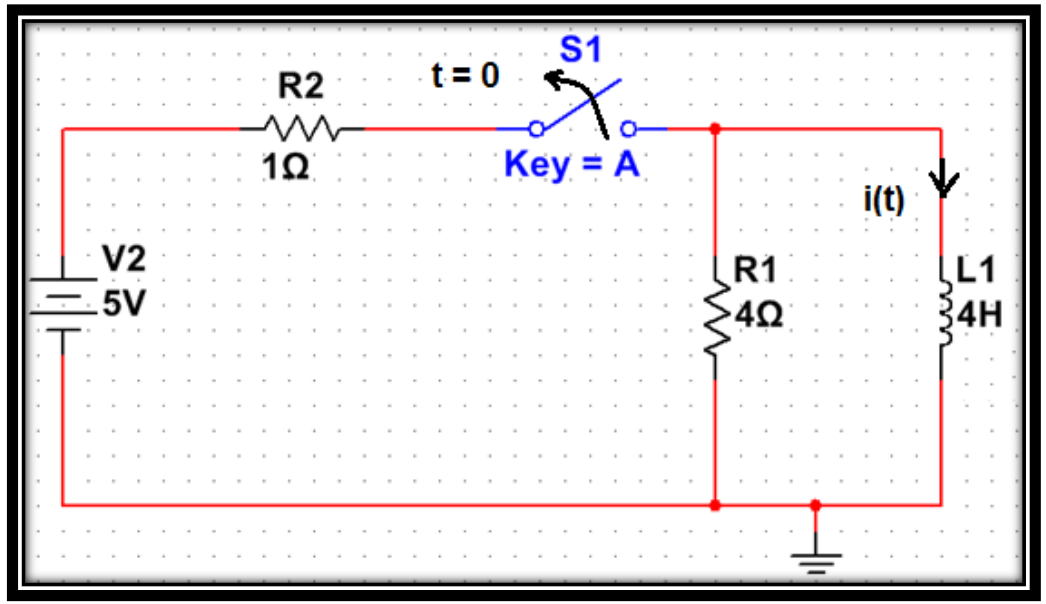


Figure 11

SOLUTION 2

Step 1: Find initial current through inductor, I_0 when t < 0.

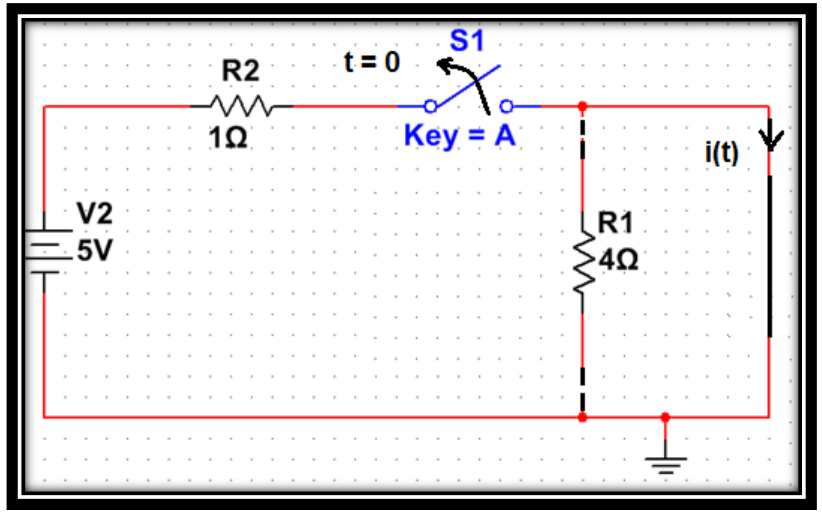


Figure 12

$$i(t) = I_0 = \frac{V_2}{R} = \frac{5V}{1\Omega} = 5A$$

Tips 1:

Initially, inductor is not charged. When t < 0, it acts like a short circuit.

Tips 2:

Current will flow to the less resistance.

Tips 3:

$i(0^-) = i(0)$

SOLUTION 2

Step 2: Find time constant, τ for t > 0.

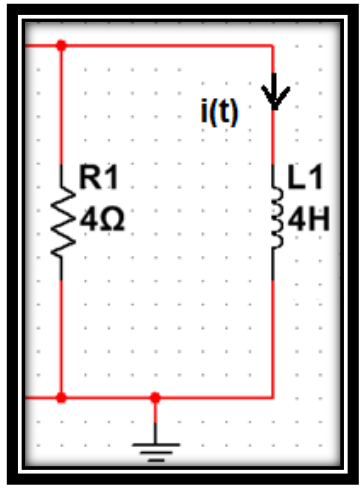


Figure 13

$$\tau = \frac{L}{R_1} = \frac{4H}{4\Omega} = 1s$$

Tips 4:

When t > 0, circuit in Figure 11 is reduced to Figure 13.

SOLUTION 2

Step 3: Find current through inductor, i(t) for t > 0.

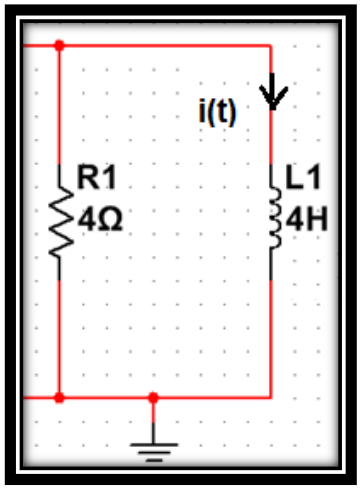


Figure 14

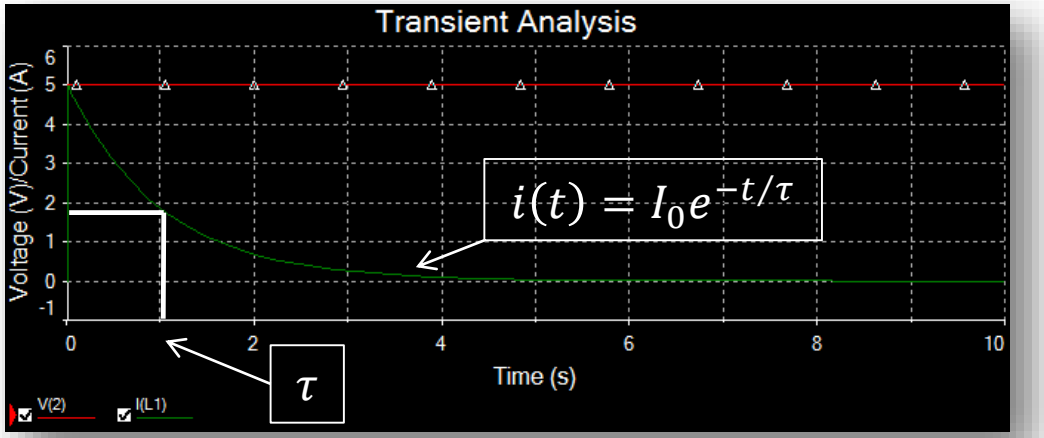
$$i(t) = I_0 e^{-t/\tau} = 5e^{-t} A$$

OUTPUT RESPONSE 2

Natural response of an RL circuit in Figure 11:

I(L1): Current across inductor

V(2): Voltage source



Graph 2

SELF REVIEW QUESTIONS

1. A source-free circuit has no external sources of excitation.

   TRUE FALSE

2. Name a storage element in a first order source-free RC circuit.

   Answer: _____

3. Forced response is the behavior of a first order source-free circuit.

   TRUE FALSE

4. Given R = 4 Ω and C = 5 F. What is the time constant for this RC circuit?

   a) 1 s b) 3s

   c) 10 s d) 20 s

5. An inductor in a source-free RL circuit with L = 2 H and R = 10 Ω is being charged. What is the time required for the inductor current to decay 50 % of its initial value?

   a) 0.20 s b) 0.05 s

   c) 0.14 s d) 0.35 s

ANSWERS

1. TRUE
2. Capacitor
3. FALSE
4. d
5. c