MECHANICAL VIBRATION

BMCG 3233

CHAPTER 4: HARMONIC FORCED VIBRATION (PART 2)

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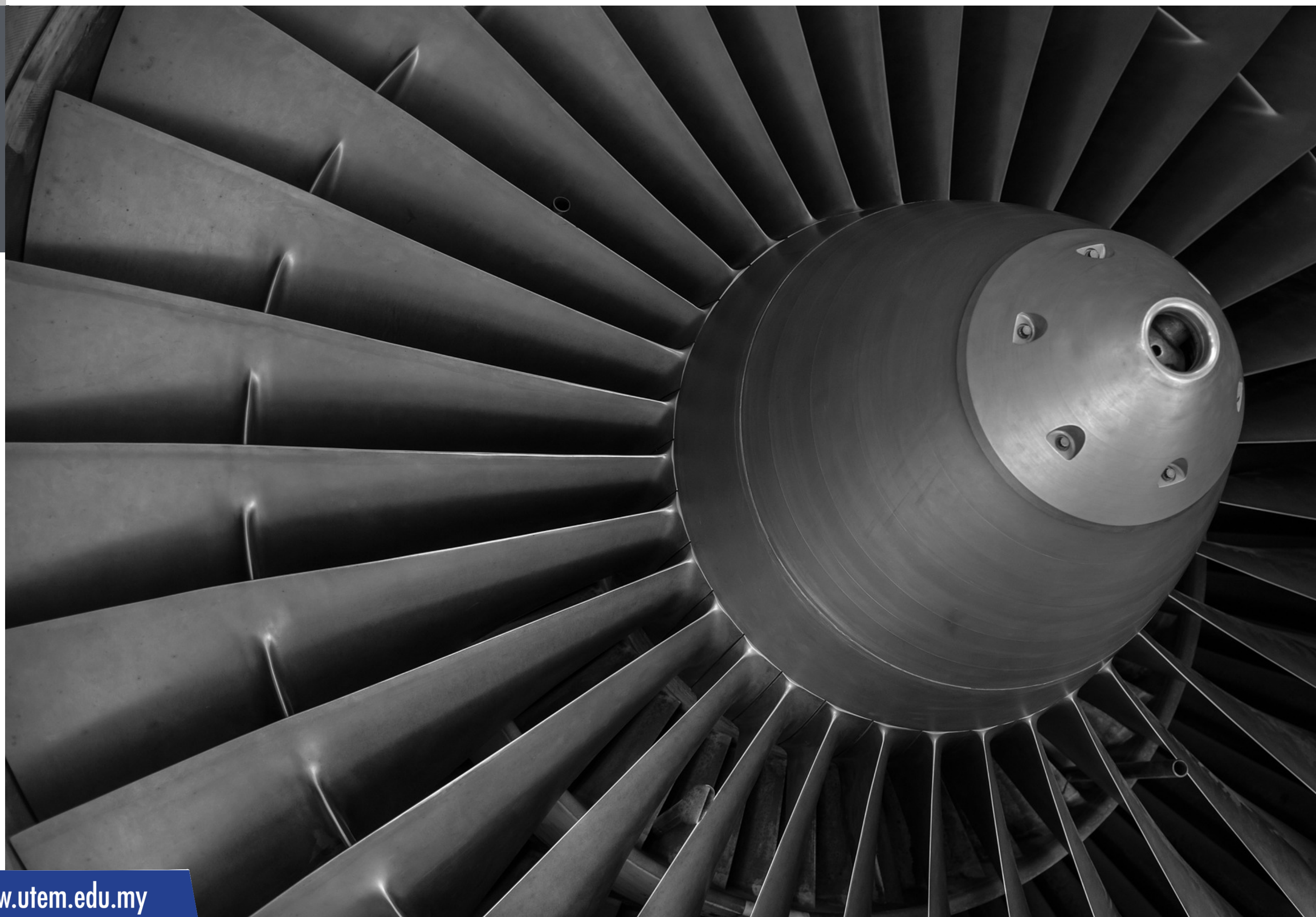
CONTENTS

LEARNING OBJECTIVES

1. Solve vibration problem due rotating unbalance
2. Recognise instrumentations in vibration



4.3

ROTATING UNBALANCE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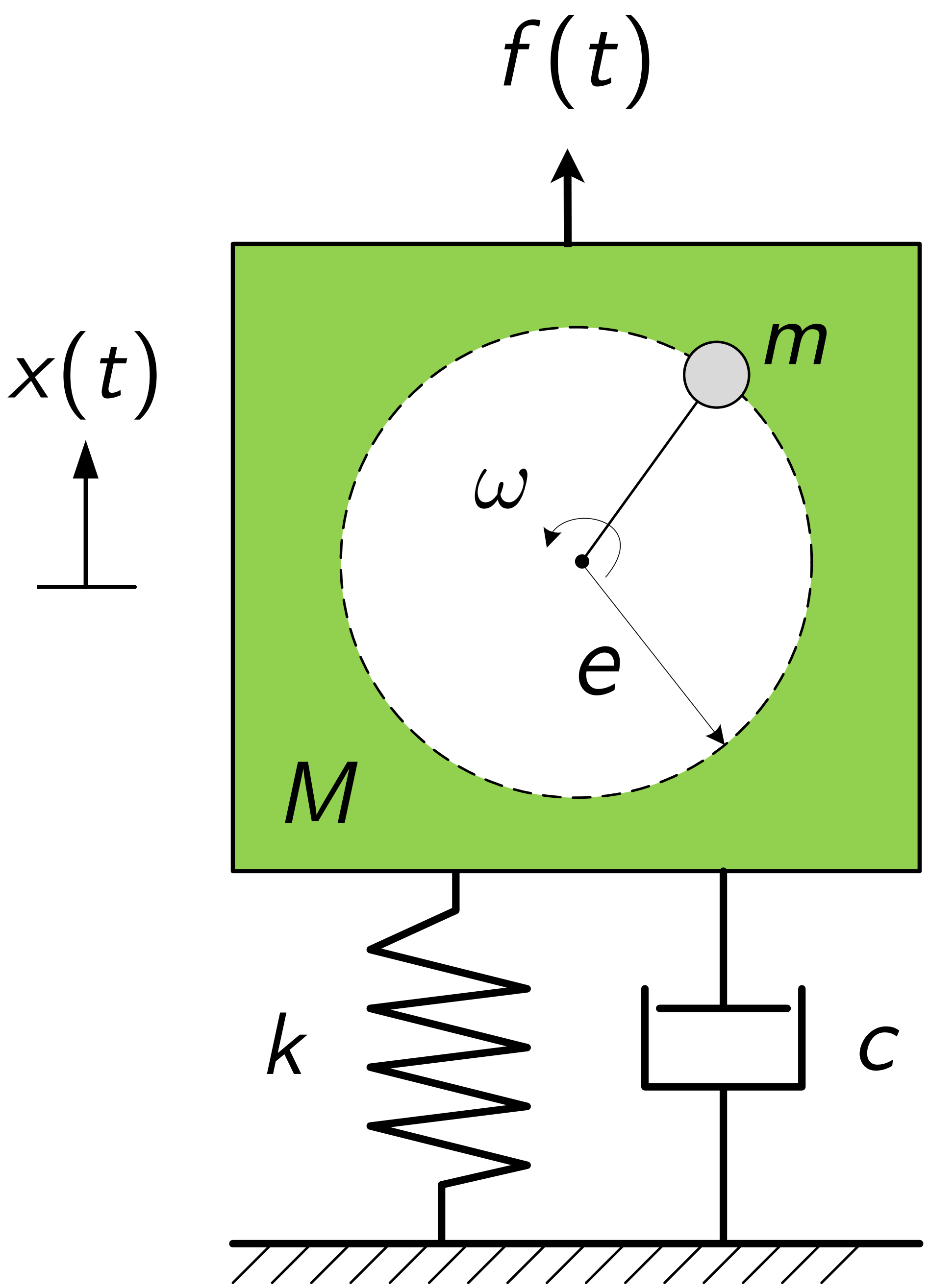
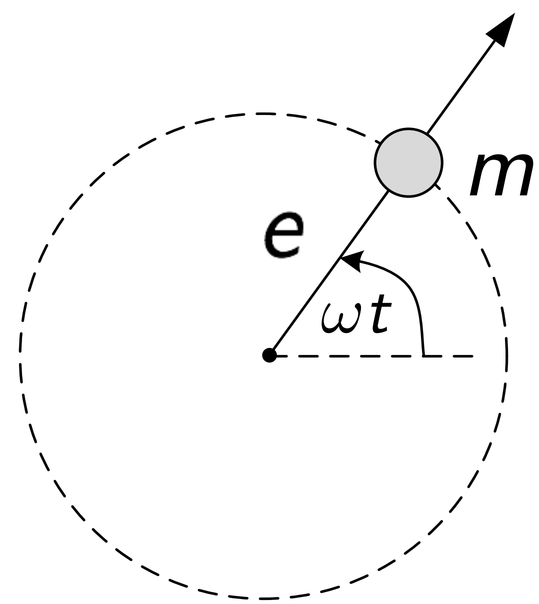
m : **Unbalance mass**

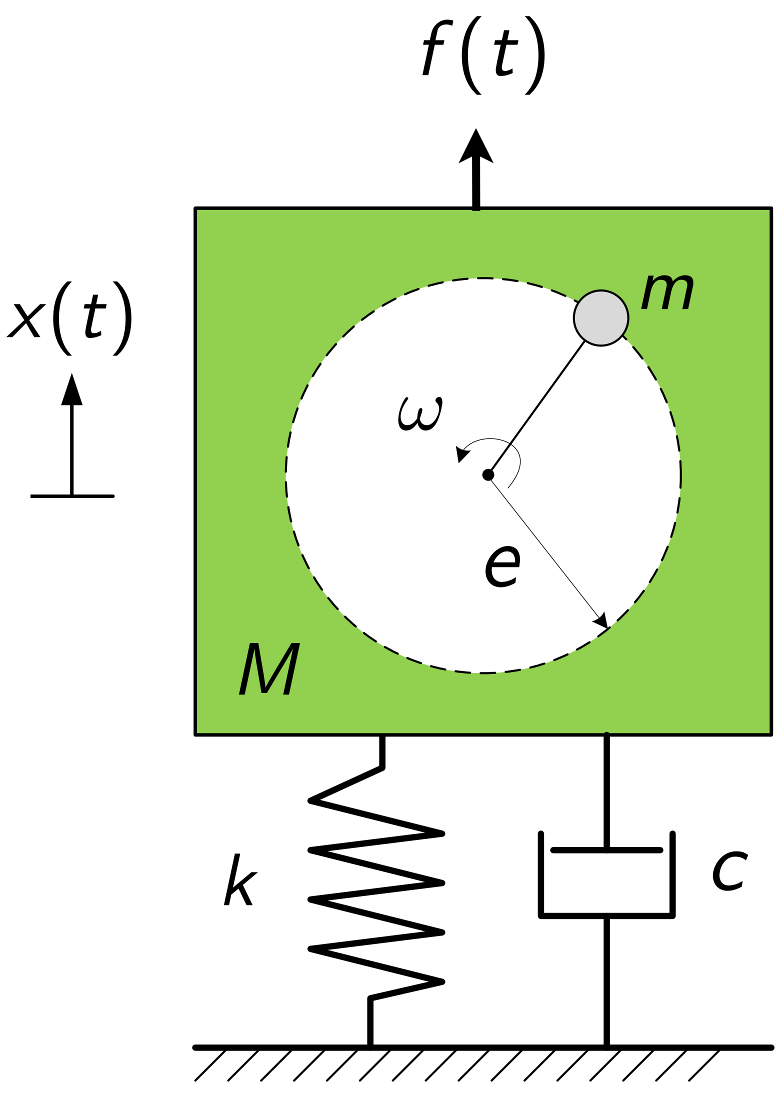
e : **Distance of the unbalance mass to the centre of rotation**



Centrifugal force produced by a rotating object:

$$F_c = me\omega^2$$





*Assume only vertical motion

Equation of motion:

$$M\ddot{x} + c\dot{x} + kx = f(t) = F_c e^{j\omega t}$$

where

$$F_c = me\omega^2$$

Substituting: $x(t) = Xe^{j\omega t}$

$$X = \frac{me}{M}\left(\frac{\omega^2/\omega_n^2}{1-(\omega/\omega_n)^2 + j2\xi\omega/\omega_n}\right)$$

$$\left|\frac{XM}{me}\right| = \frac{\omega^2/\omega_n^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}}$$

Low frequency:

$$\omega \ll \omega_n \Rightarrow |MX/me| \approx 0$$

At resonance:

$$\omega = \omega_n \Rightarrow |MX/me| = 1/(2\zeta)$$

High frequency:

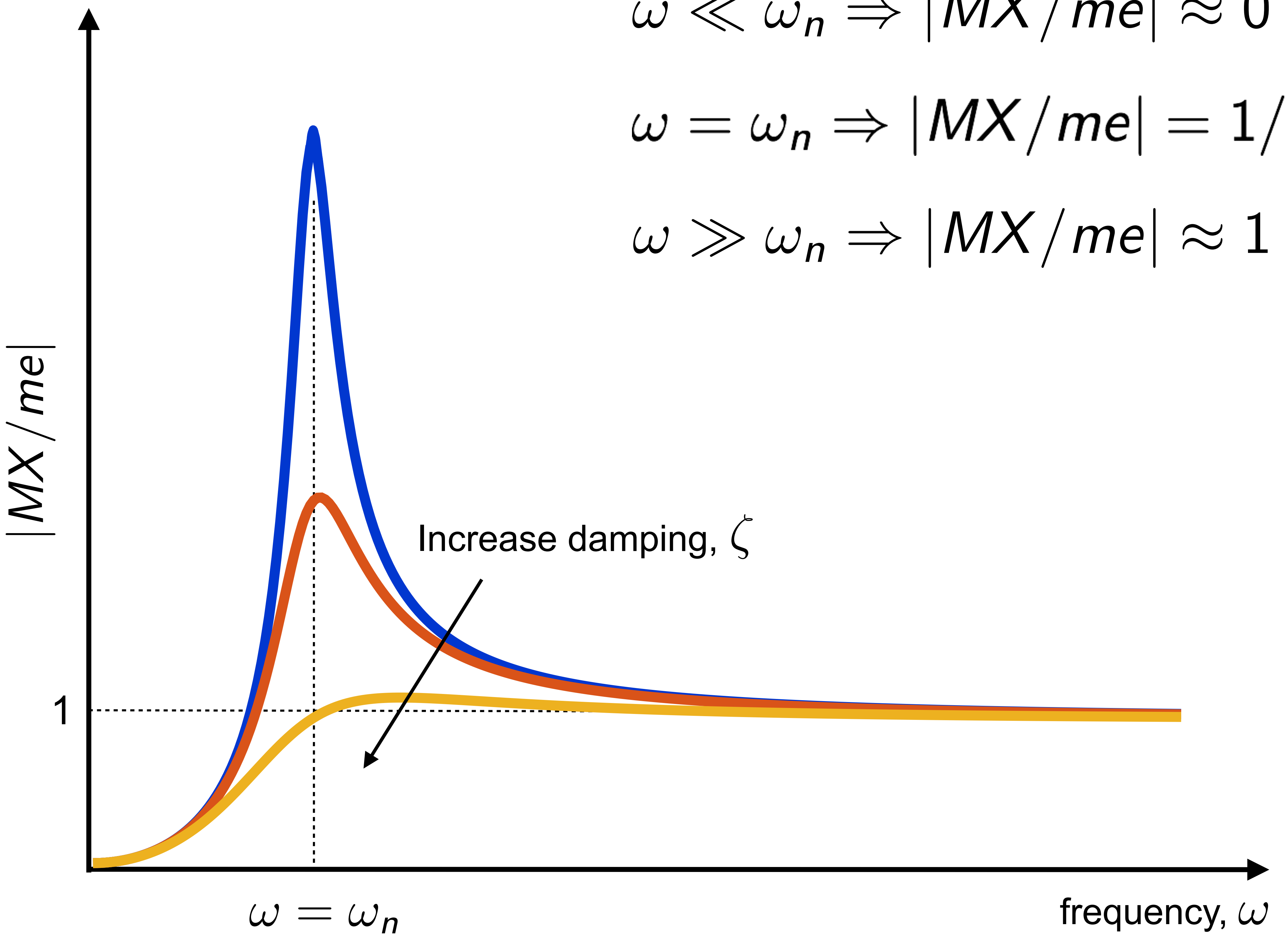
$$\omega \gg \omega_n \Rightarrow |MX/me| \approx 1$$

FRF Graph

$$\omega \ll \omega_n \Rightarrow |MX/me| \approx 0$$

$$\omega = \omega_n \Rightarrow |MX/me| = 1/(2\zeta)$$

$$\omega \gg \omega_n \Rightarrow |MX/me| \approx 1$$



Ch-47 Ground Test

Example 4.3

An electric motor has an eccentric mass of 10 kg (i.e. 10% of the total mass) and is set on two identical springs with *k* = 3200 N/m. The motor runs at 1,750 RPM and the mass eccentricity is 100 mm from the center. Neglect the damping and determine the maximum velocity of the vertical vibration.

Solution

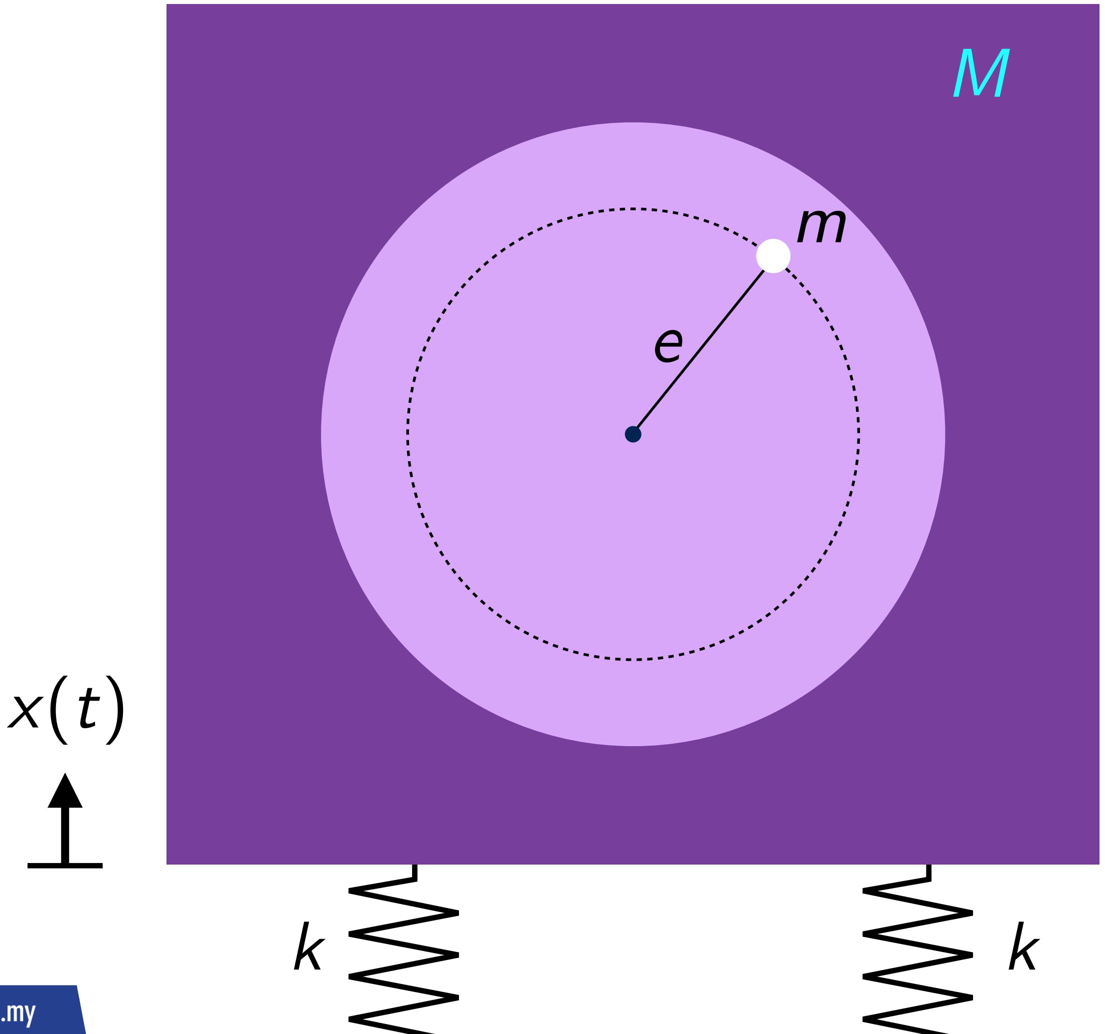
Known

$$m = 0.1M$$

$$k = 3200 \text{ N/m}$$

$$e = 0.1 \text{ m}$$

$$\omega = 2\pi(1750/60) = 29.2 \text{ rad/s}$$



From the equation with neglected damping:

$$\left|\frac{XM}{me}\right| = \frac{\omega^2/\omega_n^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} \xrightarrow{\zeta \to 0} \left|\frac{XM}{me}\right| = \frac{\omega^2/\omega_n^2}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|}$$

$$\omega_n^2 = 2k/M = 2(3200)/100 = 64 \text{ rad}^2/\text{s}^2$$

The magnitude of displacement is: $|X| = \dfrac{(\omega^2/\omega_n^2)\dfrac{me}{M}}{\left|1-\left(\dfrac{\omega}{\omega_n}\right)^2\right|} = 7.5\times 10^{-4}\text{m} = 0.75 \text{ mm}$

The maximum velocity is therefore: $v_{\max} = \omega|X| = 29.2(0.75) = 22 \text{ mm/s}$

Example 4.4

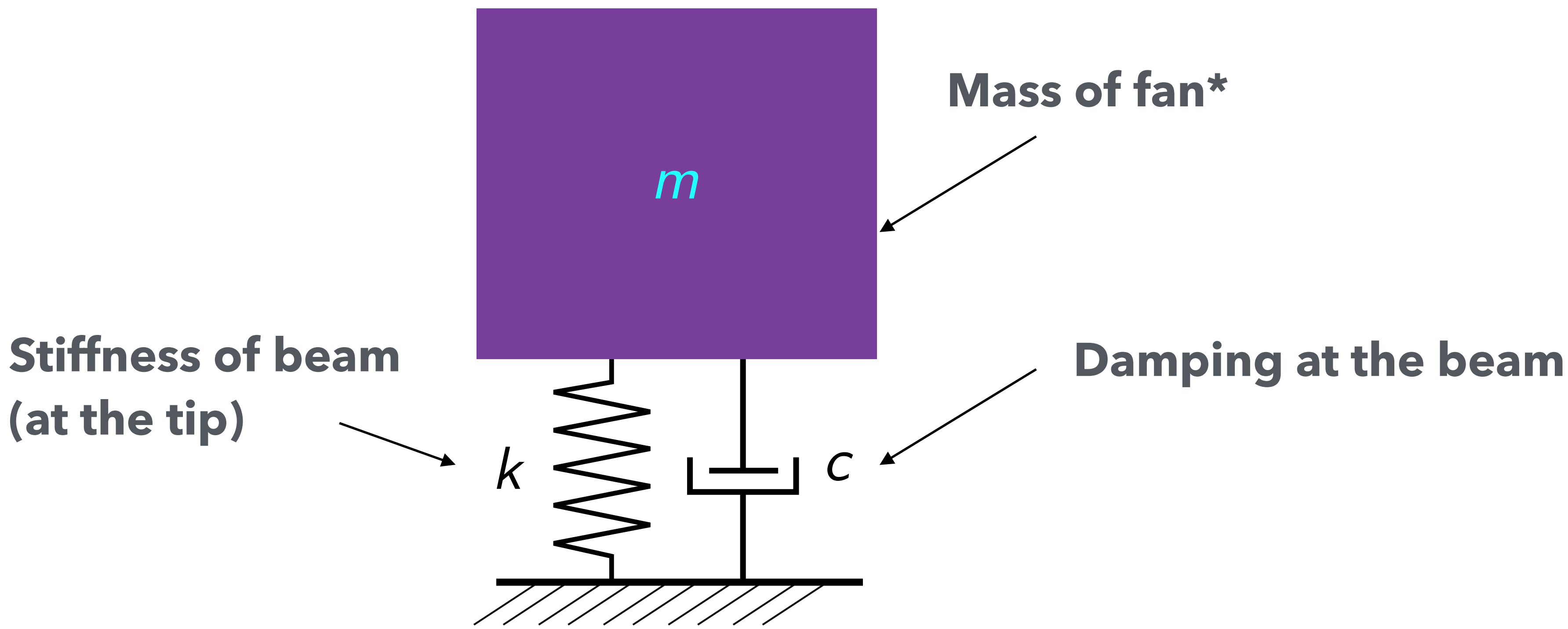
A 30 kg fan is installed on a cantilever, massless steel beam with stiffness 6 x 10^5 N/ has rotating unbalance of 0.15 kg-m.

As the speed of the fan is varied, it is observed that it has maximum amplitude of 10 mm.

a. Draw the mass-spring-damper system.
b. What is the magnitude of velocity if the fan is operated at 500 RPM?

Solution

a. Mass-spring-damper system

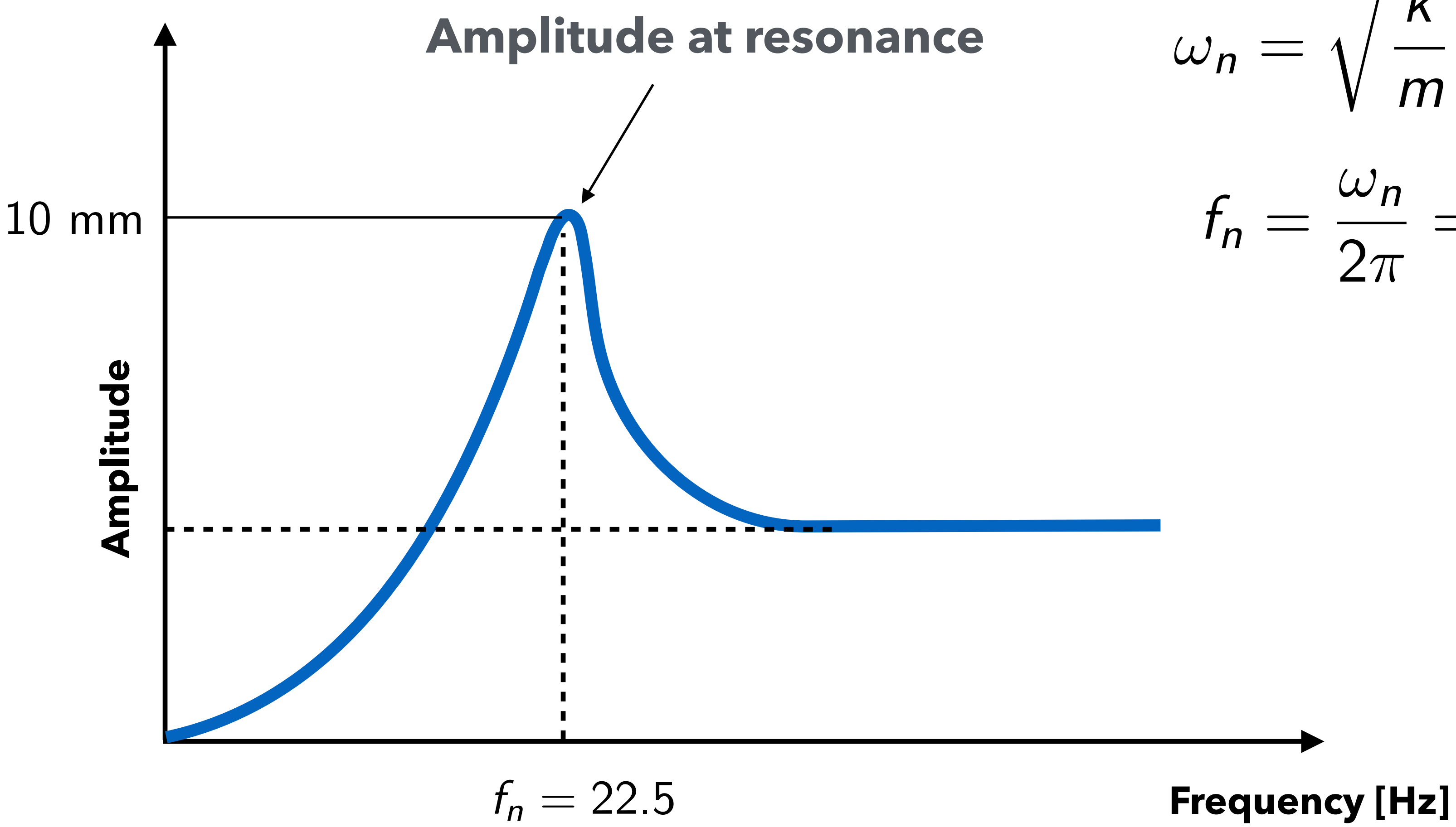


*The beam is massless, otherwise the effective mass of the beam must be included in the mass component

b. Known $m = 30$ kg

$k = 6 \times 10^5$ N/m

$\zeta = 0.1$



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6 \times 10^5}{30}} = 141.4 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 22.5 \text{ Hz}$$

At resonance:

Amplitude at resonance

$$\left|\frac{XM}{me}\right| = \frac{\omega^2/\omega_n^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} \xrightarrow{\omega=\omega_n} \left|\frac{XM}{me}\right| = \frac{1}{2\zeta} \longrightarrow \zeta = \frac{me}{2XM}$$

$$\zeta = \frac{0.15}{2(0.01)(30)} = 0.25$$

The displacement at 500 RPM = 8.3 Hz: $\omega = 2\pi f = 2\pi(8.3) = 52 \text{ rad/s}$

$$\left|\frac{XM}{me}\right| = \frac{\omega^2/\omega_n^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} \longrightarrow X = 0.765 \text{ mm}$$

$\omega = 52$ rad/s

$\zeta = 0.25$

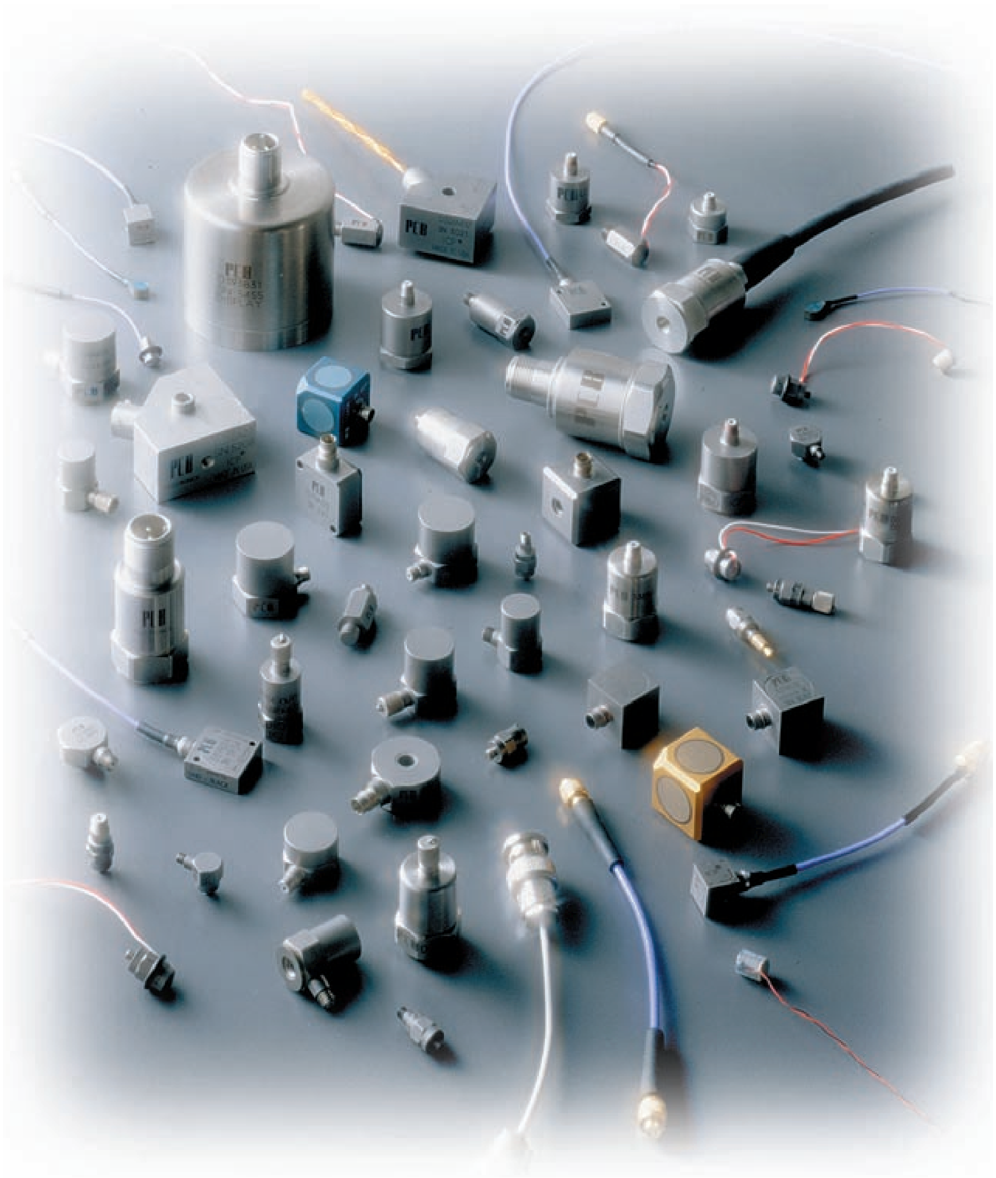
$\omega_n = 141.4$ rad/s

The vibration velocity is therefore: $v = \omega X = 52(0.765) = 39.8 \text{ mm/s}$

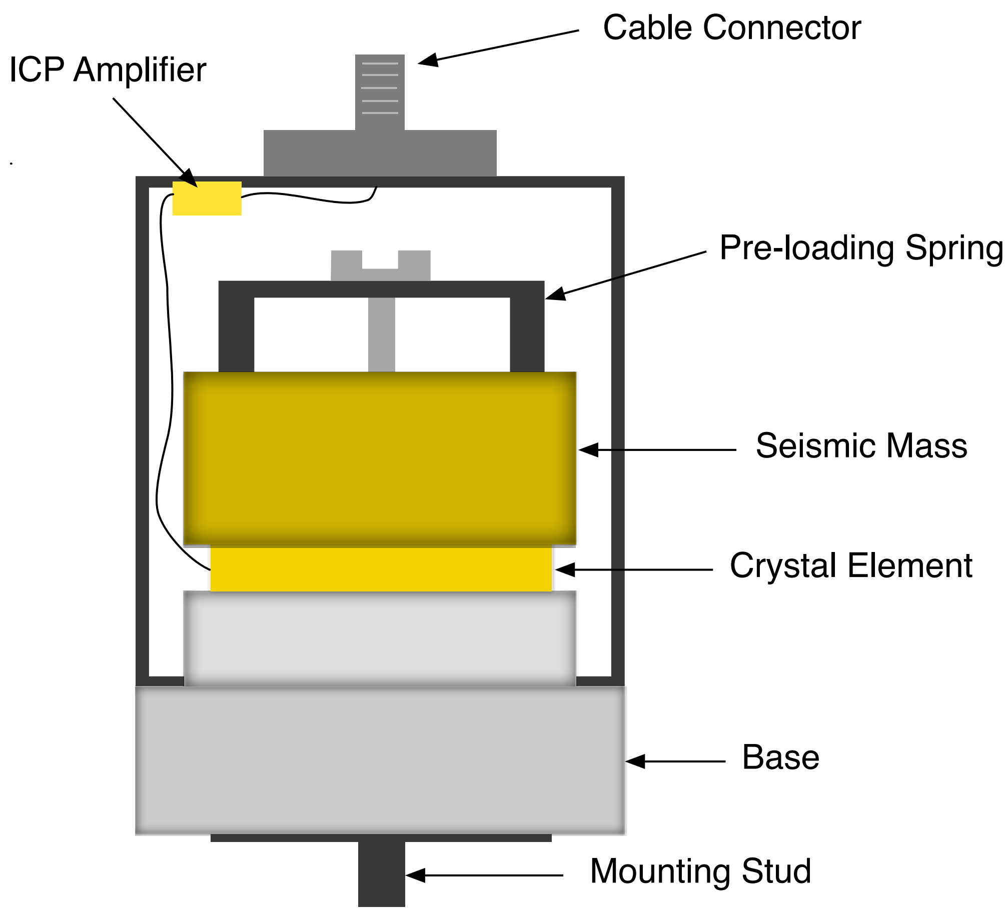
4.4

VIBRATION INSTRUMENTATION

to measure acceleration of a vibrating object (output: mV/g)



Credited to PCB® sensors: www.pcb.com



***The output can also be converted into displacement and velocity**

The basic principle: the accelerometer has to detect the same amount of vibration amplitude of the vibrating surface.

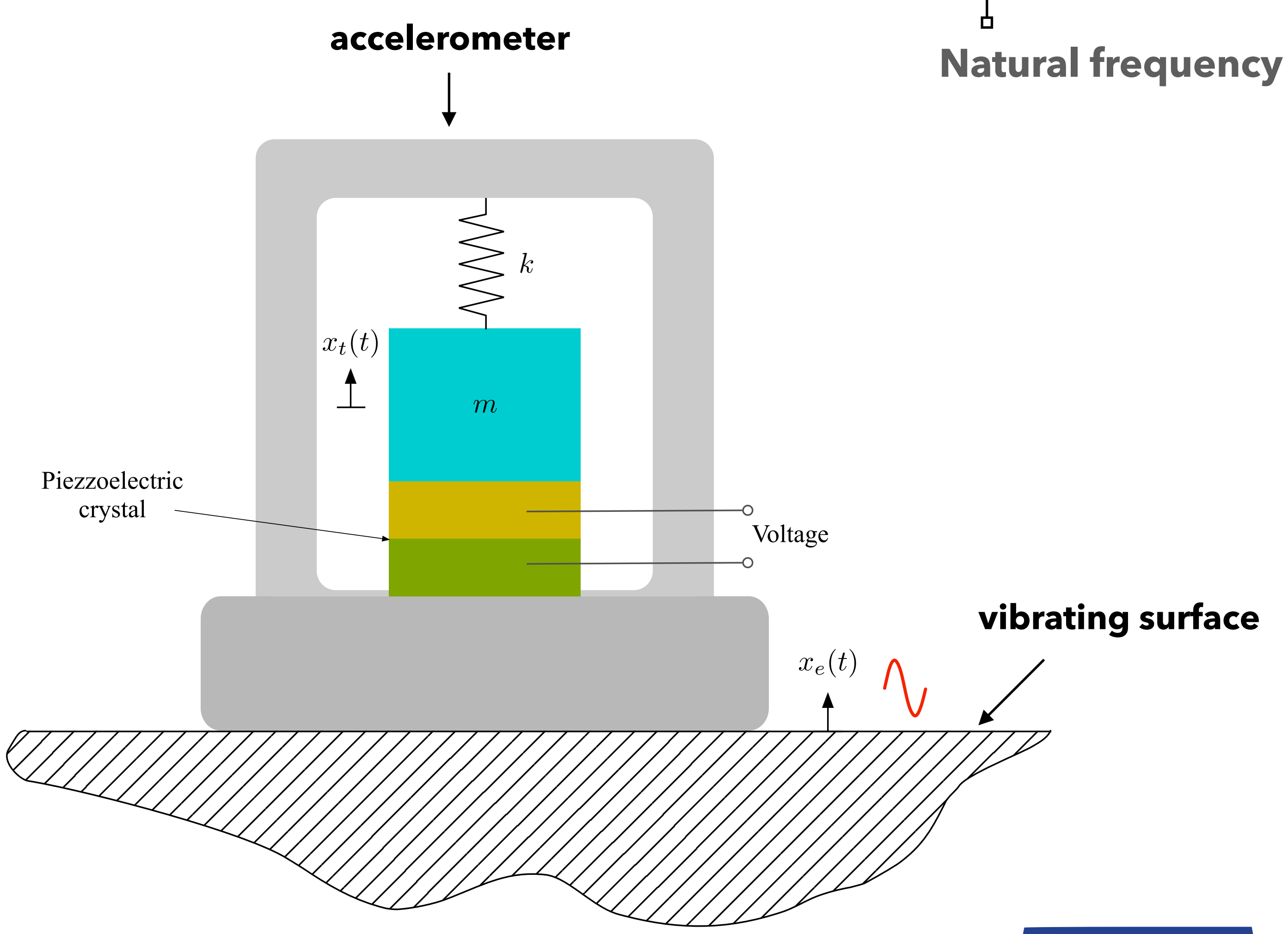
$$x_t = x_e$$

Requirement:

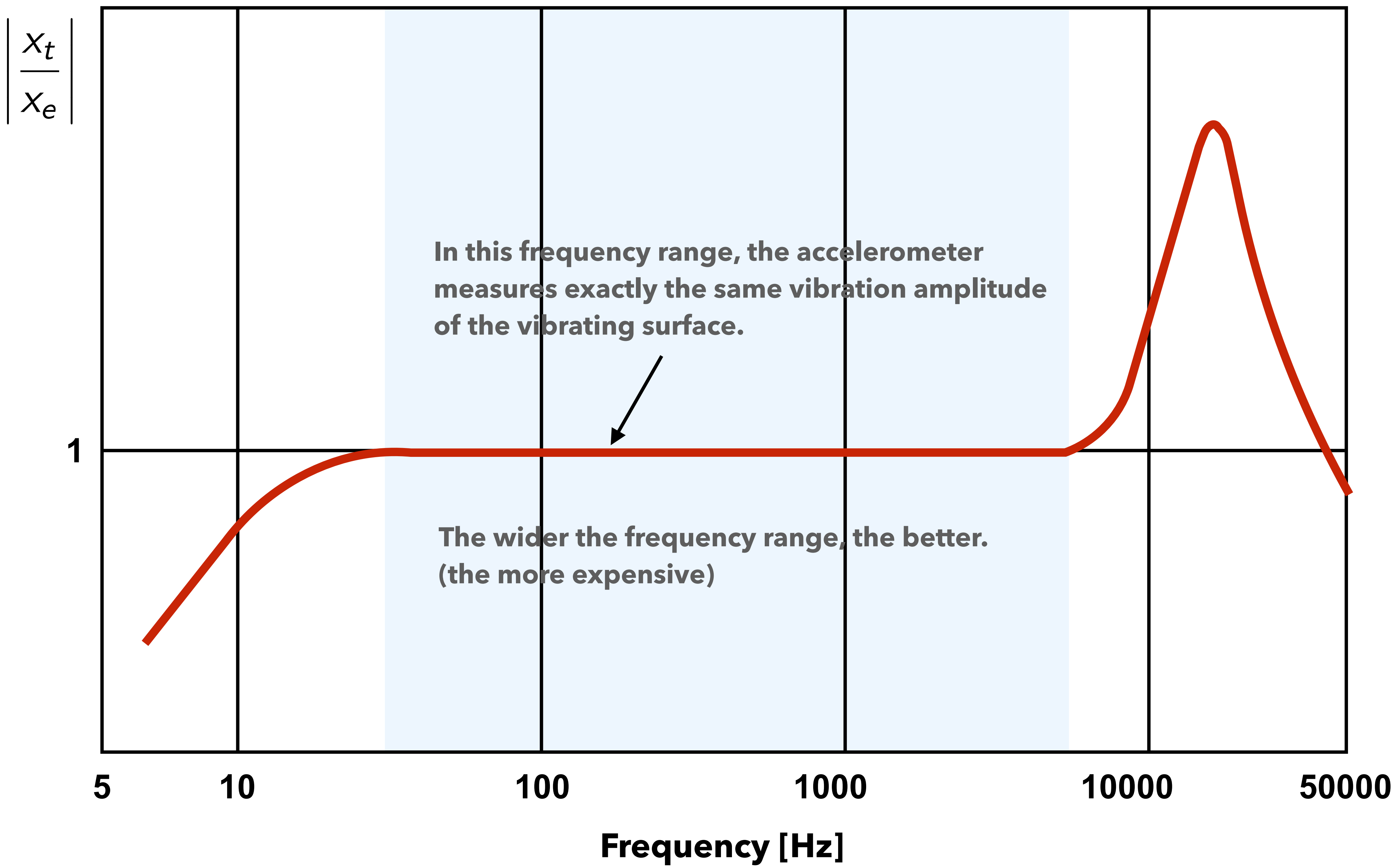
$$\omega_n \gg \omega$$

Frequency of the vibrating surface (ω)

Natural frequency of accelerometer, $\omega_n = \sqrt{\frac{k}{m}}$ (ω_n)



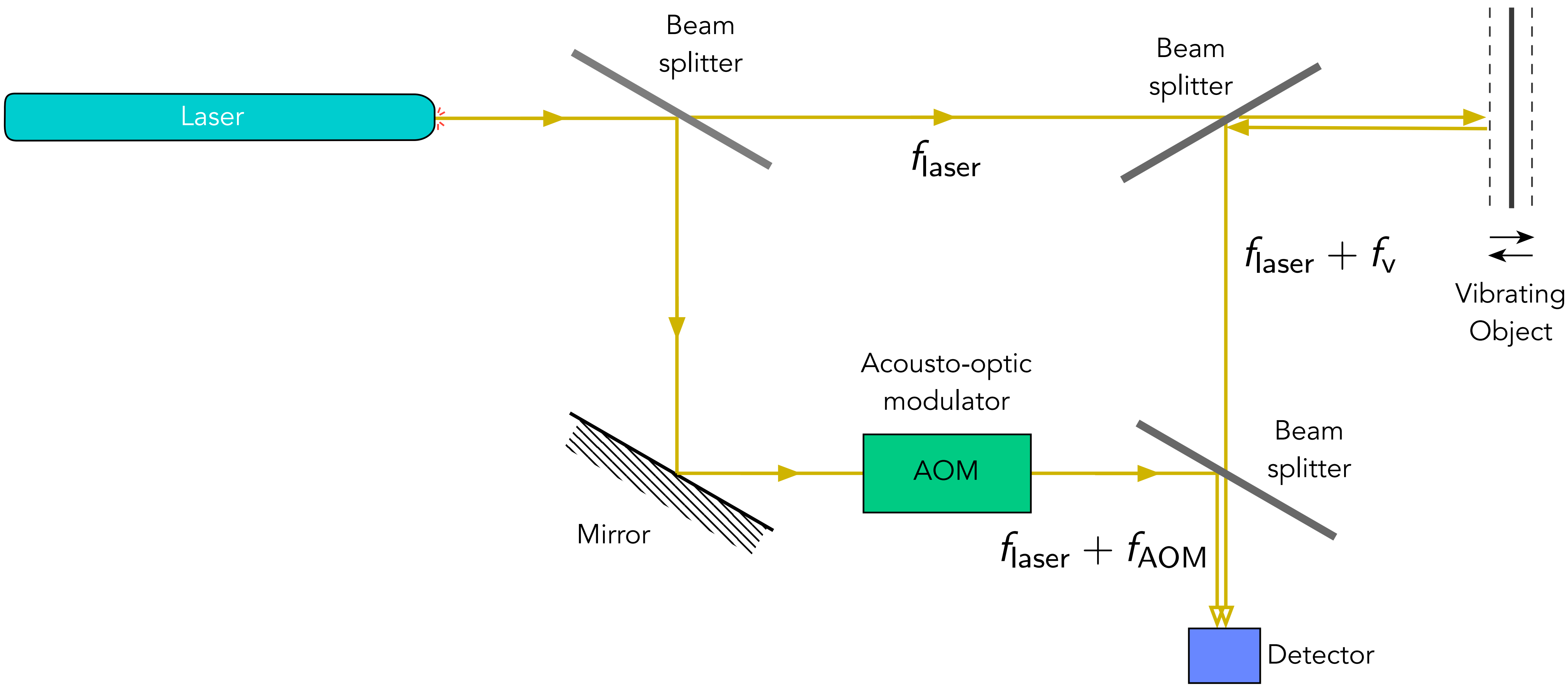
***Will discuss in details in Chapter 6**


$\left|\frac{x_t}{x_e}\right|$
1
In this frequency range, the accelerometer measures exactly the same vibration amplitude of the vibrating surface.
The wider the frequency range, the better. (the more expensive)
5
10
100
1000
10000
50000
Frequency [Hz]

- same function as accelerometer, but using laser beam
- non-contact surface measurement
- use the concept of Doppler effect

Credited to Polytec: www.polytec.com

Laser Scanning Head Vibrometer



Doppler effect: Principle of Laser Scanning Vibrometer

Electromagnetic shaker

to provide force excitation to a test structure

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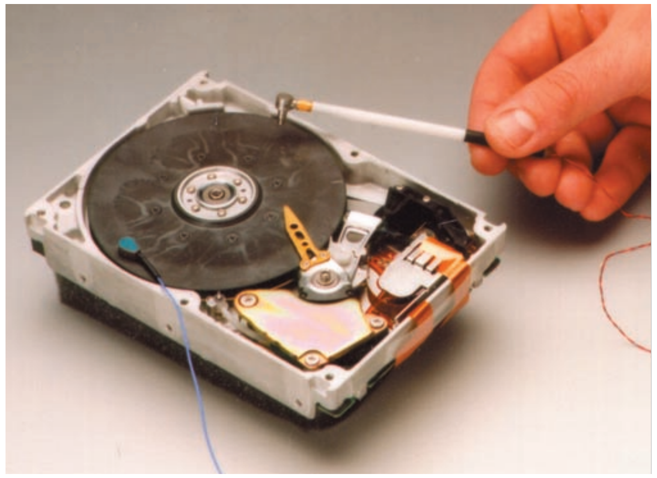
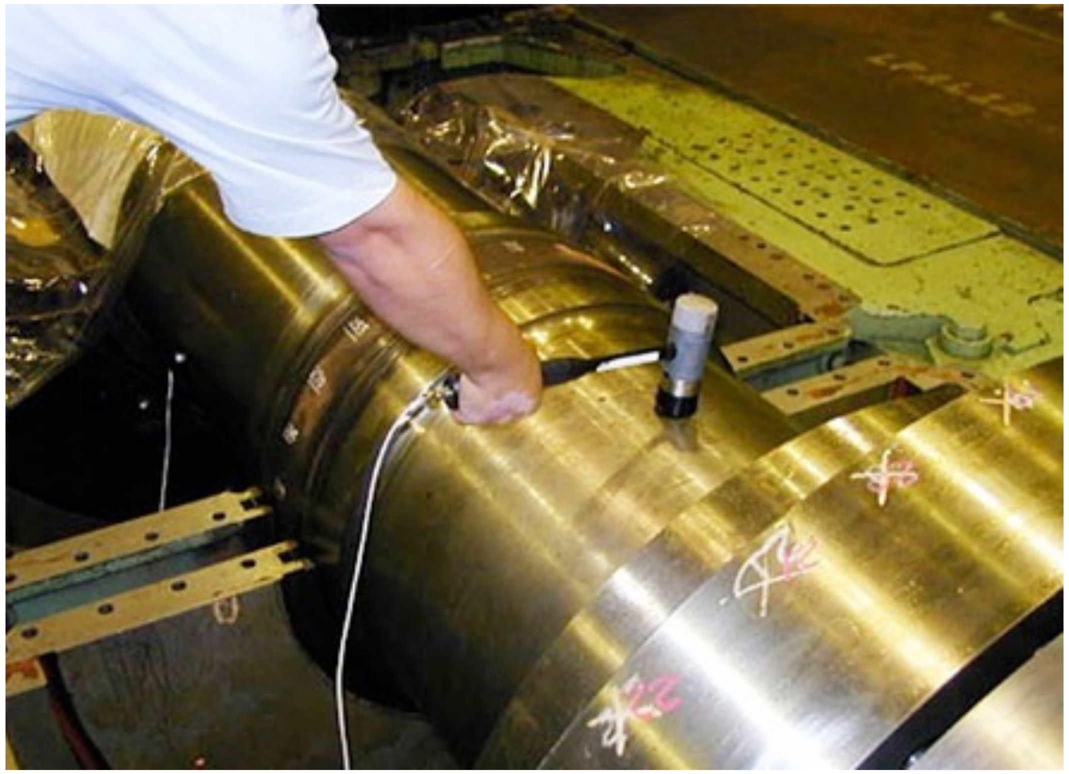
to provide force excitation (impulse) to a test structure

(output: N, lbf, ...)





to measure force acting on a test structure

(output: N, lbf, ...)

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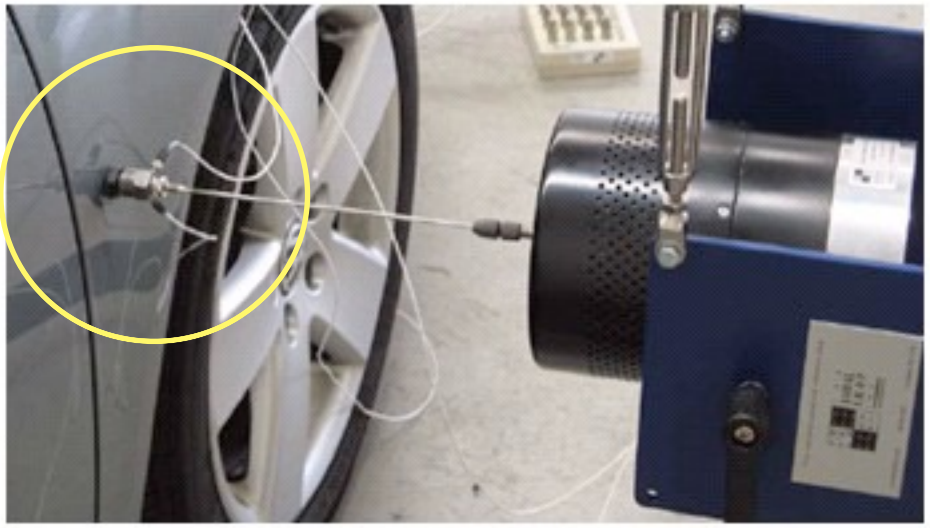
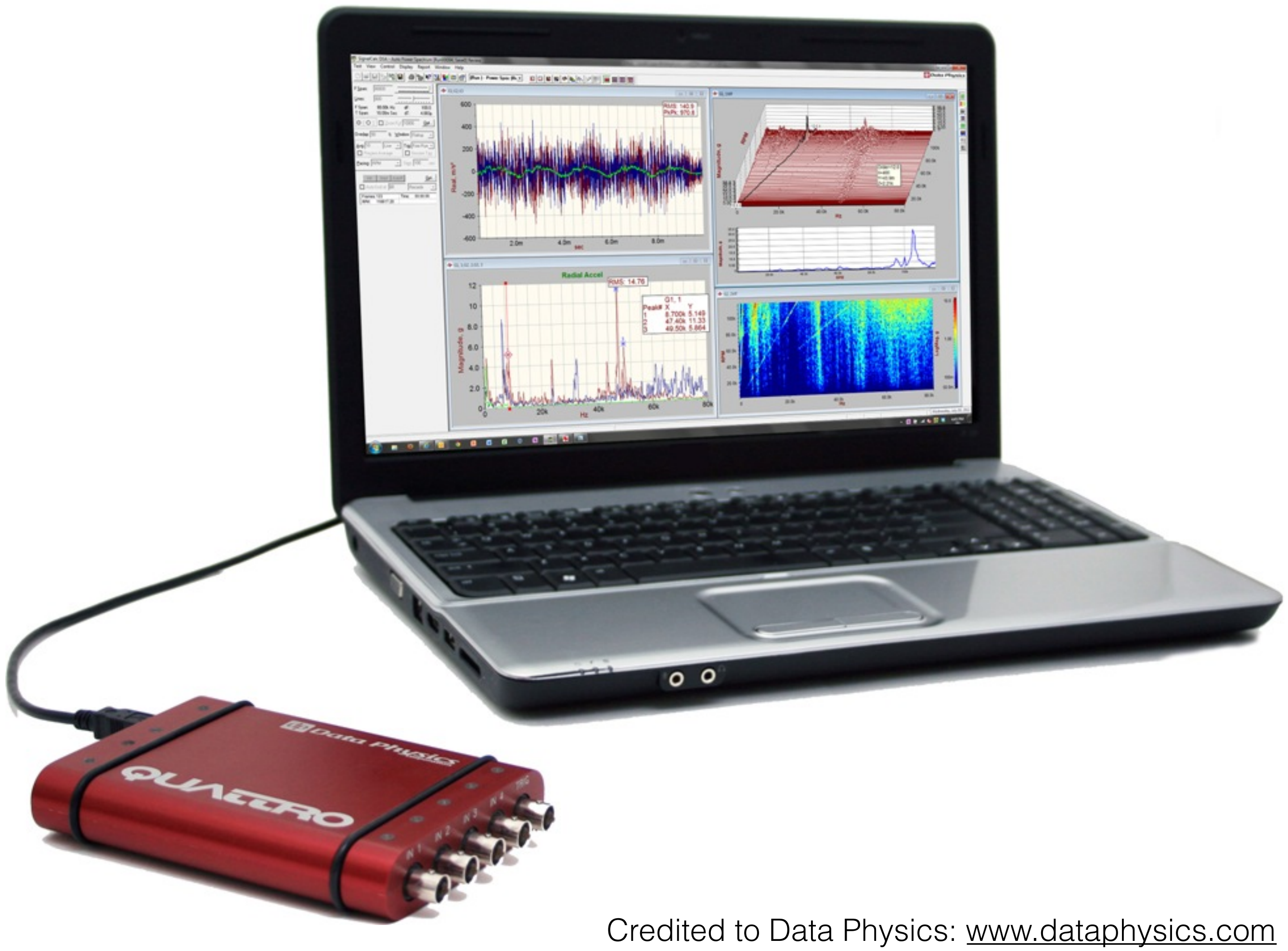


Signal Analyzer

to process the signals recorded from vibration sensors (accelerometer, impact hammer, force gauge, etc)



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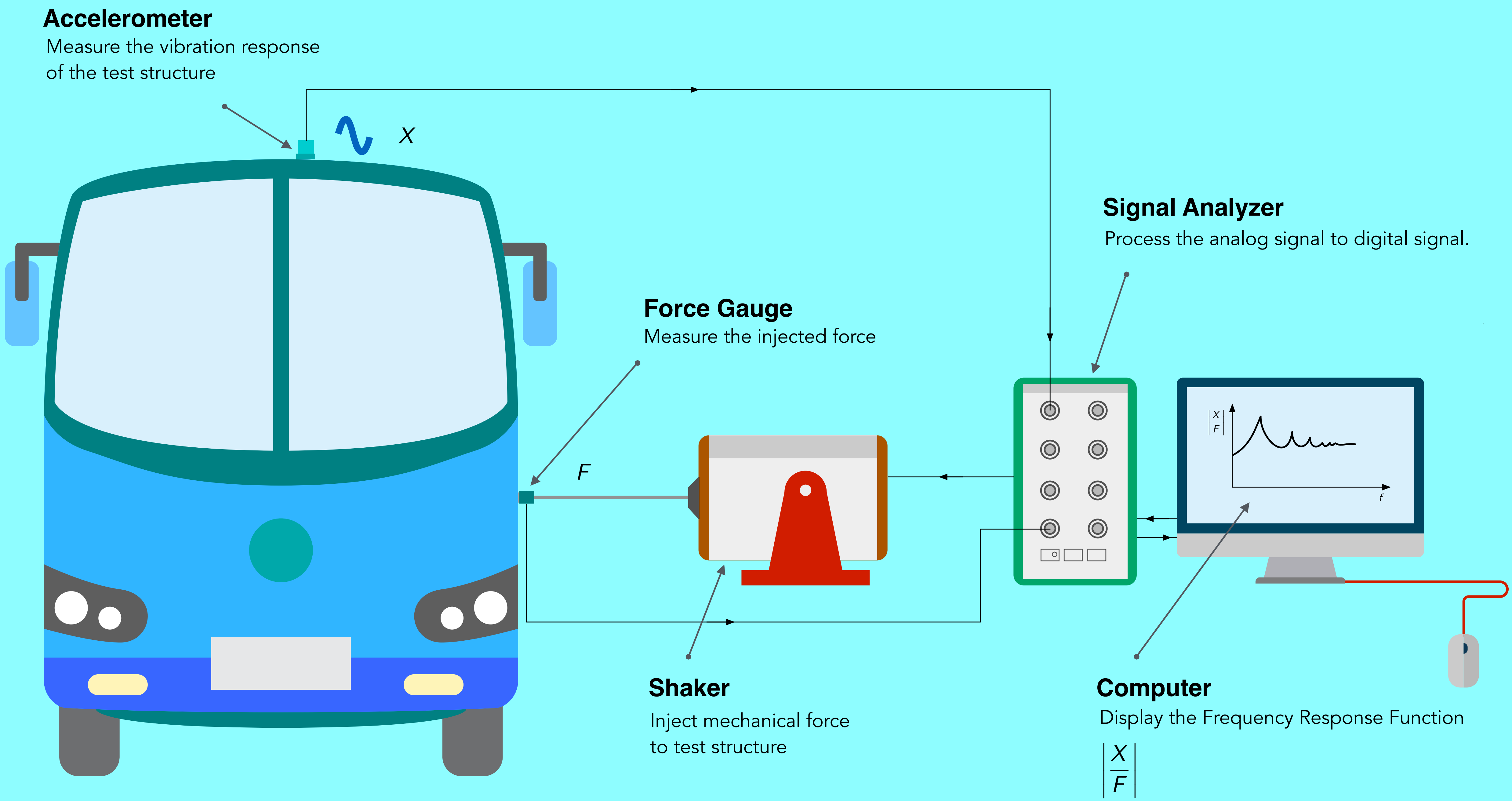
Instrumentation Setup



My website:
http://www.azmaputra.com

My white-board animation videos:
http://www.youtube.com/c/AzmaPutra-channel

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