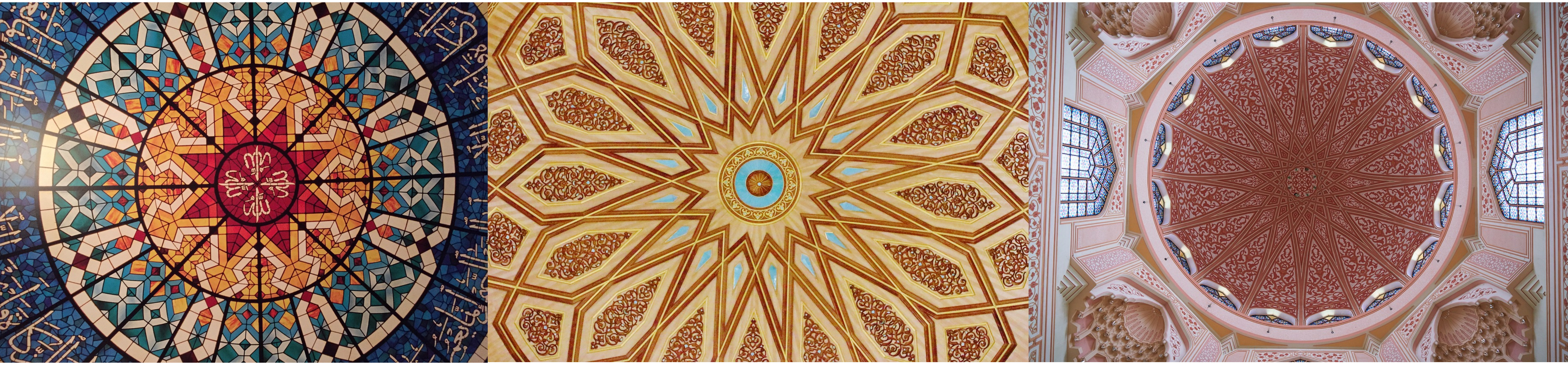

MECHANICAL VIBRATION

BMCG 3233

CHAPTER 4: HARMONIC FORCED VIBRATION (PART 1)

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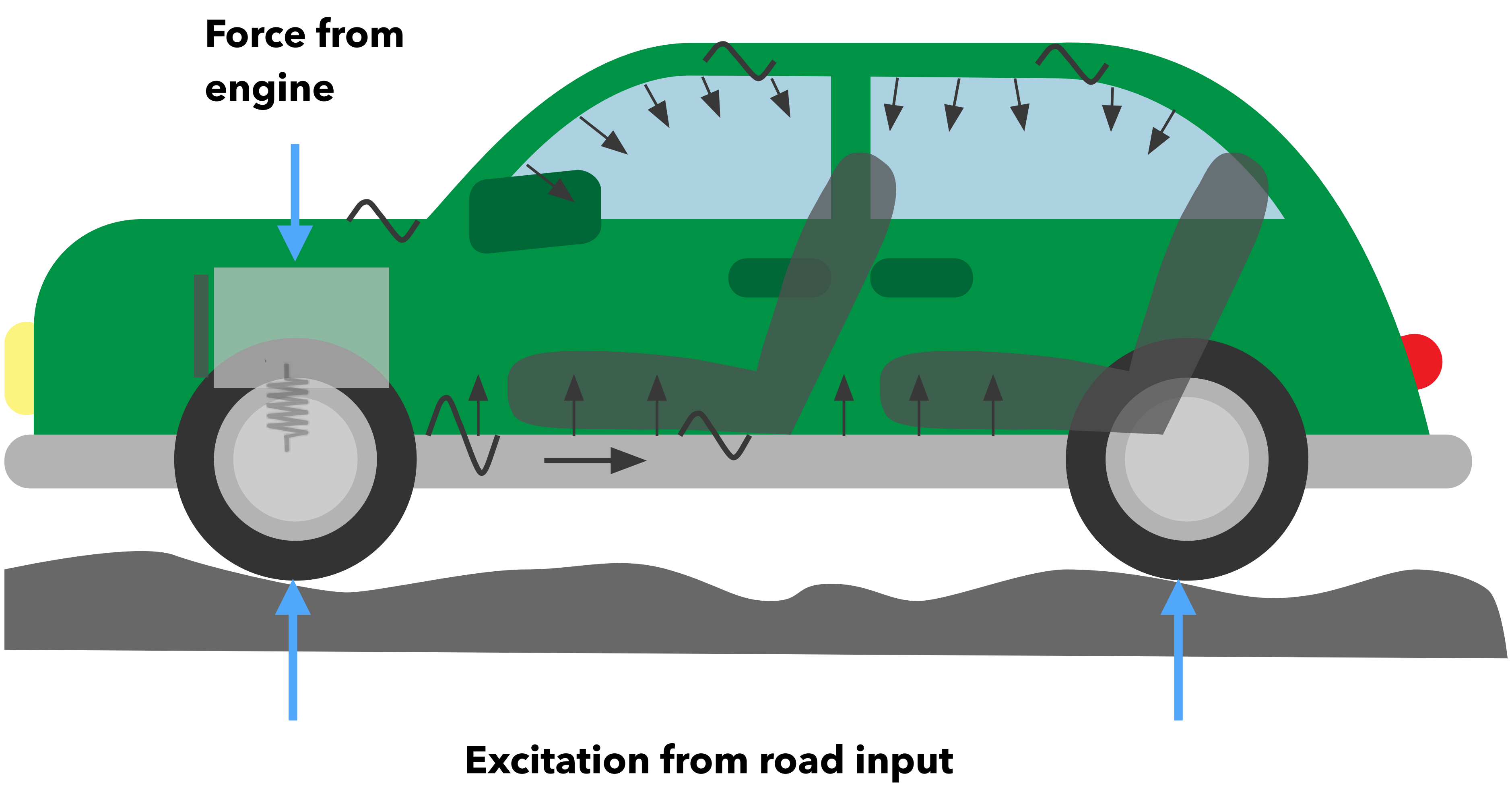
CONTENTS

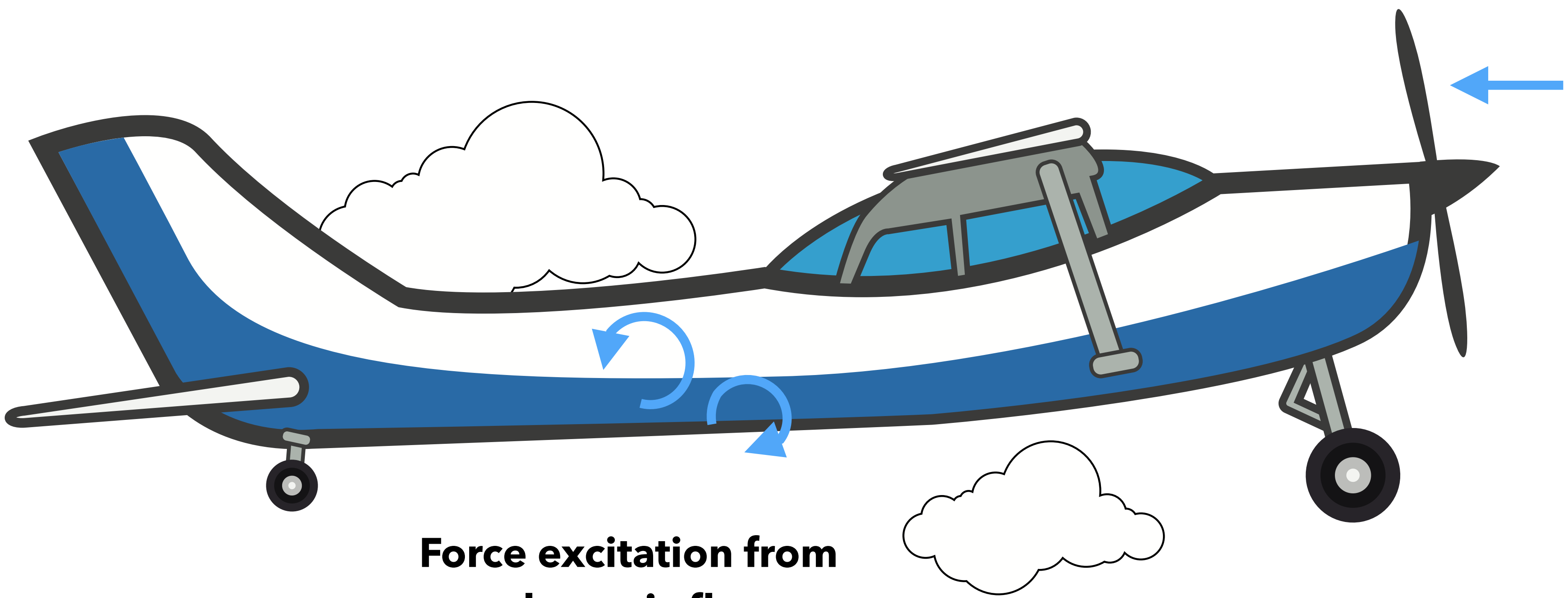
LEARNING OBJECTIVES

1. Derive the Frequency Response Function
2. Solve vibration problem due to damped forced vibration.

Vibration caused by force exciting the structure

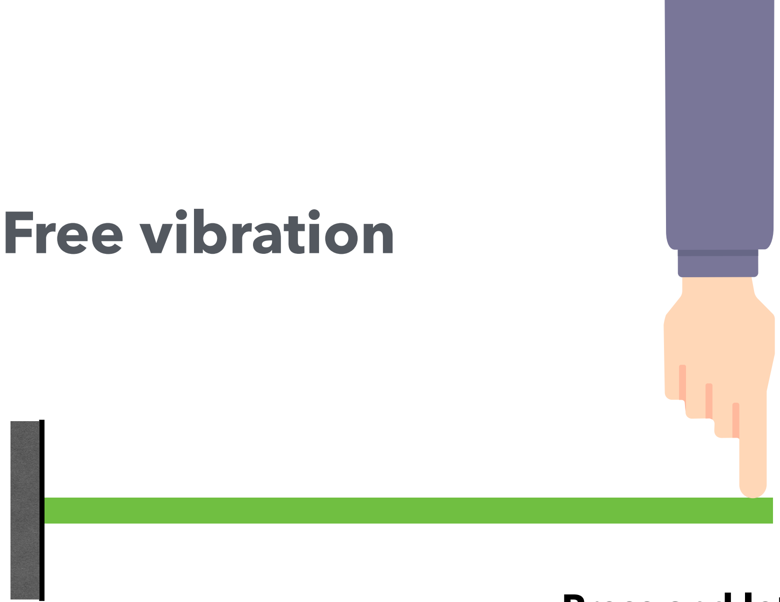


Force excitation from propeller

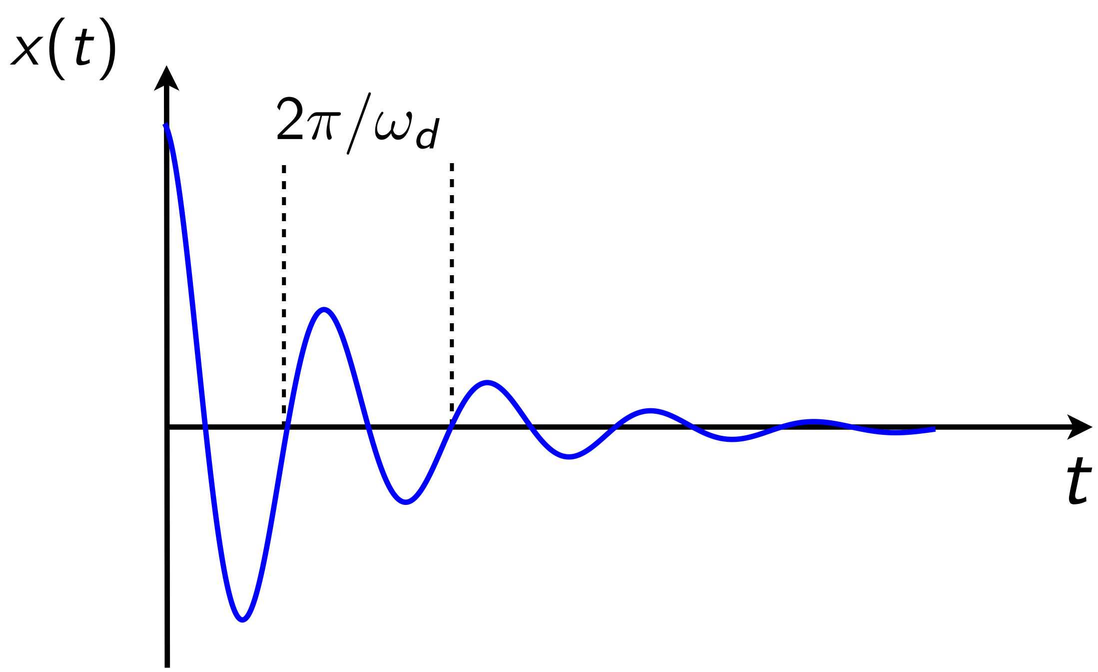


Force excitation from
aerodynamic flow

Free vibration

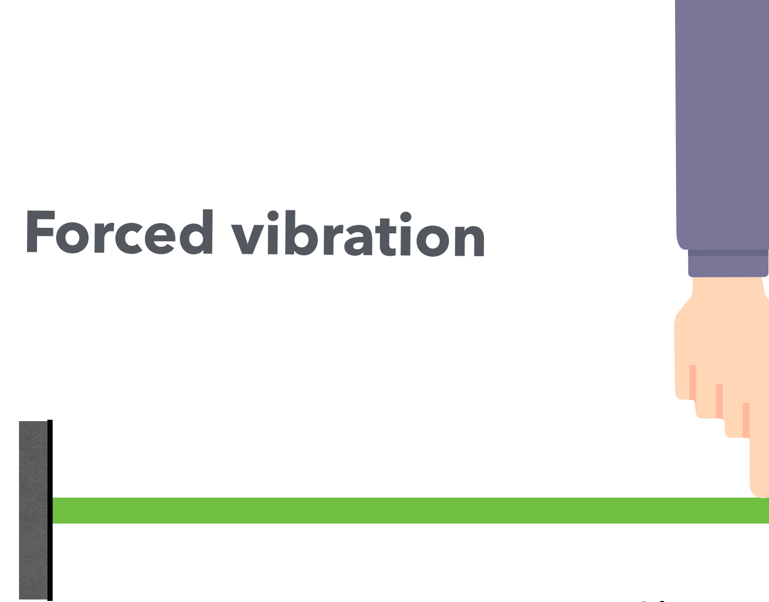


Press and let go

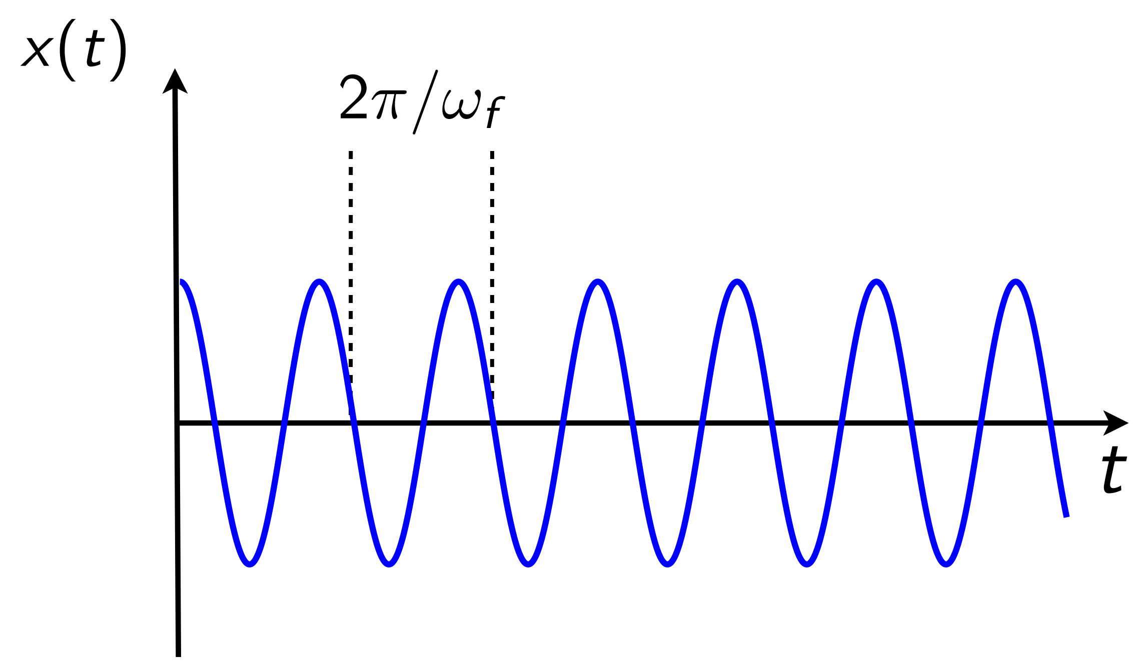


Vibration decays to zero

Forced vibration



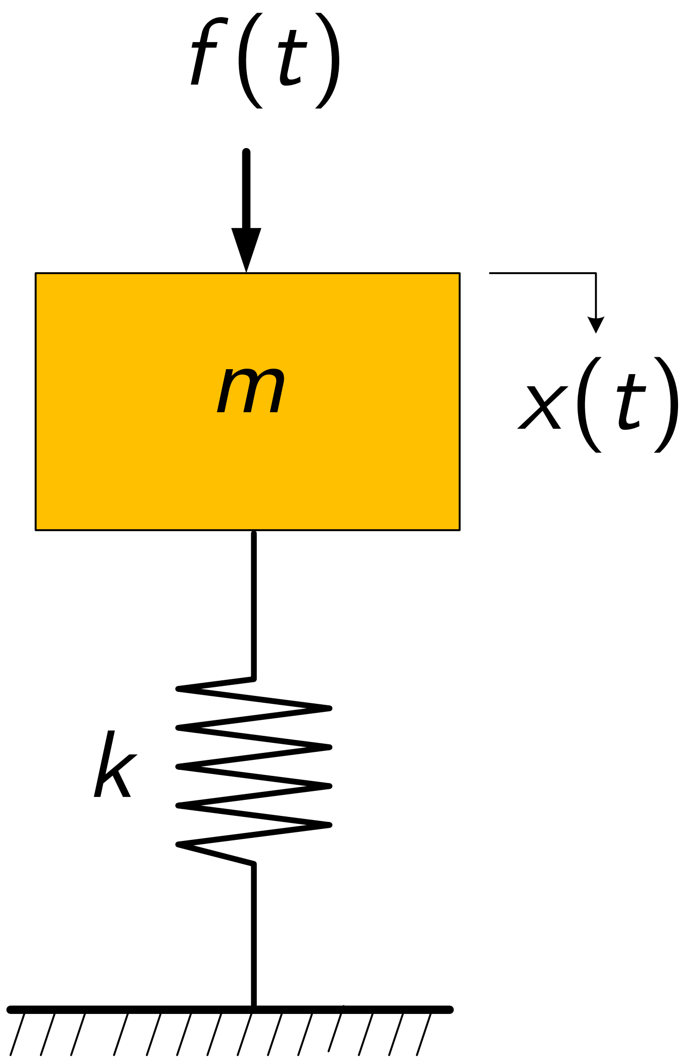
Give constant frequency



Vibration keeps going forever, as long as the force still exists

4.1

UNDAMPED FORCED VIBRATION



Equation of motion:

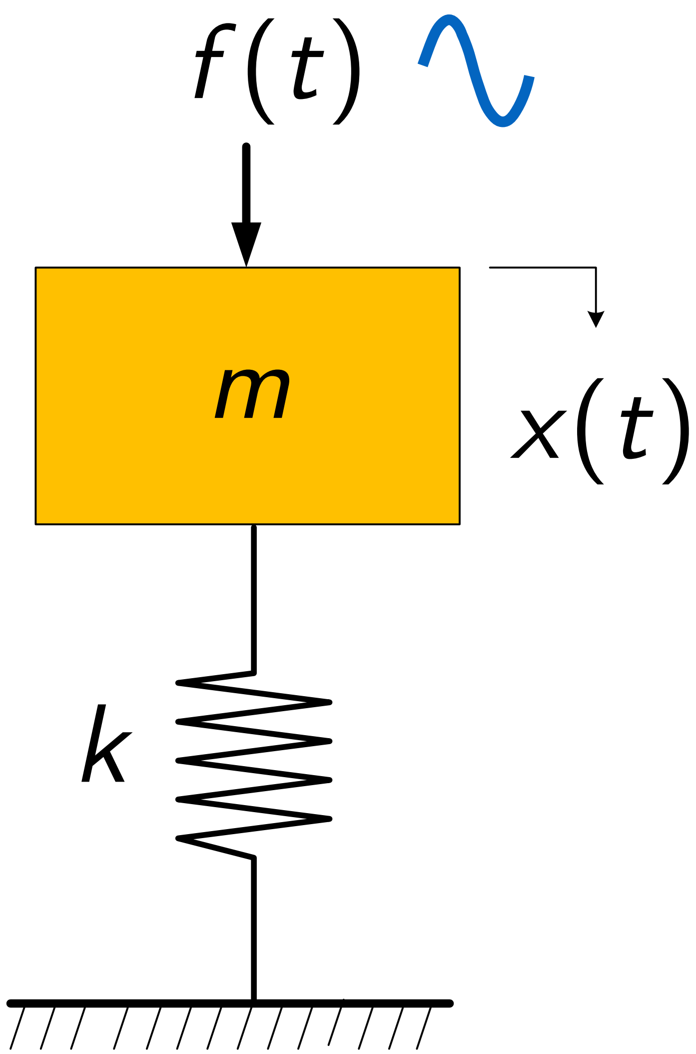
$$m\ddot{x}(t) + kx(t) = f(t)$$

Force $f(t)$ can be:

- **Time harmonic** - rotating machine
- **Transient** (finite duration) - bump or impact
- **Random** (unpredictable) - wind load

We will focus on time harmonic excitation:

$$f(t) = F\sin(\omega t + \phi)$$



For harmonic force, equation of motion:

$$m\ddot{x}(t) + kx(t) = F\sin(\omega t + \phi)$$

Response will also be harmonic:

$$x(t) = \underbrace{X}_{\text{Magnitude of } x(t)}\sin(\omega t + \phi)$$

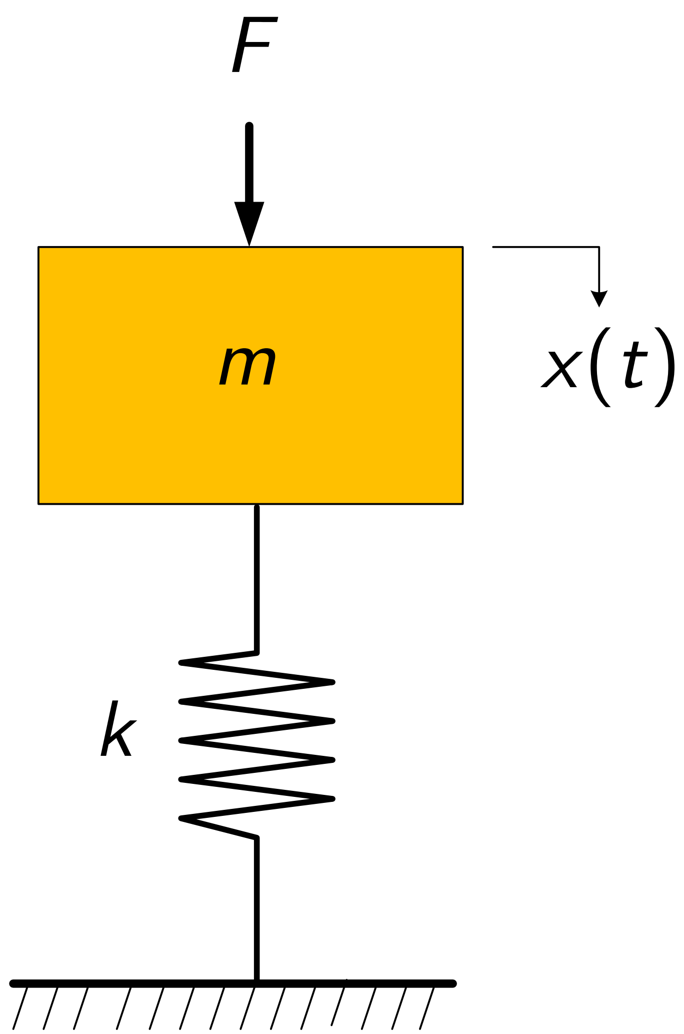
This gives steady-state vibration amplitude:

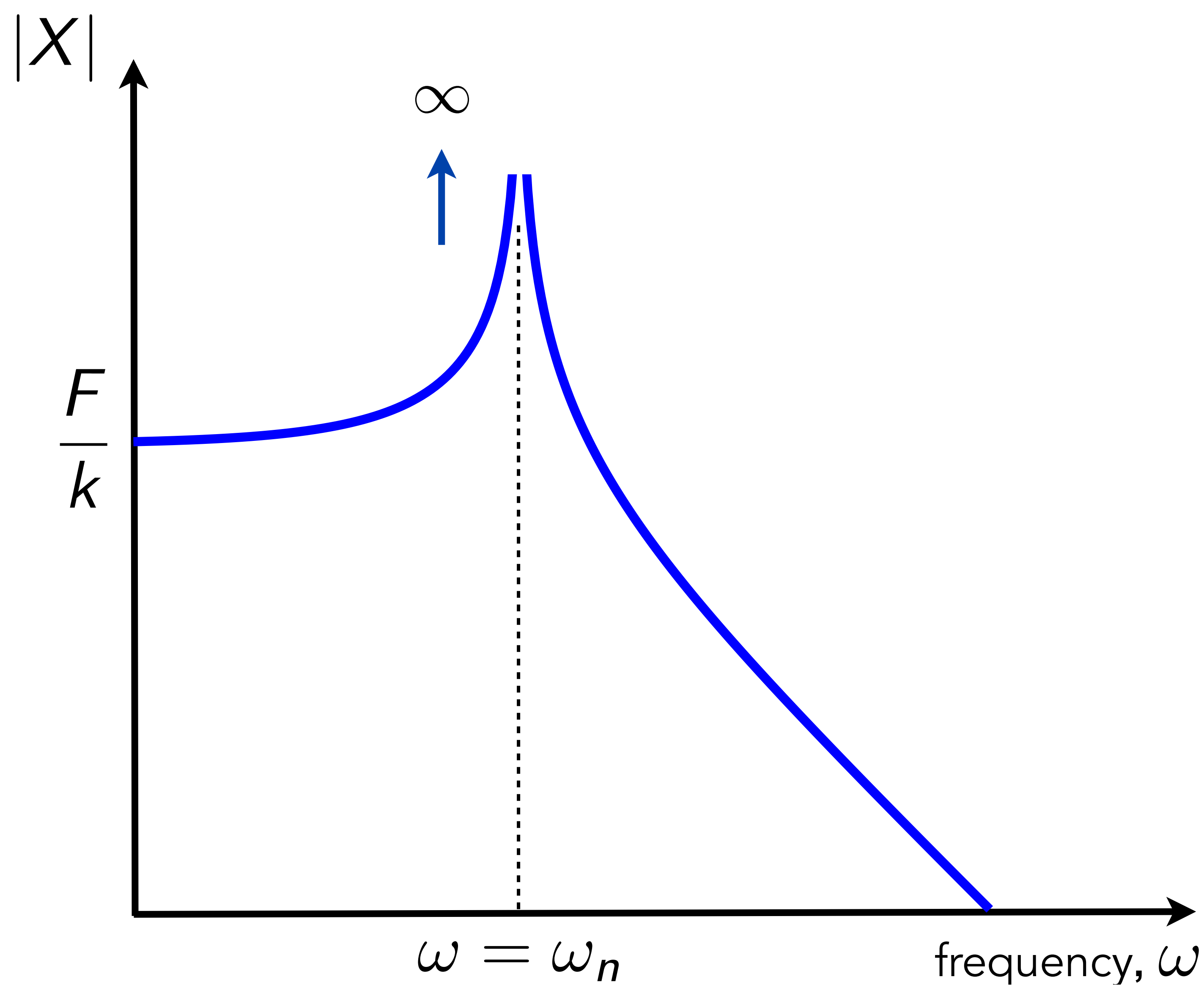
$$X = \frac{F}{k - \omega^2 m}$$

$$X = \frac{F}{k - \omega^2 m} = \frac{F}{k}\left(\frac{1}{1-(\omega/\omega_n)^2}\right)$$

Excite this system with whole frequency of excitation

Characteristic of the SDOF system when excited with dynamic force





Static force ($\omega = 0$):

$$X = \frac{F}{k}$$

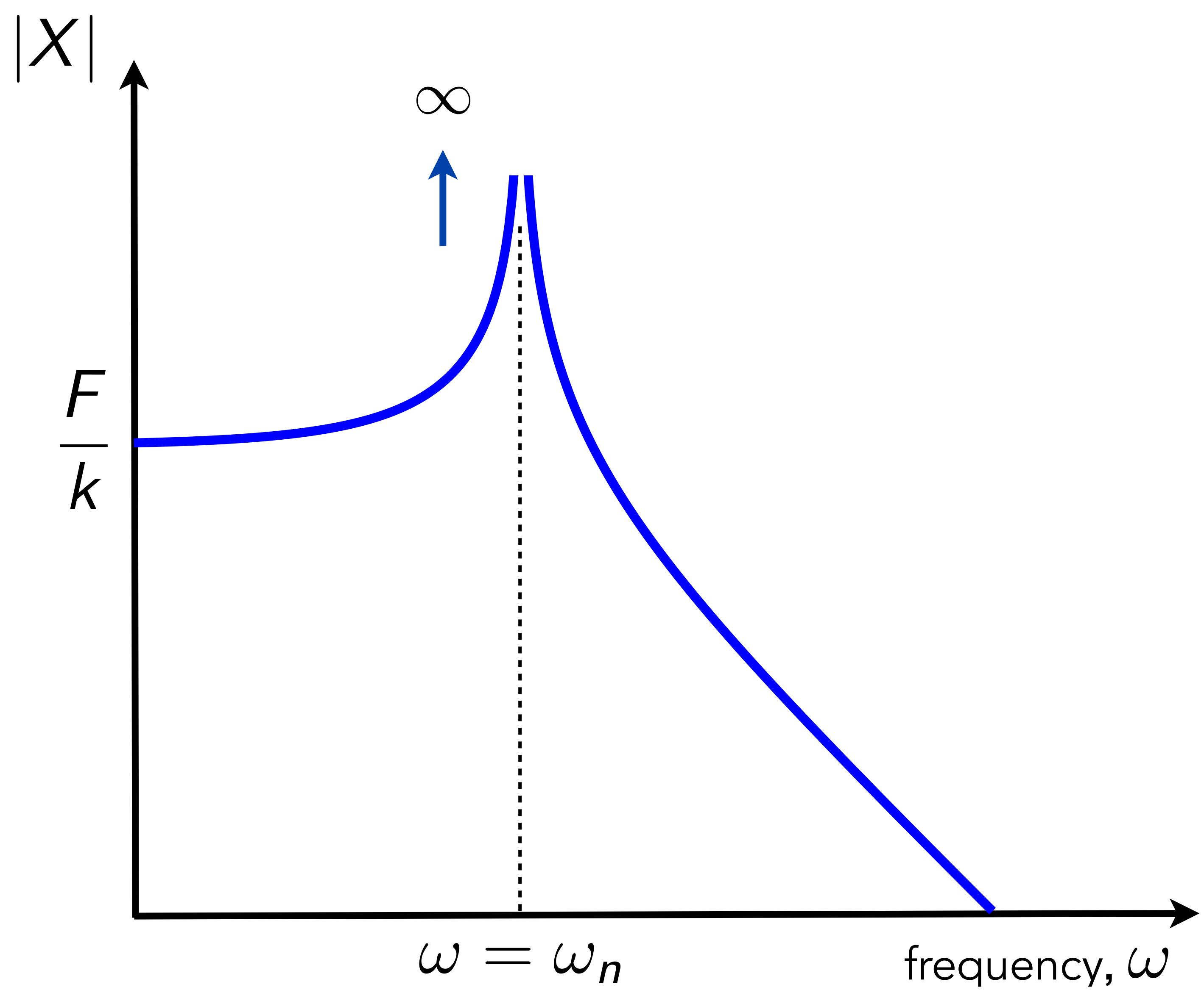
Dynamic force:

$\boxed{X \text{ depends on } \omega}$

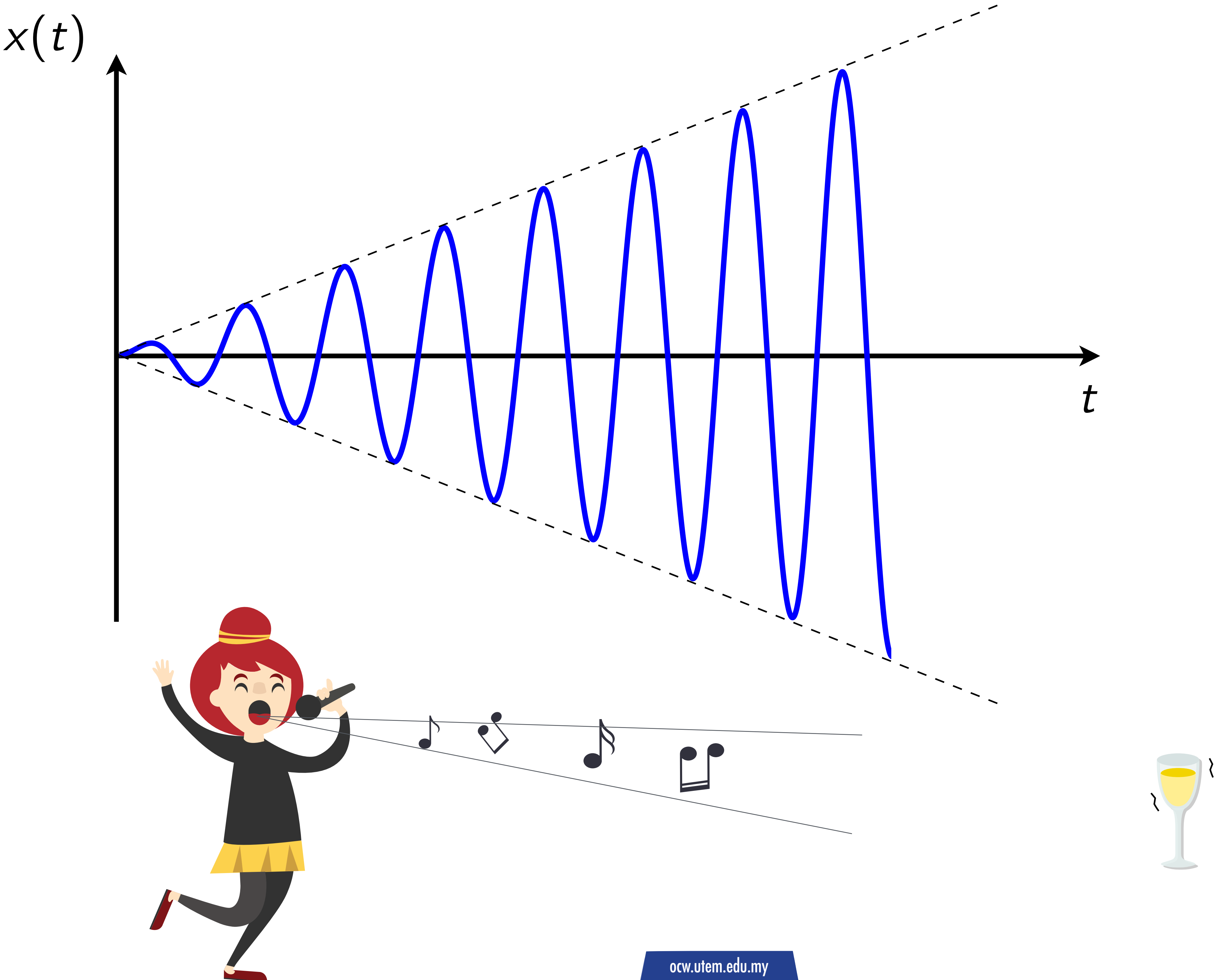
If $\omega = \omega_n$:

$\boxed{X \text{ becomes very LARGE!!}}$

RESONANCE

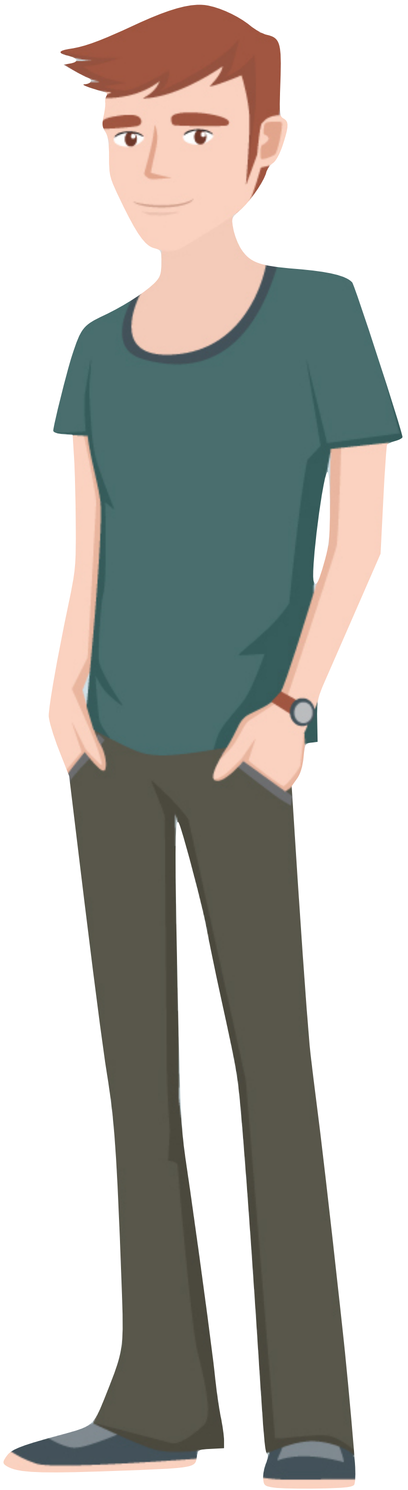


Time response of undamped vibration at resonance.




When the frequency of the voice from
a Soprano singer matches
the natural frequency of the glass.

It happened in 1940, when resonance from the wind destroyed this massive structure - Tacoma Bridge, Washington, USA.

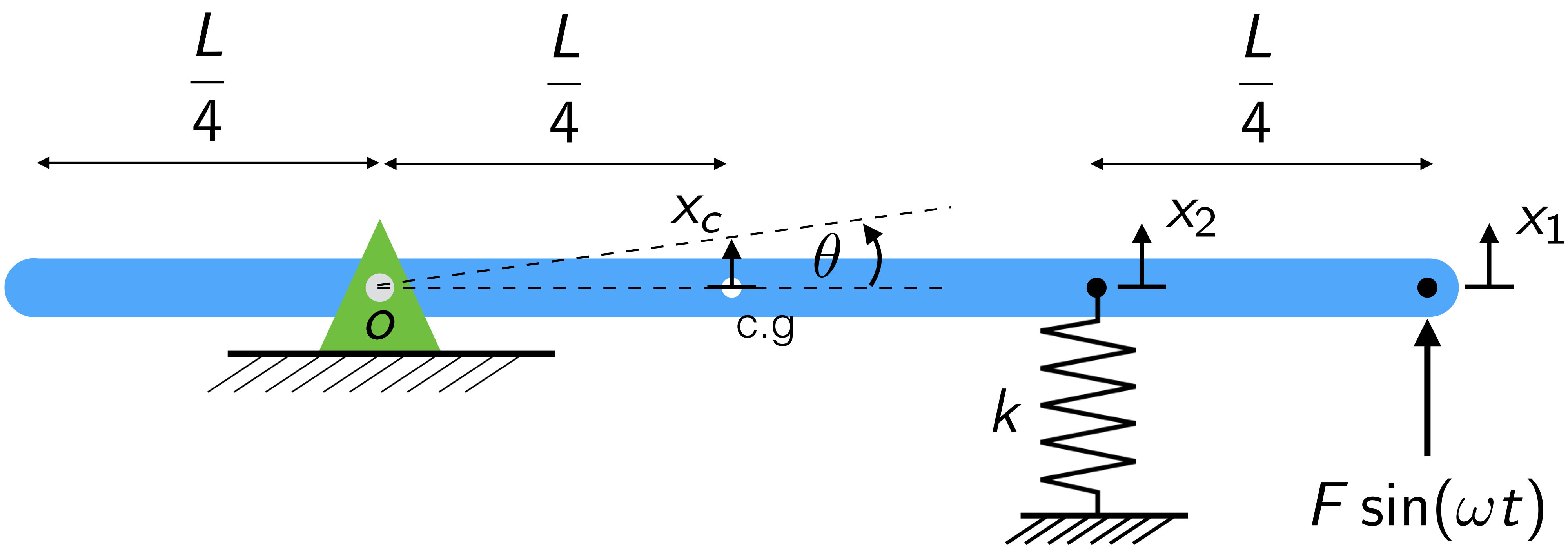


As a vibration engineer, you job is
to ensure that a designed structure has
natural frequency away from
the frequency of the excitation force.

Example 4.1

Calculate the magnitude of the Frequency Response Function (FRF)!

Use x_1 as the generalised coordinate.



Use the D'Alembert principle

The external moment at point O: $\sum (M_{ext})_o = -kx_2\left(\frac{L}{2}\right) + F\sin(\omega t)\left(\frac{3L}{4}\right)$

The inertial moment at point O: $\sum (M_{int})_o = J\ddot{\theta} + m\ddot{x}_c\left(\frac{L}{4}\right)$

The D'Alembert principle:

$$\sum (M_{\text{ext}})_o = \sum (M_{\text{int}})_o$$

$$-kx_2\left(\frac{L}{2}\right) + F\sin(\omega t)\left(\frac{3L}{4}\right) = J\ddot{\theta} + m\ddot{x}_c\left(\frac{L}{4}\right)$$

Re-arrange:

$$x_2 = \tfrac{1}{3}x_1$$

$$\boxed{J\ddot{\theta} + m\ddot{x}_c\left(\frac{L}{4}\right) + kx_2\left(\frac{L}{2}\right) = F\left(\frac{3L}{4}\right)\sin(\omega t)}$$

$$\ddot{x}_c = \tfrac{1}{3}\ddot{x}_1$$

$$\ddot{\theta} = \left(\frac{4}{3L}\right)\ddot{x}_1$$

After substitution:

$$\left(\frac{4J}{3L}\right)\ddot{x}_1 + \left(\frac{mL}{12}\right)\ddot{x}_1 + \left(\frac{kL}{6}\right)x_1 = F\left(\frac{3L}{4}\right)\sin(\omega t)$$

Simplified:

$$\left(\frac{16J}{L^2} + m\right)\ddot{x}_1 + 2kx_1 = 9F\sin(\omega t)$$

We are interested in steady-state response, we can use C.E. N.

The force can be expressed as $f(t) = Fe^{j\omega t}$ **and substitute:** $x_1(t) = X_1 e^{j\omega t}$

$$\boxed{-\omega^2\left(\frac{16J}{L^2} + m\right)X_1 + 2kX_1 = 9F} \qquad (*)$$

From (*), the Frequency Response Function (FRF):

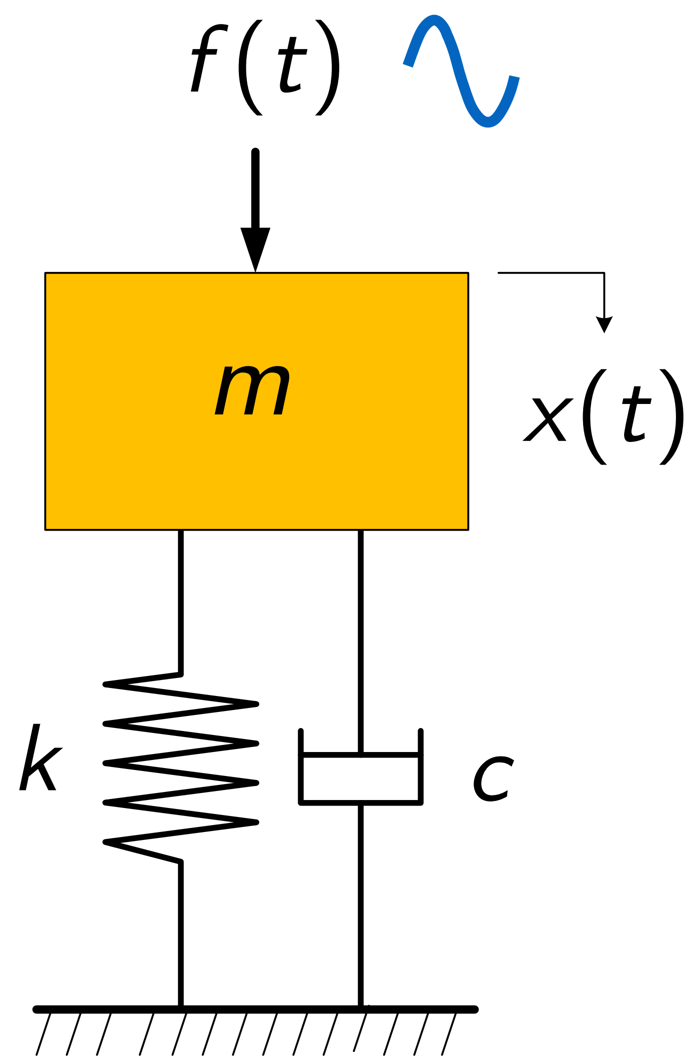
$$\left|\frac{X_1}{F}\right| = \frac{9}{2k - \omega^2\left(\frac{16J}{L^2} + m\right)}$$

The displacement per unit force.

This FRF is called **RECEPTANCE**.

4.2

DAMPED FORCED VIBRATION

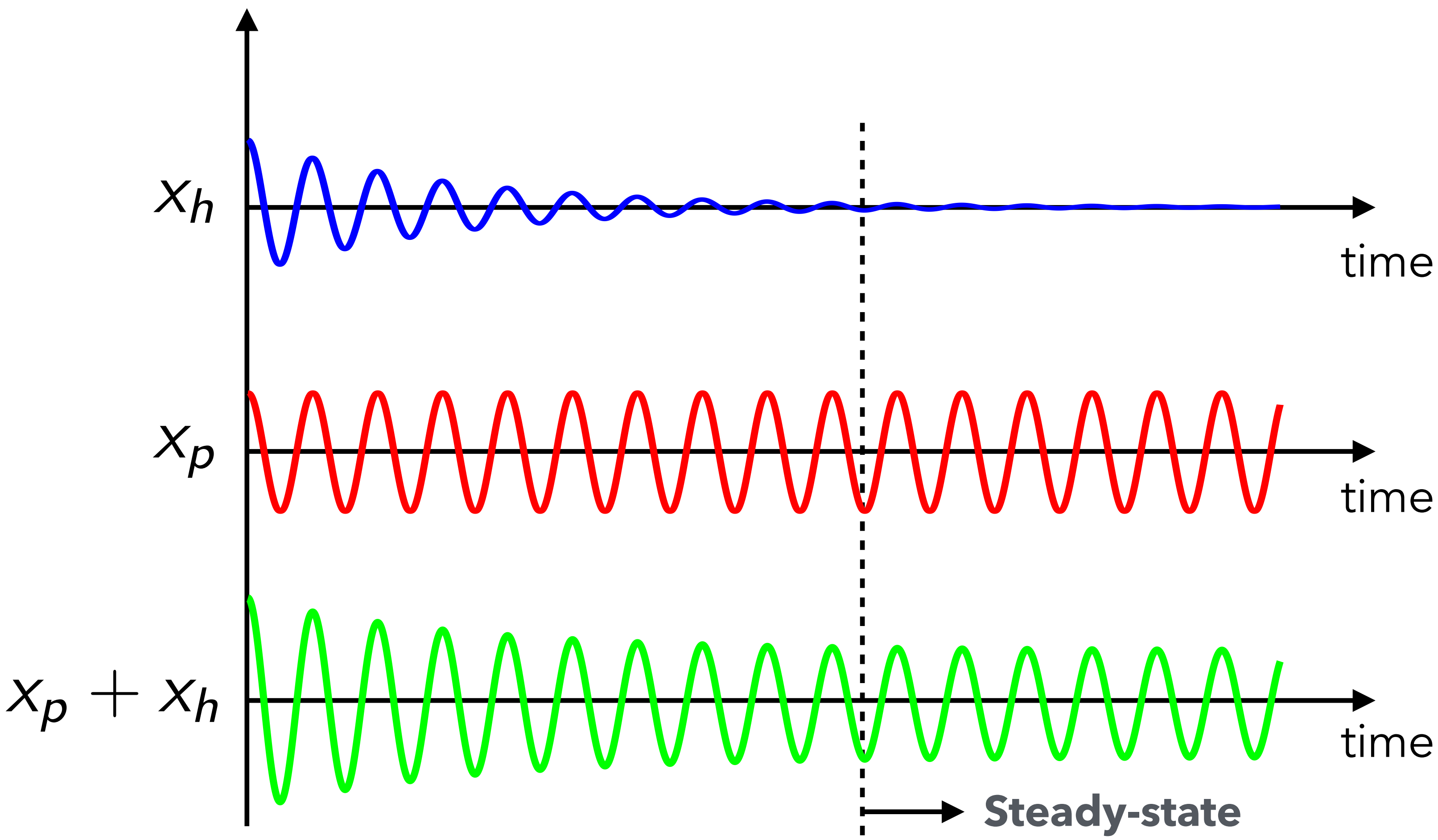


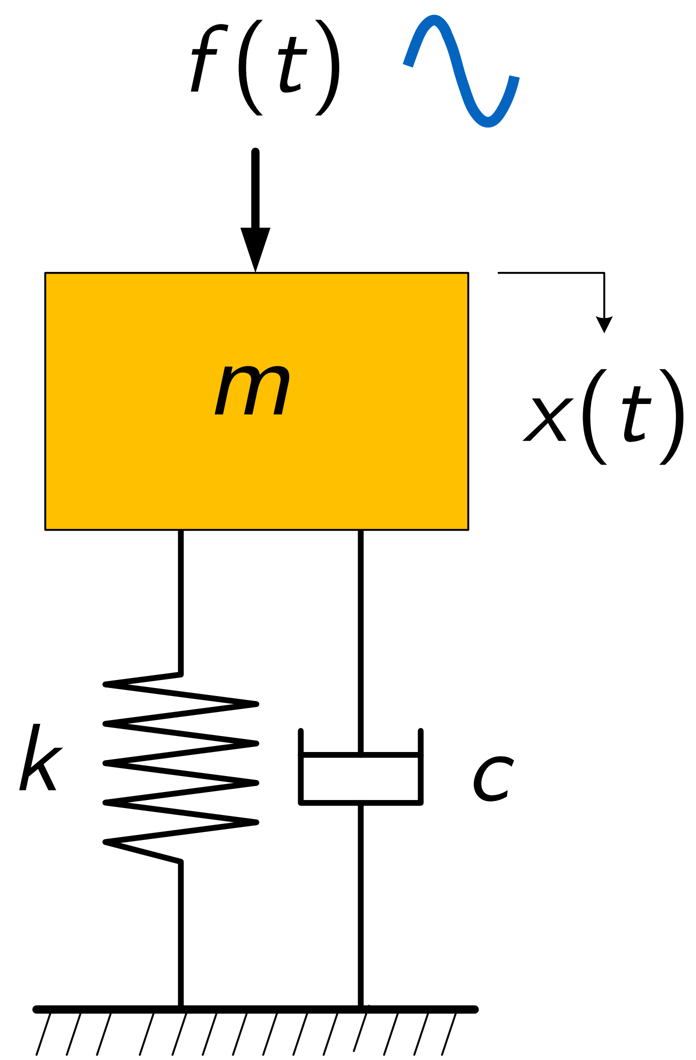
Equation of motion:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

Homogeneous solution, x_h

Particular solution, x_p





Equation of motion:

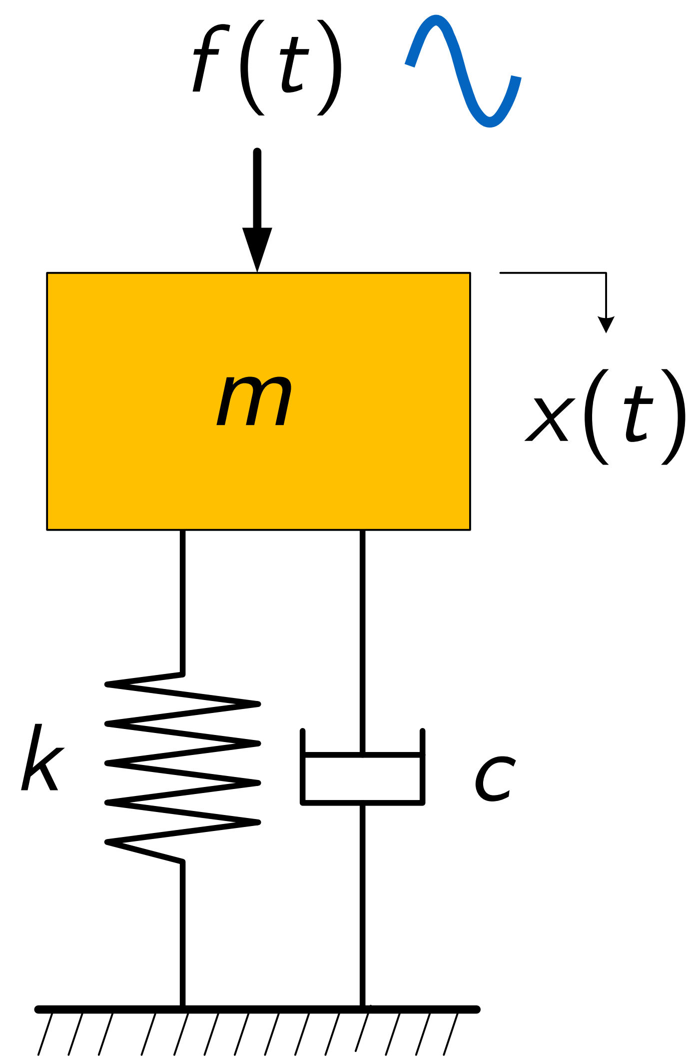
$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

We are interested in the steady-state response.

For convenience, we can use the complex exponential notation:

$$f(t) = Fe^{j\omega t}, \quad x(t) = Xe^{j\omega t}$$

Complex amplitude of $f(t)$ Complex amplitude of $x(t)$



We obtain ratio of displacement to force:

$$\boxed{\frac{X}{F} = \frac{1}{k - m\omega^2 + j\omega c}}$$

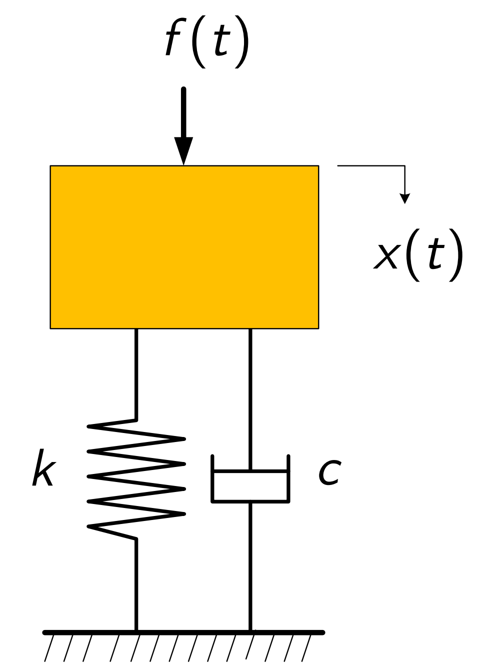
Using $\omega_n^2 = k/m$ **and** $c = 2\zeta\omega_n m$

We can have another expression:

(*) $$\boxed{\frac{X}{F} = \frac{1}{k}\left(\frac{1}{1 - (\omega/\omega_n)^2 + j2\zeta\omega/\omega_n}\right)}$$

The magnitude of * as a **function of frequency**:

$$\left|\frac{X}{F}\right| = \frac{1/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}}$$



Usually called **Frequency Response Function (FRF)**

Low frequency: $\omega \ll \omega_n \Rightarrow |X/F| = 1/k$ **Stiffness controlled**

Resonance: $\omega = \omega_n \Rightarrow |X/F| = 1/(2\zeta k)$ **Damping controlled**

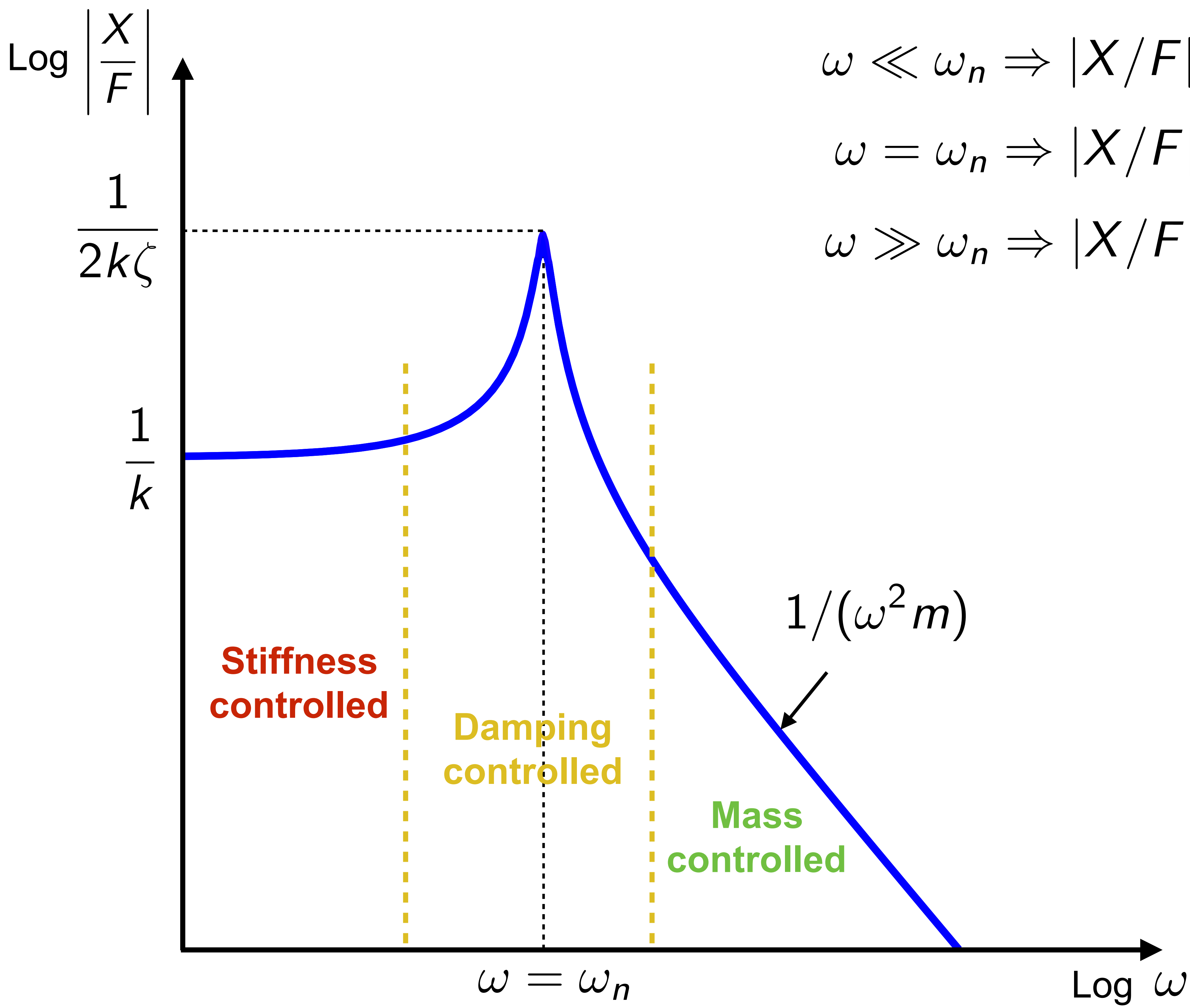
High frequency: $\omega \gg \omega_n \Rightarrow |X/F| = 1/(\omega^2 m)$ **Mass controlled**

FRF graph

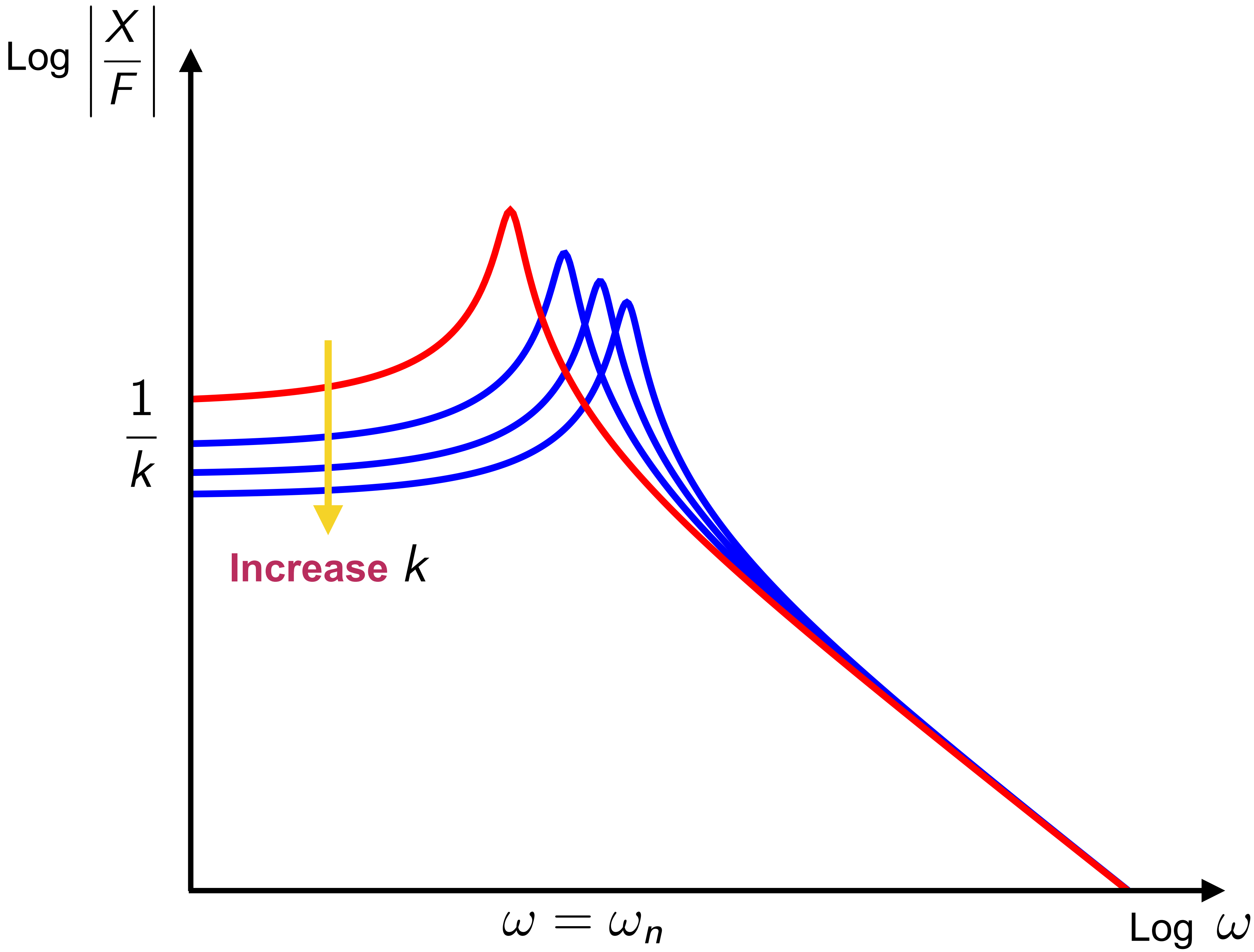
$$\omega \ll \omega_n \Rightarrow |X/F| = 1/k$$

$$\omega = \omega_n \Rightarrow |X/F| = 1/(2\zeta k)$$

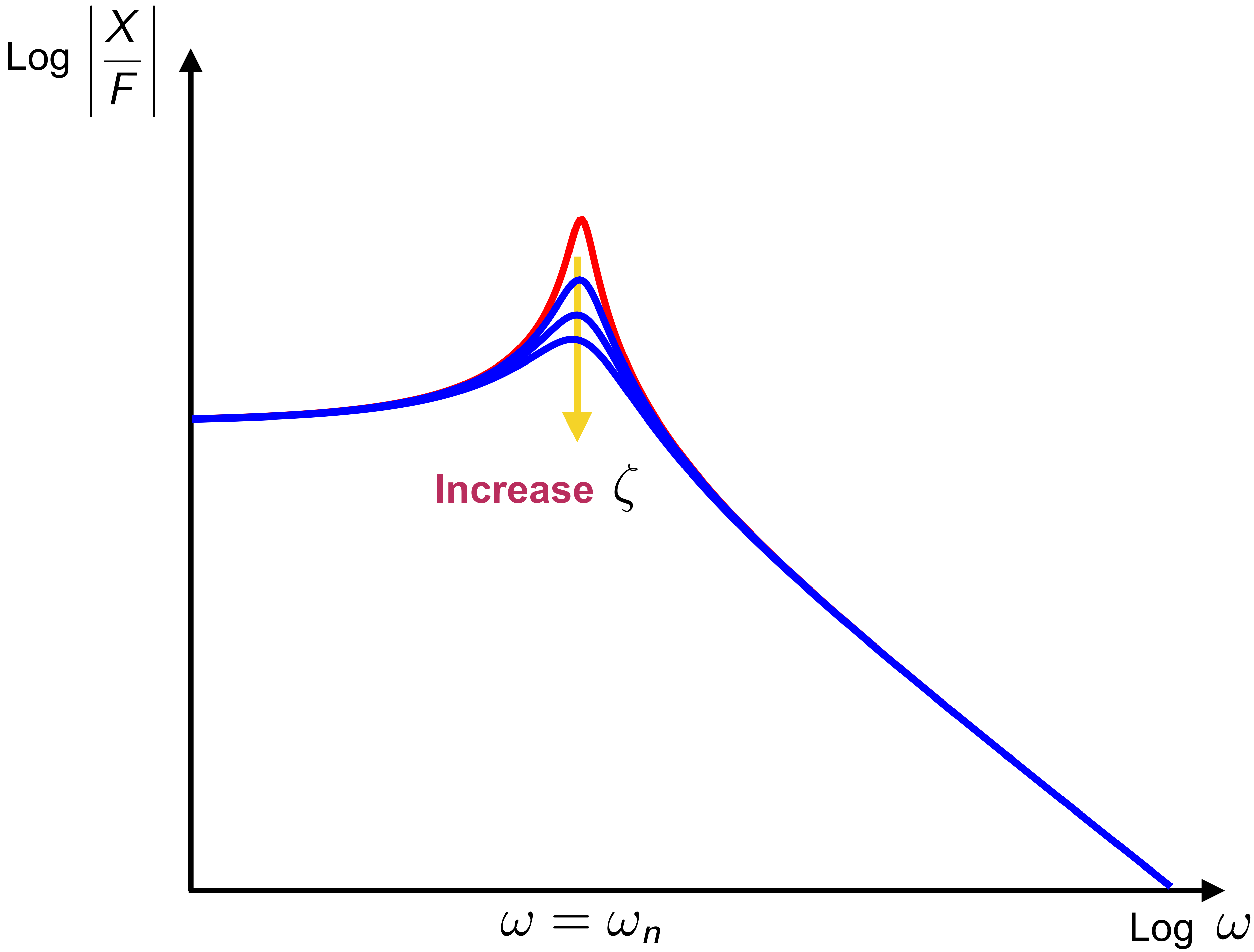
$$\omega \gg \omega_n \Rightarrow |X/F| = 1/(\omega^2 m)$$



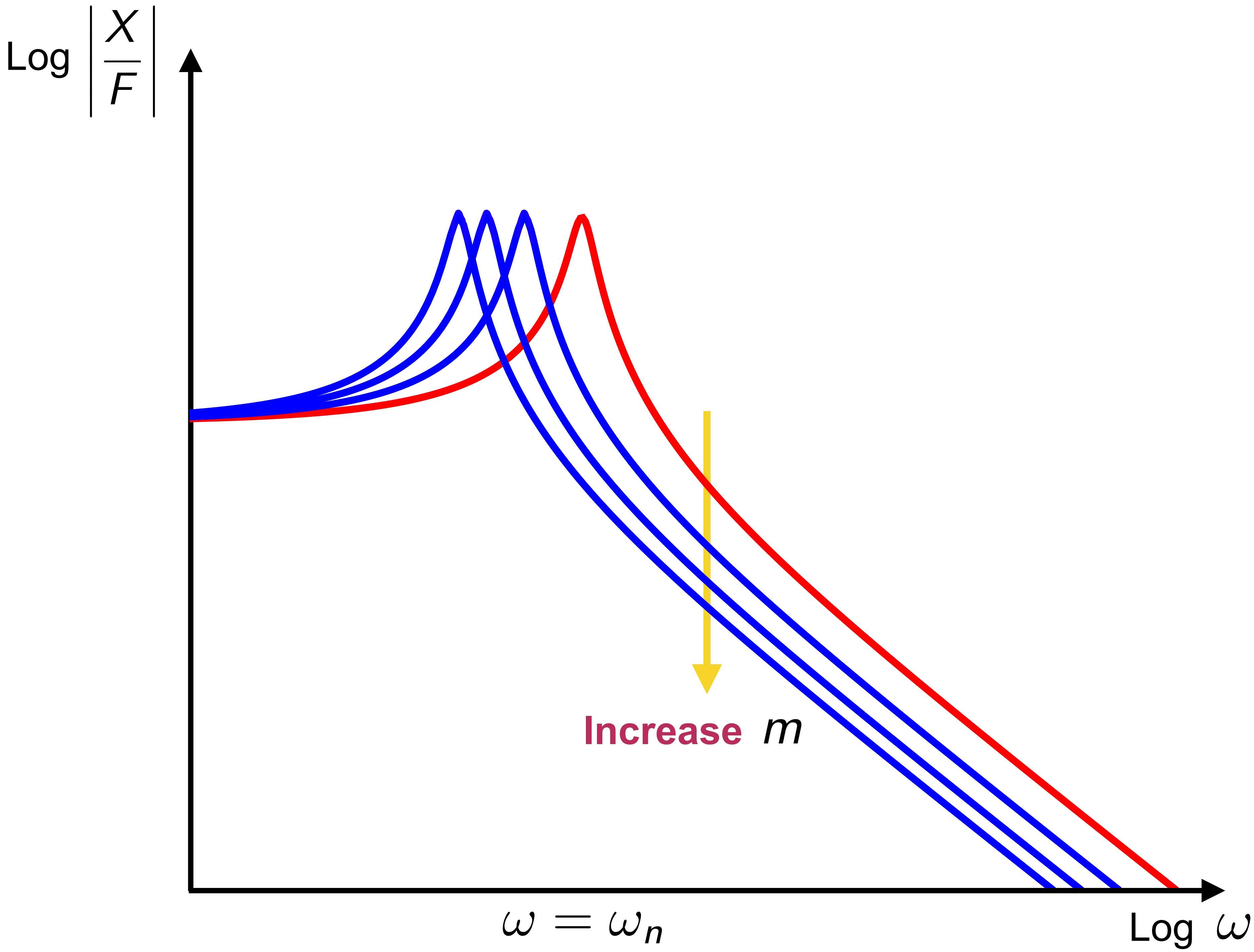
Stiffness controlled



Damping controlled



Mass controlled



How to know the phase between the force and the response?

From (*)

$$\angle\left|\frac{X}{F}\right| = \phi = -\tan^{-1}\left[\frac{\frac{2\zeta\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

For small ζ :

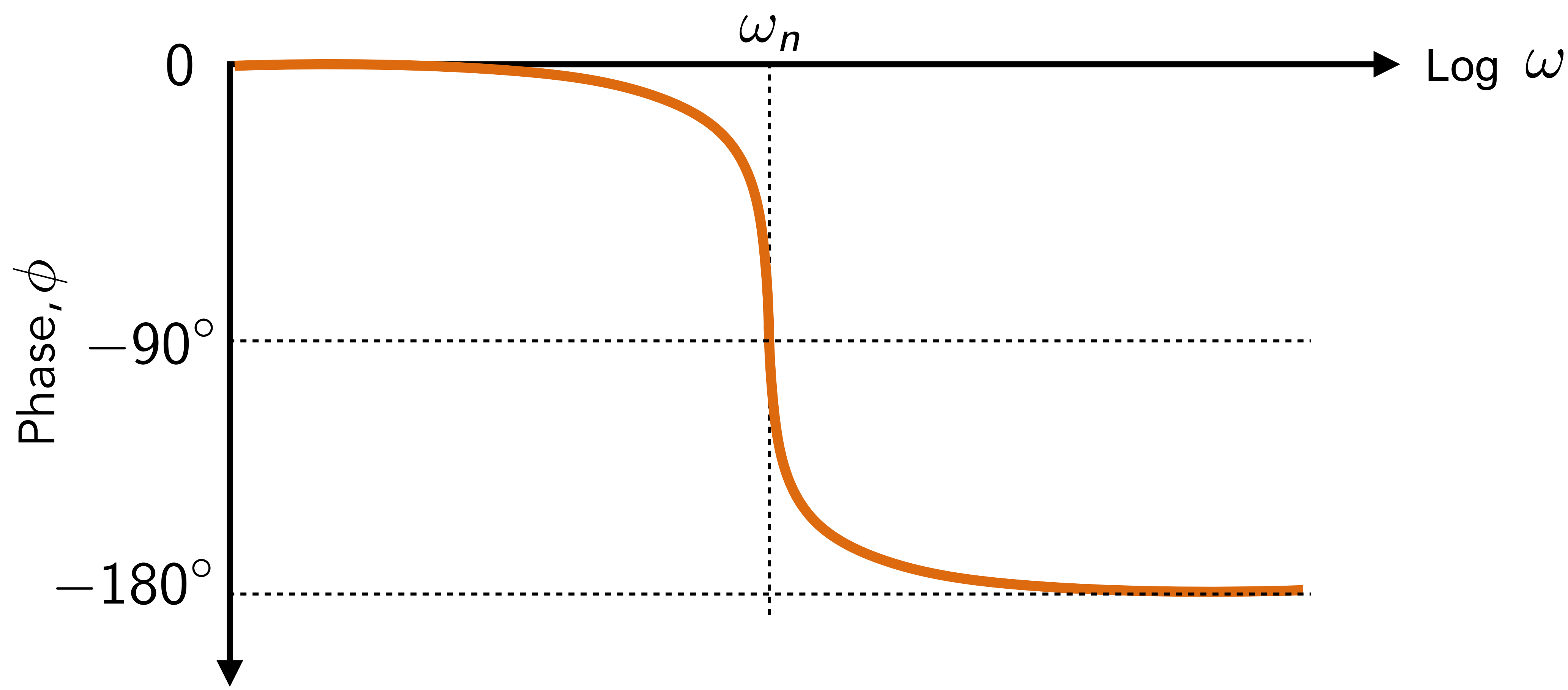
$$\omega \ll \omega_n \Rightarrow \phi \approx -\tan^{-1}(0) = 0$$

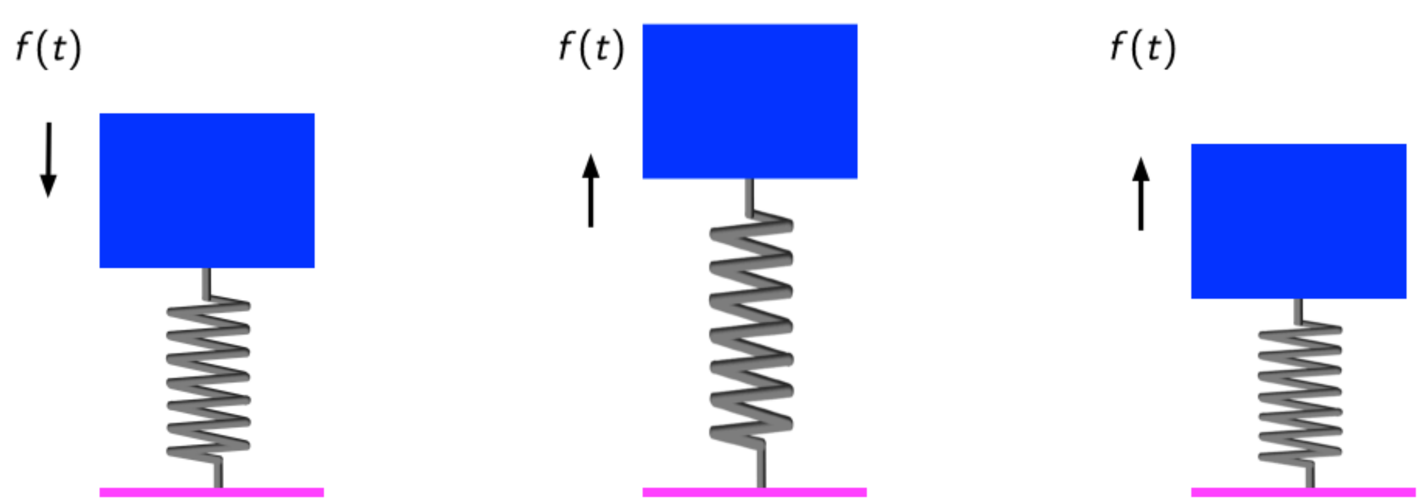
$$\omega = \omega_n \Rightarrow \phi \approx -\tan^{-1}(\infty) = -90^\circ$$

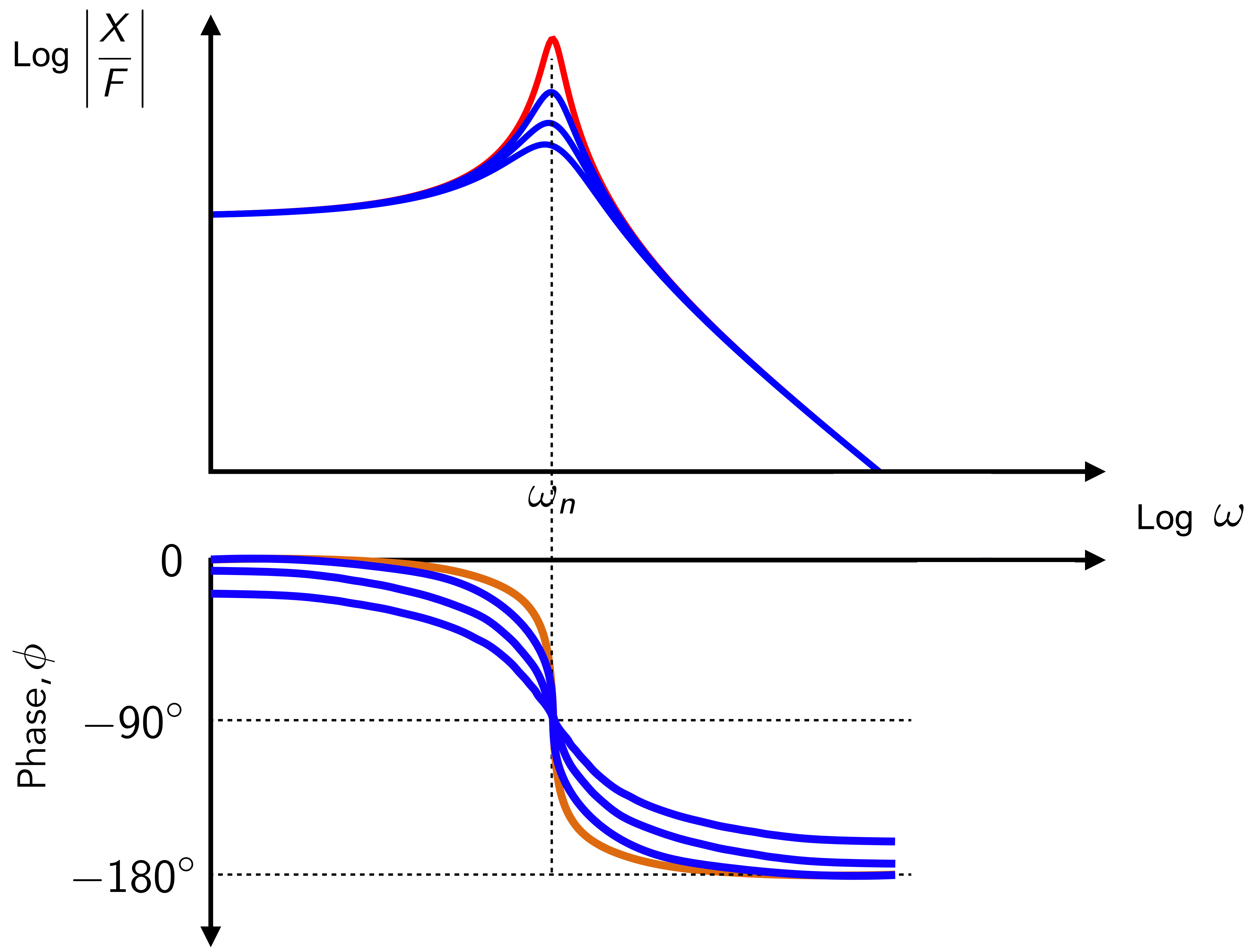
$$\omega \gg \omega_n \Rightarrow \phi \approx \tan^{-1}(0) = -180^\circ$$

Note: If $z = 1/(a+jb)$, the phase is $\phi = -\tan^{-1}(b/a)$

Phase graph





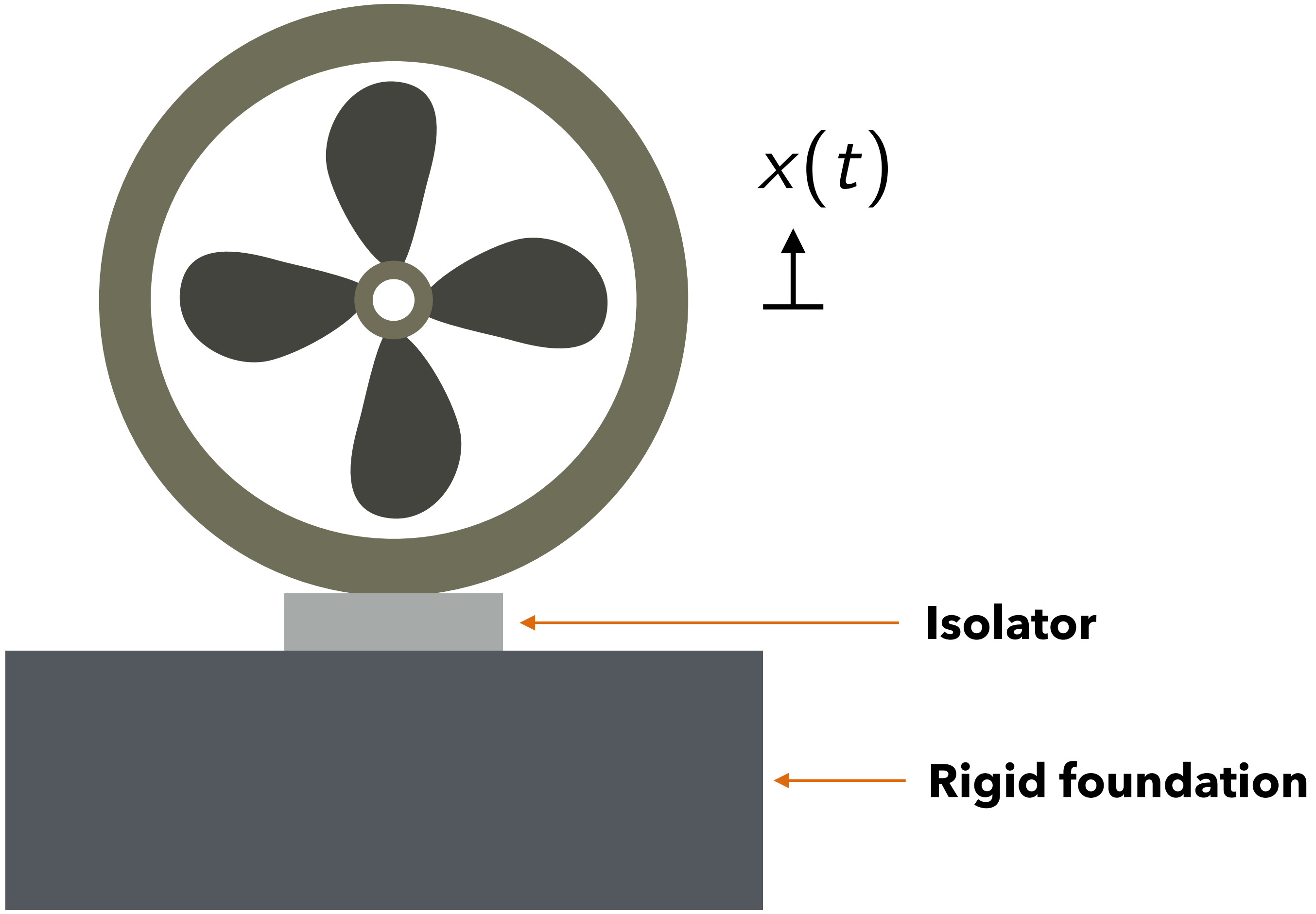


$\text{Log}\left|\frac{X}{F}\right|$
ω_n
Log ω
0
Phase, ϕ
$-90°$
$-180°$

Example 4.2

A 50 kg rotating machine is known to have vertical r.m.s velocity of 8 mm/s. The machine runs at operating frequency of 800 RPM. The machine is rested on rubber isolator with total stiffness constant of 10,000 N/m.

How to effectively reduce the vibration of the machine?



The operating frequency of the machine:

$$f = \frac{900}{60} = 15 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi(15) = 94.2 \text{ rad/s}$$

The natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10,000}{50}} = 14.14 \text{ rad/s}$$

The condition: $\omega \gg \omega_n$ **(Mass controlled)**

Thus the vibration can be effectively reduced by increasing the mass of the machine.

Interact with my animations:
http://www.azmaputra.com/animations/

My white-board animation videos:
http://www.youtube.com/c/AzmaPutra-channel

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