MECHANICAL VIBRATION

BMCG 3233

CHAPTER 3: HARMONIC FREE VIBRATION (PART 1)

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CONTENTS

LEARNING OBJECTIVES

1. Calculate the energy from mass and spring elements

2. Calculate the equivalent spring constant, and equivalent mass

3.1

SPRING ELEMENT

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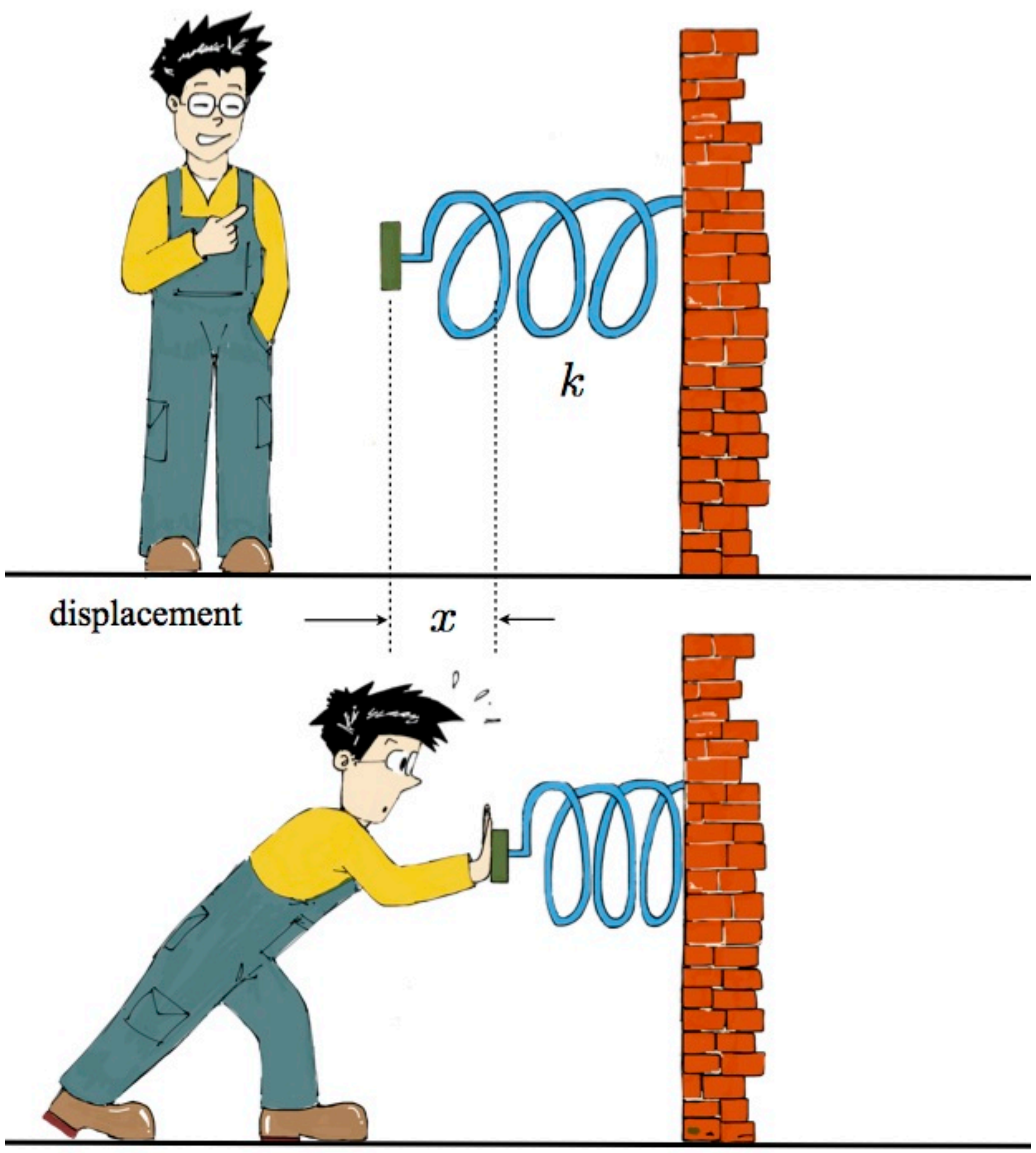
Spring element

Any flexible element where it deforms if an external force is applied, and returns to its original shape when the force is absence.

$$F = k \times x$$



Mathematical symbol:

k

Stiffness constant [N/m]

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Helical spring is the easiest spring element to visualise.

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Rubber isolator, usually used as engine mounting.

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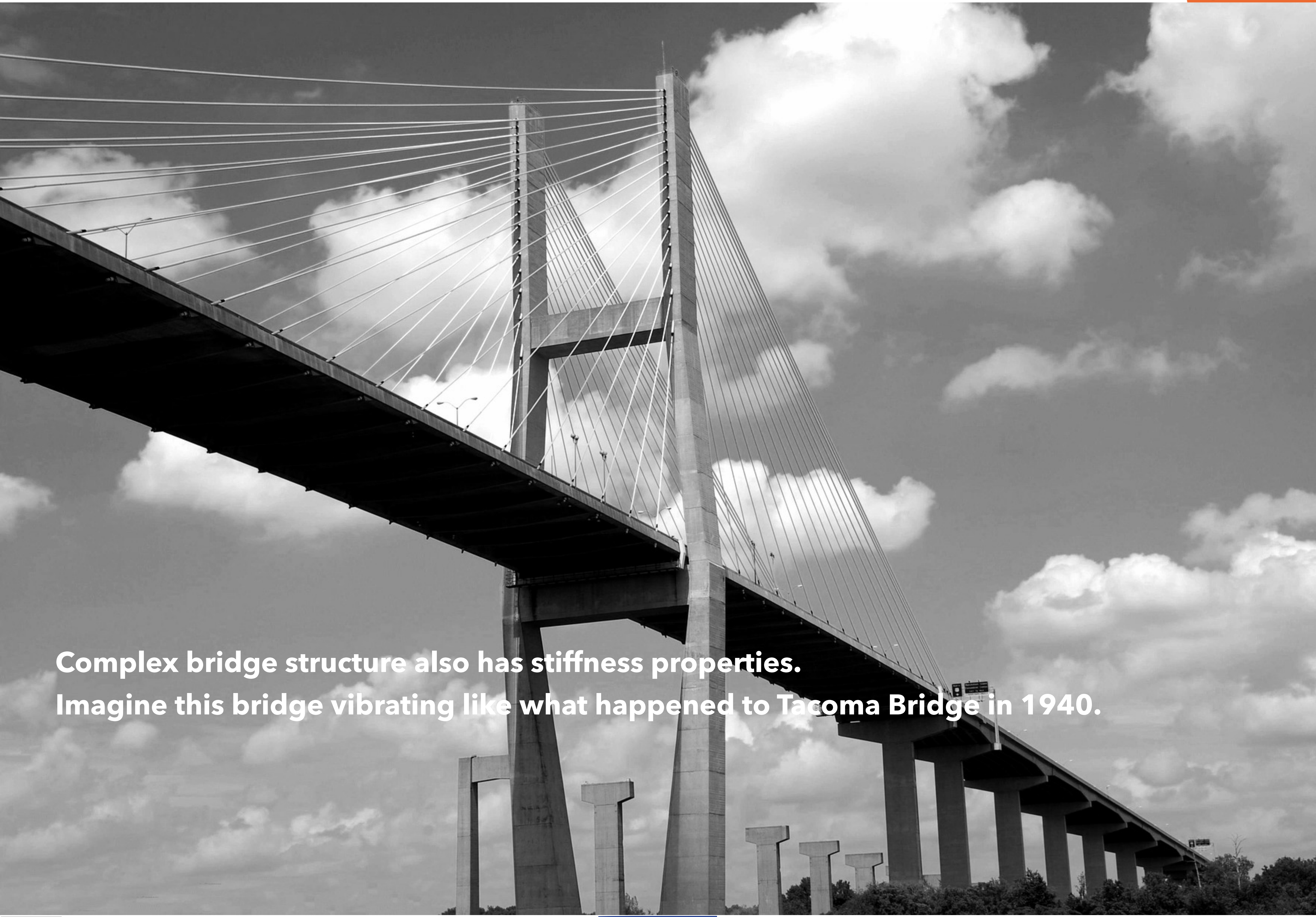
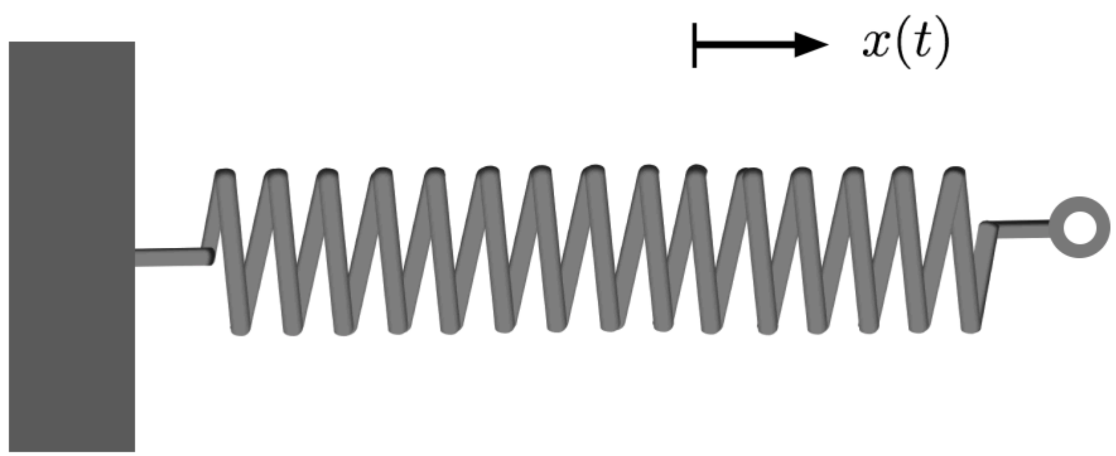
432 HZ
Looks like a rigid structure, a tuning fork is actually flexible when it vibrates. That is why we can hear the emitted sound.



Complex bridge structure also has stiffness properties.
Imagine this bridge vibrating like what happened to Tacoma Bridge in 1940.

Translational Spring



Unit: N/m

Force:

$$F = kx(t)$$

Potential energy:

$$U = \frac{1}{2}kx(t)^2$$

Unit: Joule

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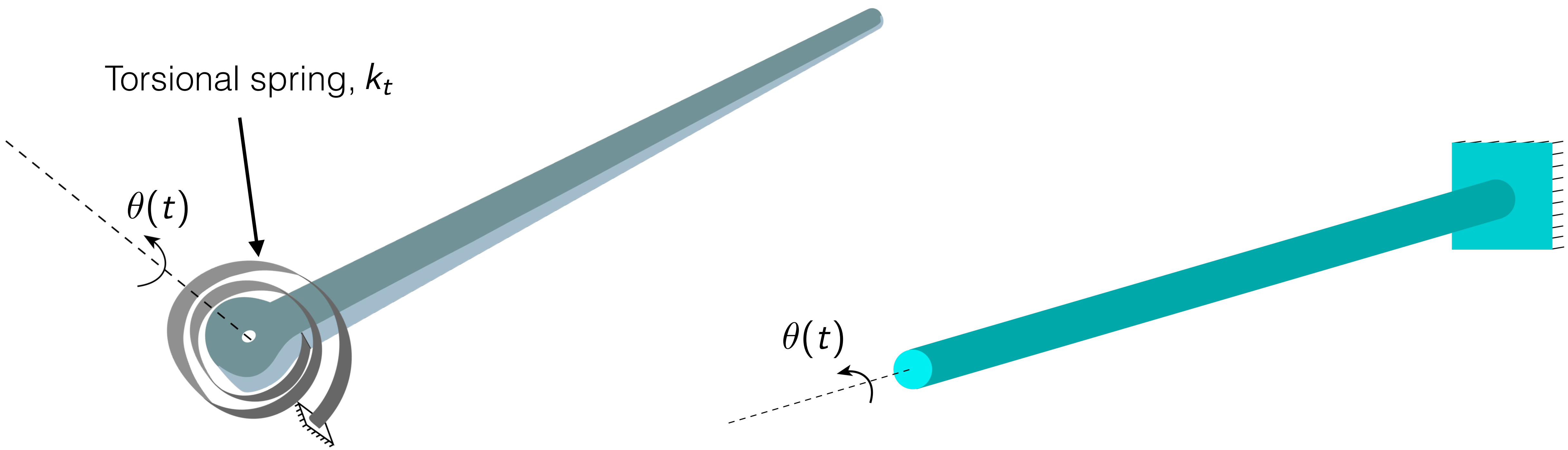
Rotational Spring



Torsion:

$$T = k_t \theta(t)$$

Unit: N.m

Potential energy:

$$U = \frac{1}{2} k_t \theta(t)^2$$

Unit: Joule

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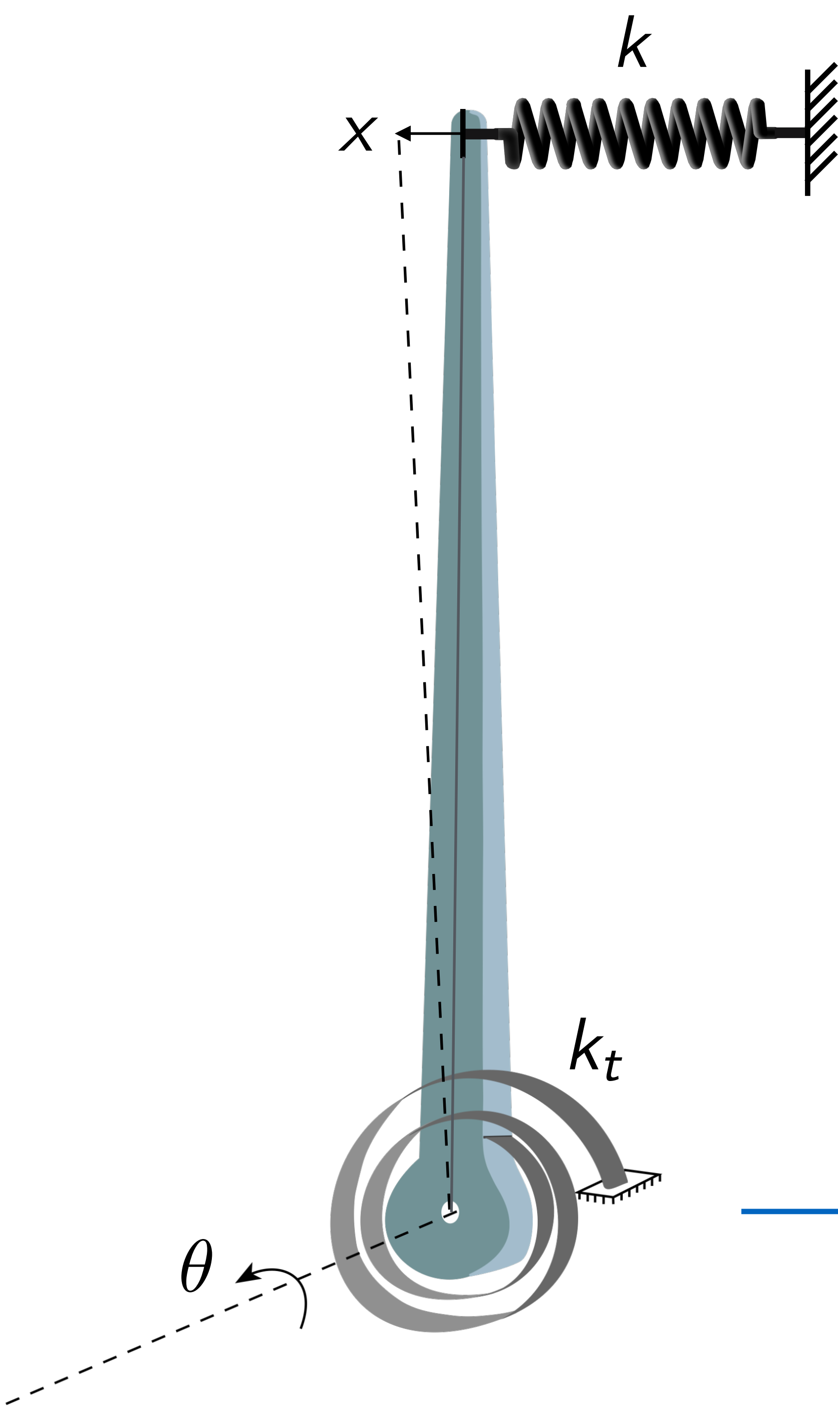
We can **sum up** the potential energy from both translational and rotational springs.



$$U = \frac{1}{2} k x^2$$

$$U = \frac{1}{2} k_t \theta^2$$

Total Potential Energy

$$\boxed{U = \frac{1}{2} k_t \theta^2 + \frac{1}{2} k x^2}$$

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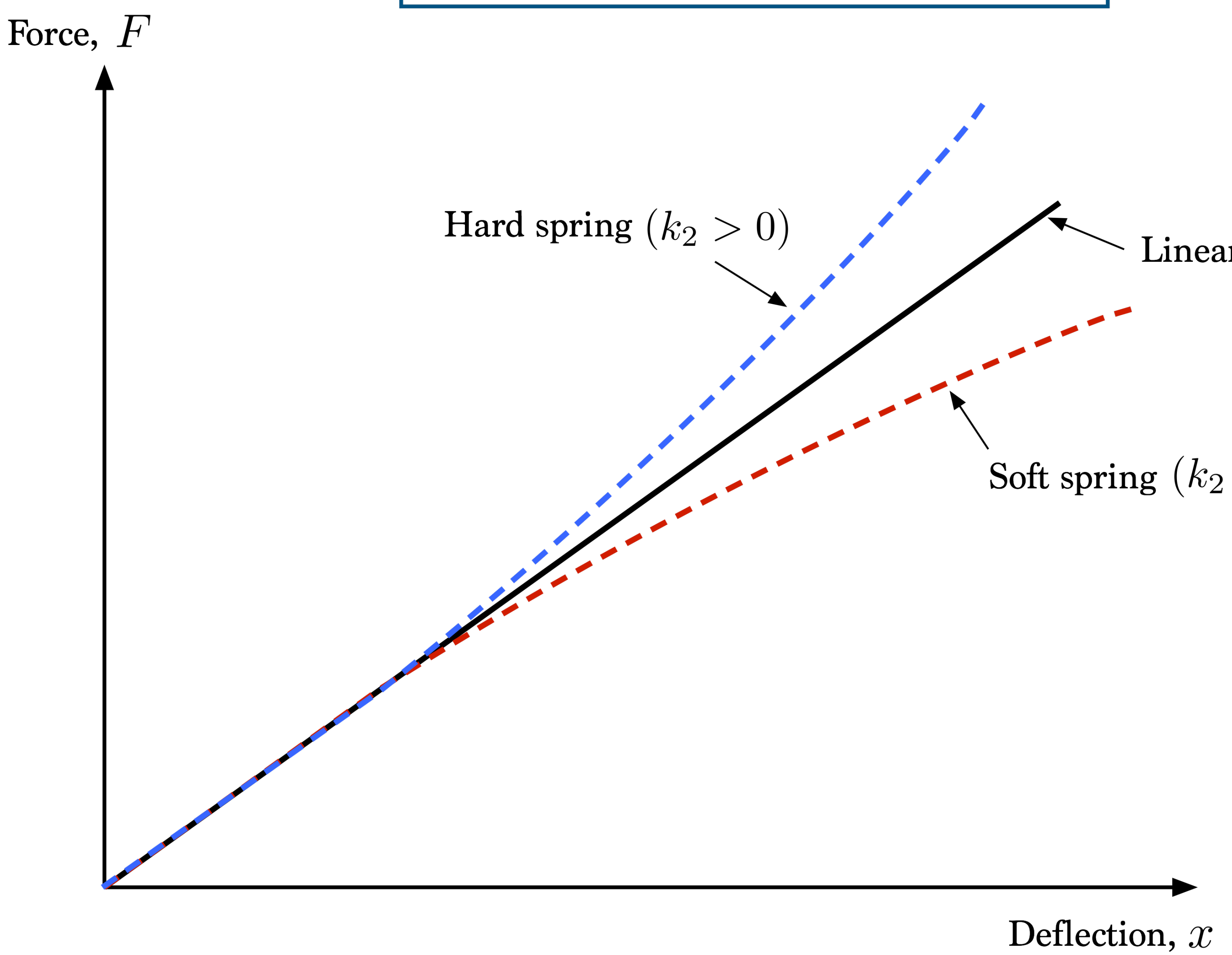
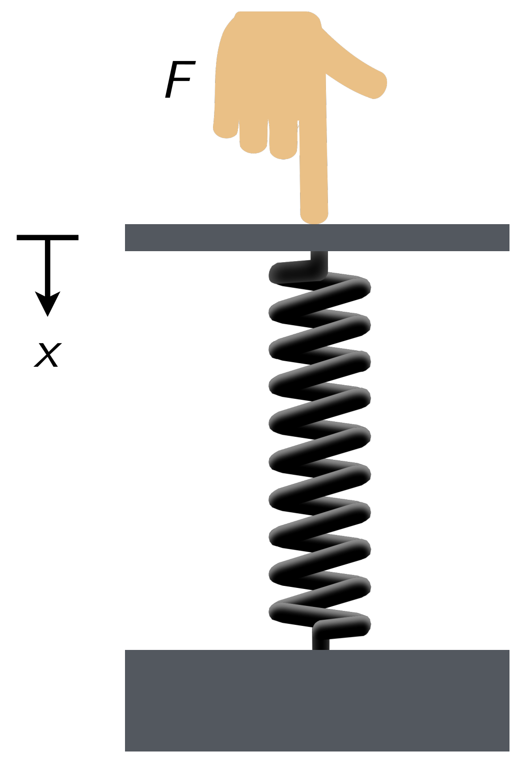
We use 'linear spring', where force is proportional to the spring deflection.

$$\boxed{F = k_1 x + k_2 x^3\,; \qquad k_1 > 0}$$





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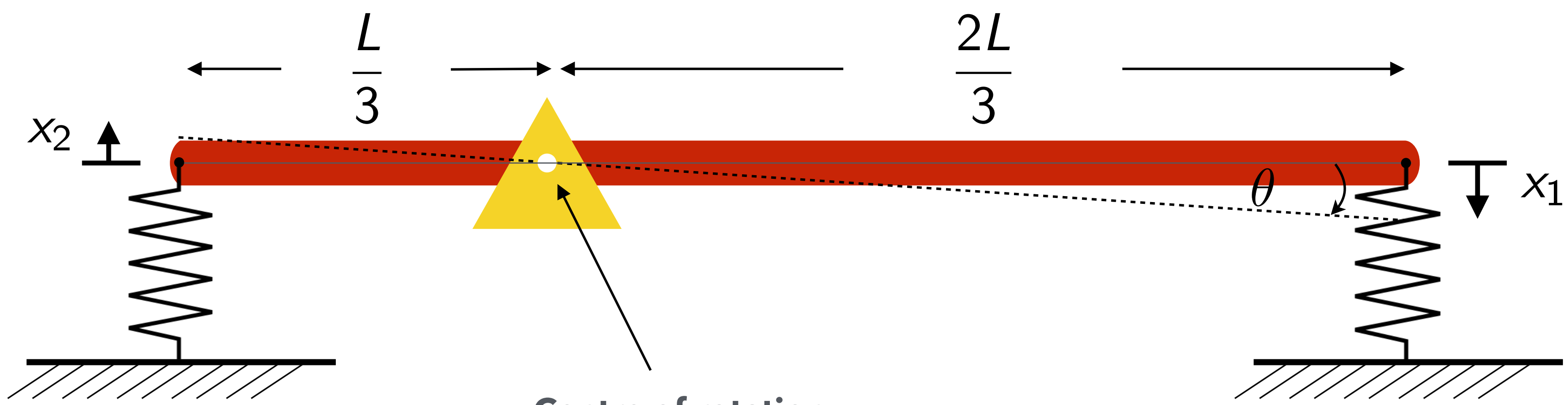
What are the deflections x_1 and x_2 in terms of θ ?



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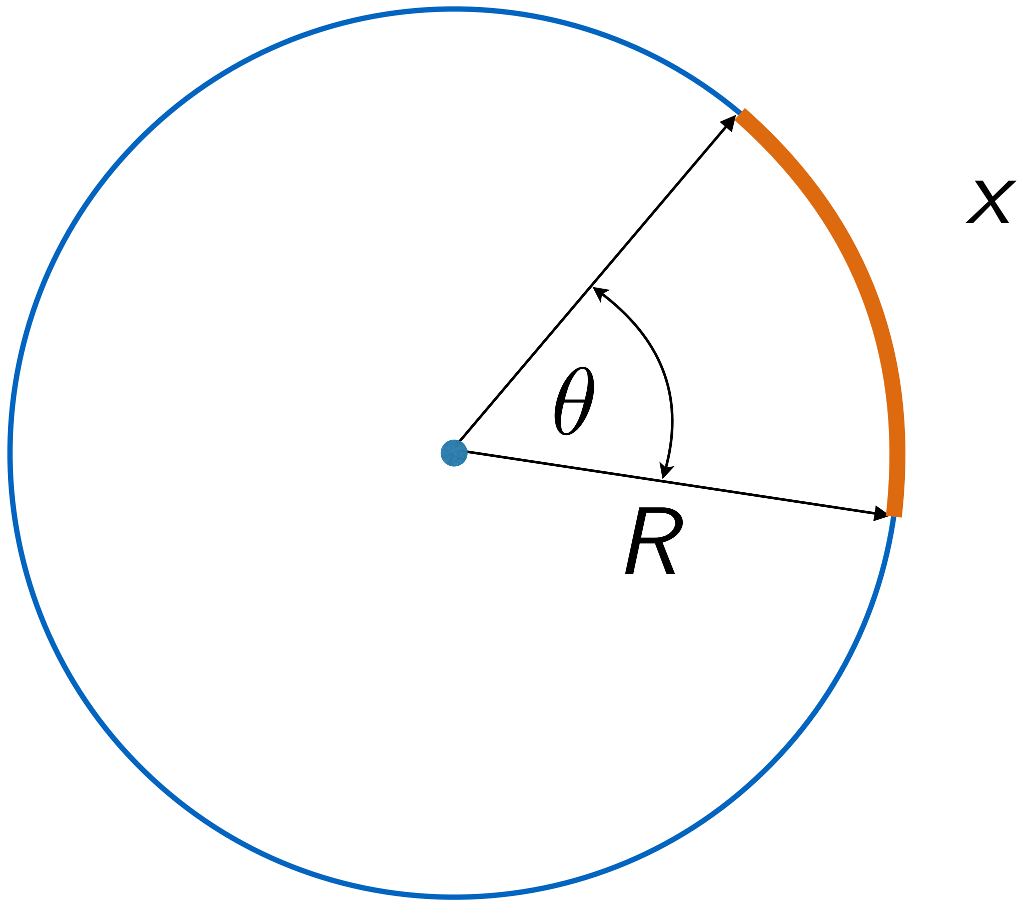
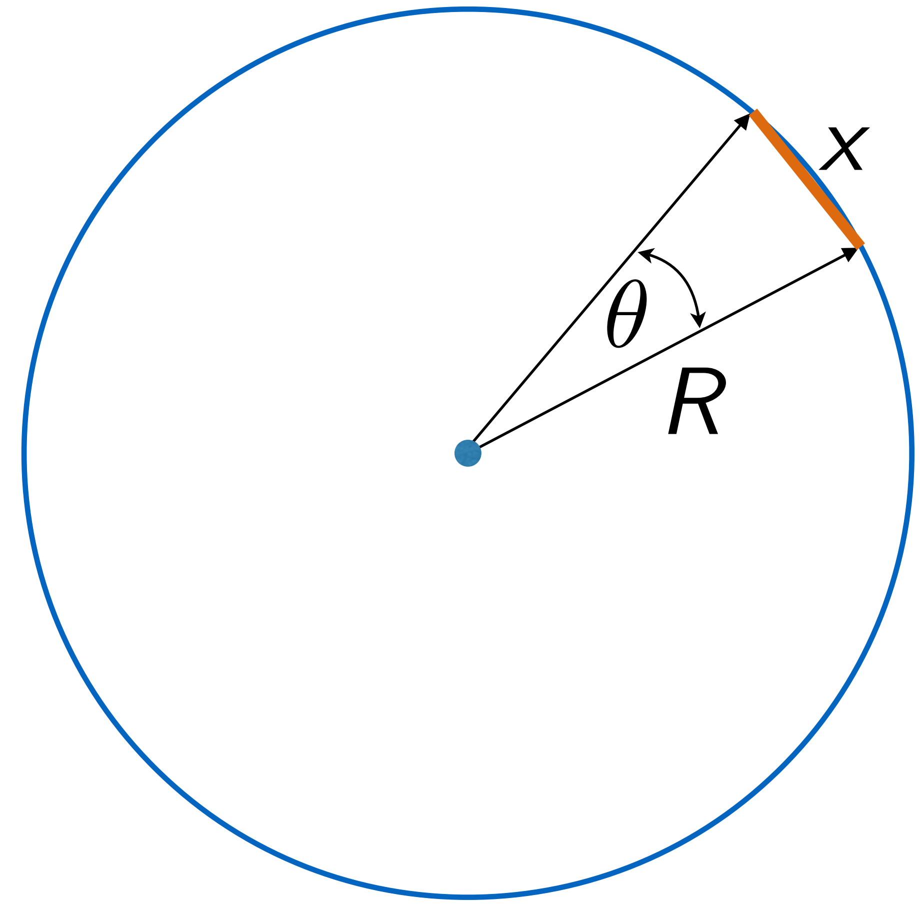




$$x = R \sin\theta$$

$$\boxed{x \approx R\theta}$$

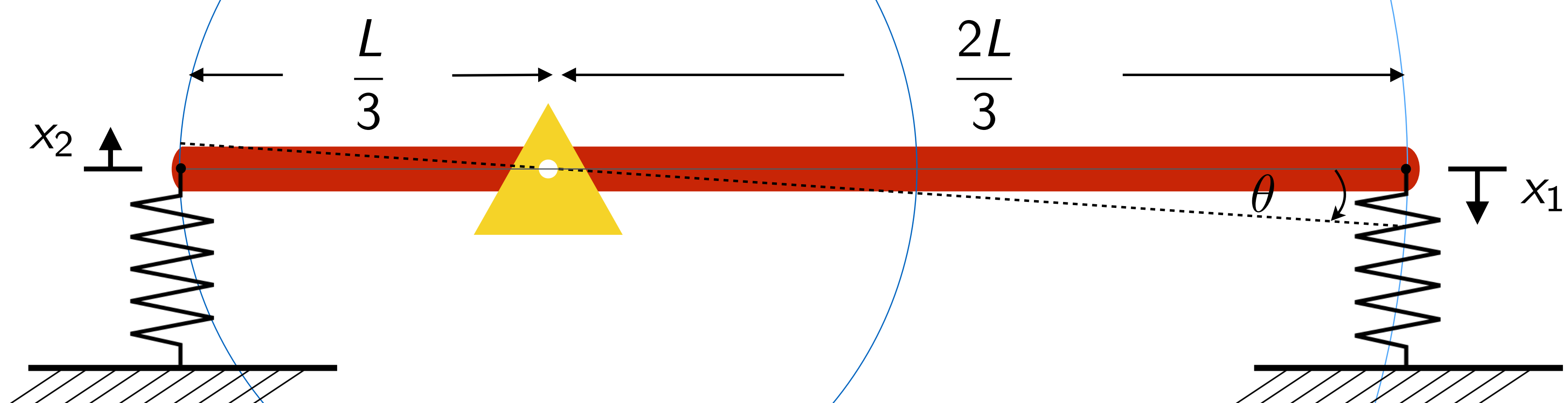
If θ is small,

$$\sin\theta \approx \theta - \frac{\theta^3}{3!} + \cdots \approx \theta$$

x **can be approximated by a <u>straight</u> line.**

Linear deflections of the spring

$$x_2 = \left(\frac{L}{3}\right)\theta$$



$$x_1 = \left(\frac{2L}{3}\right)\theta$$

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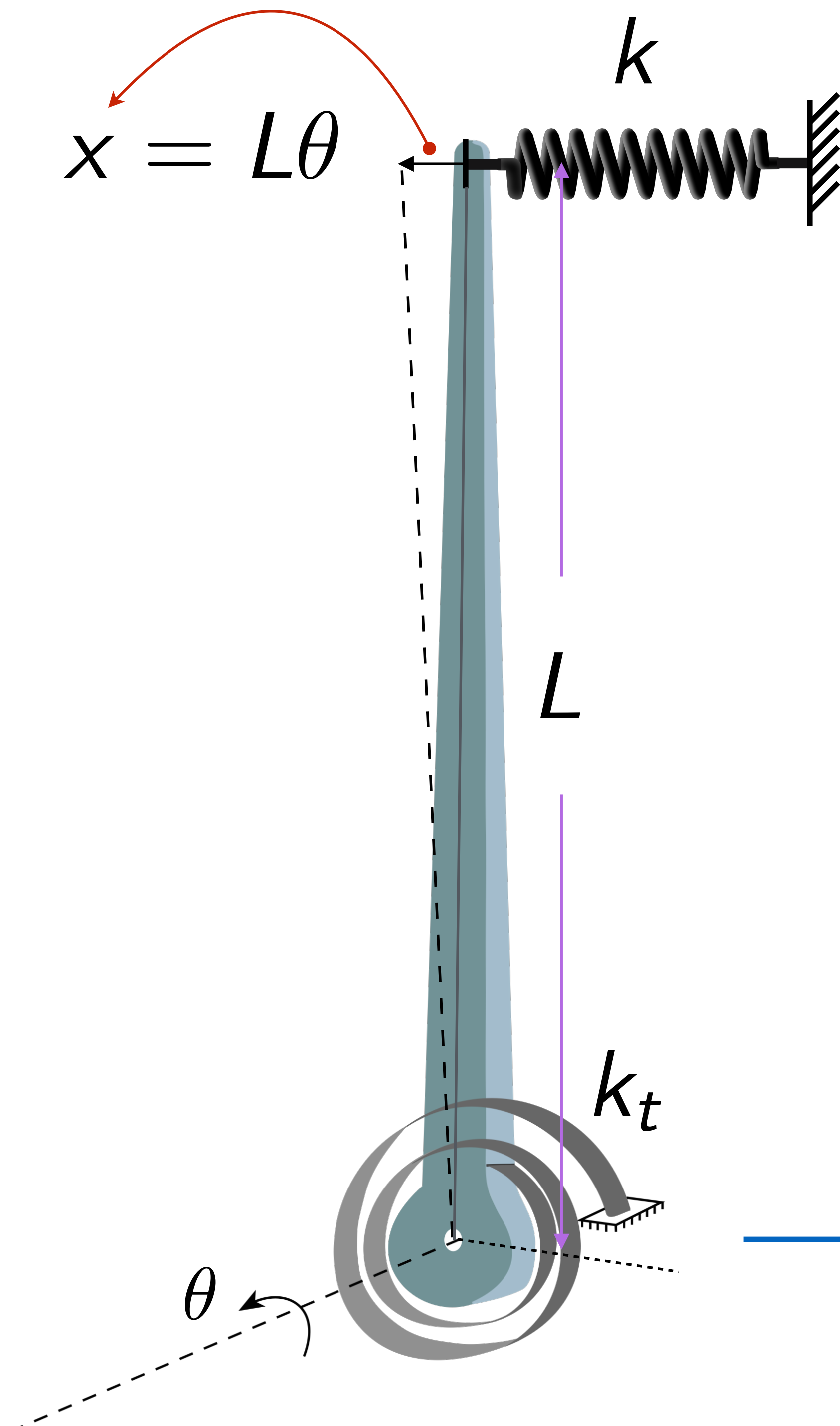
By using the principle of linearity:



$$U = \frac{1}{2}kx^2 = \frac{1}{2}kL^2\theta^2$$

$$U = \frac{1}{2}k_t\theta^2$$

Total Potential Energy

$$\boxed{U = \frac{1}{2}k_t\theta^2 + \frac{1}{2}kL^2\theta^2}$$

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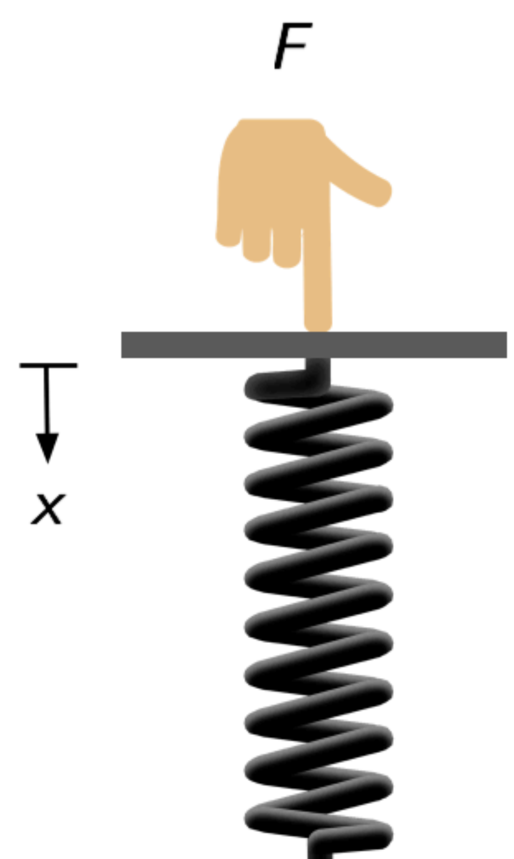
The equivalent stiffness can be found by considering:

1. **Force-Deformation relationship**
2. **Potential Energy-Deformation relationship**

$$\boxed{k = \frac{F}{x}}$$



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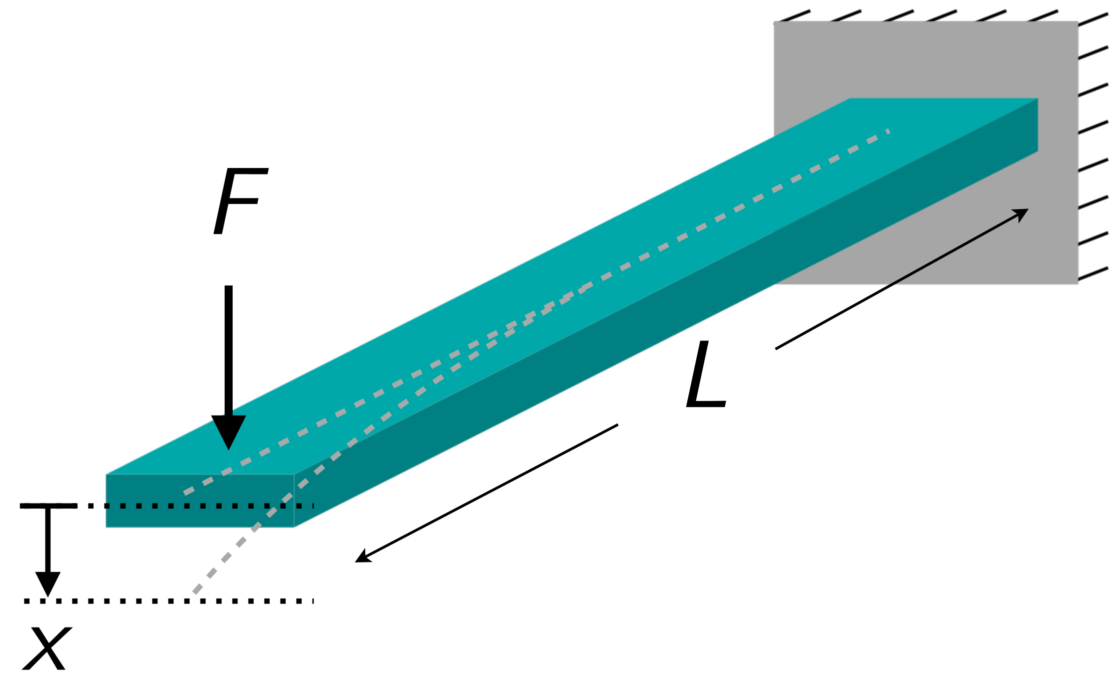
Cantilevered beam

Force required to deflect the tip of the beam by *x* is

$$F = \left(\frac{3EI}{L^3}\right) x$$



The stiffness constant is therefore:

$$\boxed{k = \frac{F}{x} = \frac{3EI}{L^3}}$$

"The stiffness at the tip of the beam"

E : **Young's Modulus**

I : **Second mass moment of inertia**

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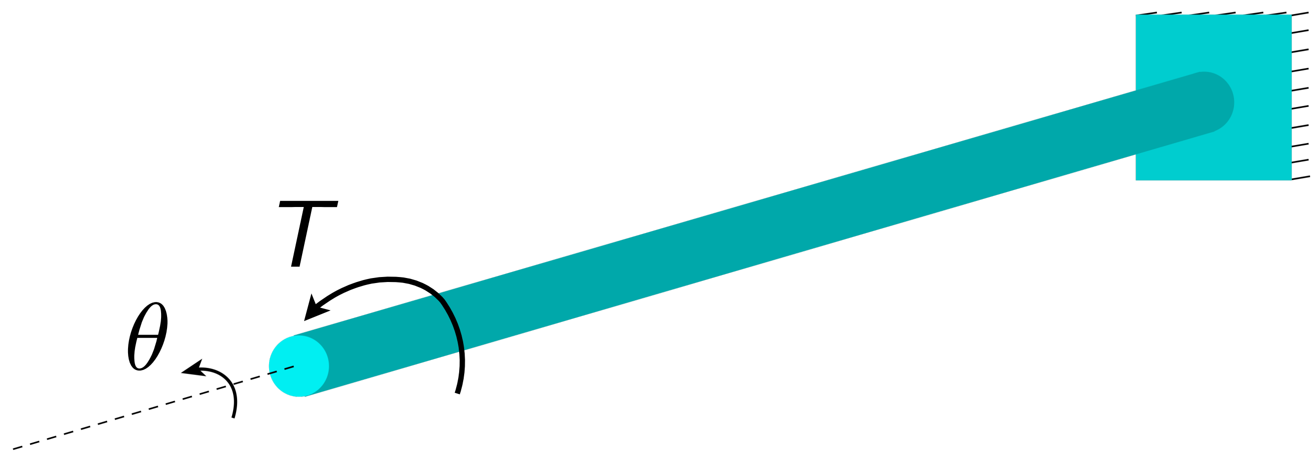
Rod in torsion

Torque required to deflect the tip of the rod by θ is

$$T = \left(\frac{GI}{L}\right)\theta$$



The torsional stiffness constant is:

$$\boxed{k = \frac{T}{\theta} = \frac{GI}{L}}$$

G : **Shear modulus**

I : **Polar moment of area**

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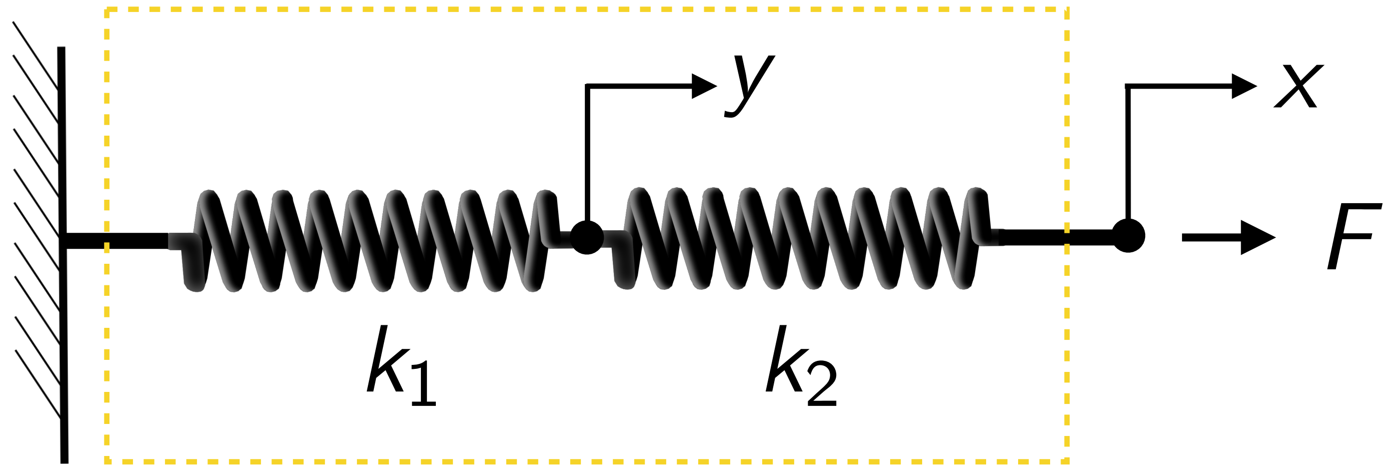
Springs in series



1. Each spring carries the same force
2. Total deflection = sum of individual deflection

$$F = k_1 y$$
$$F = k_2(x - y)$$
$$\longrightarrow \quad F\left(1 + \frac{k_2}{k_1}\right) = k_2 x$$

$$\boxed{\frac{1}{k_{eq}} = \frac{x}{F} = \frac{1}{k_1} + \frac{1}{k_2}}$$

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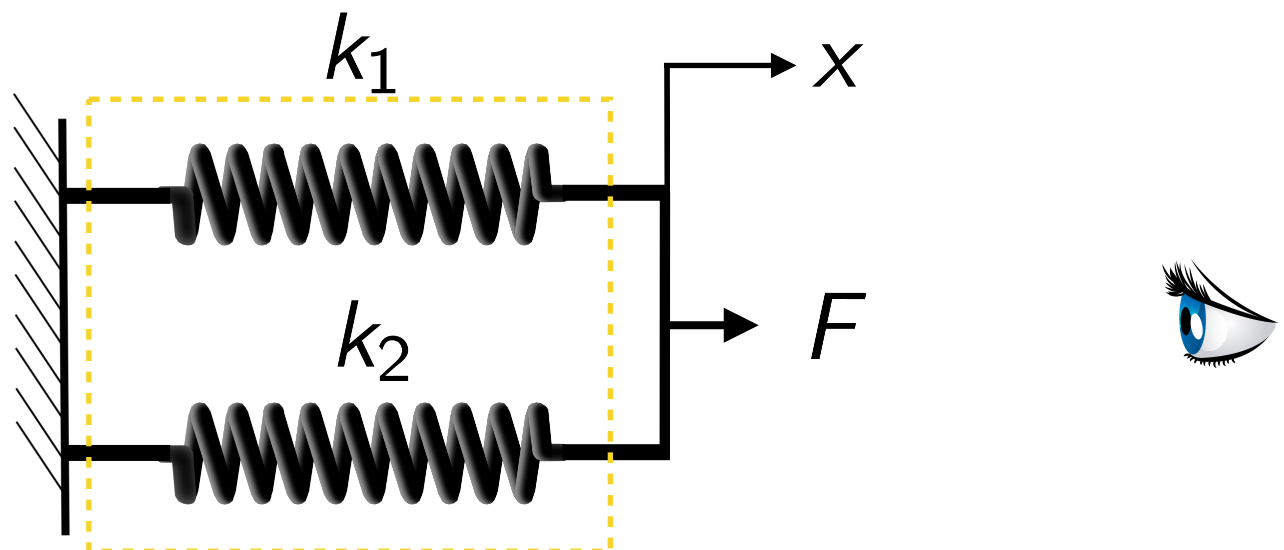
Springs in parallel



1. All springs have the same deflection
2. Total force = sum of force for each spring

$$F_1 = k_1 x \; ; \qquad F_2 = k_2 x \; ; \qquad F = F_1 + F_2$$

$$k_{eq} = \frac{F}{x} = k_1 + k_2$$

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k_b **and** k_s**, series or parallel?**

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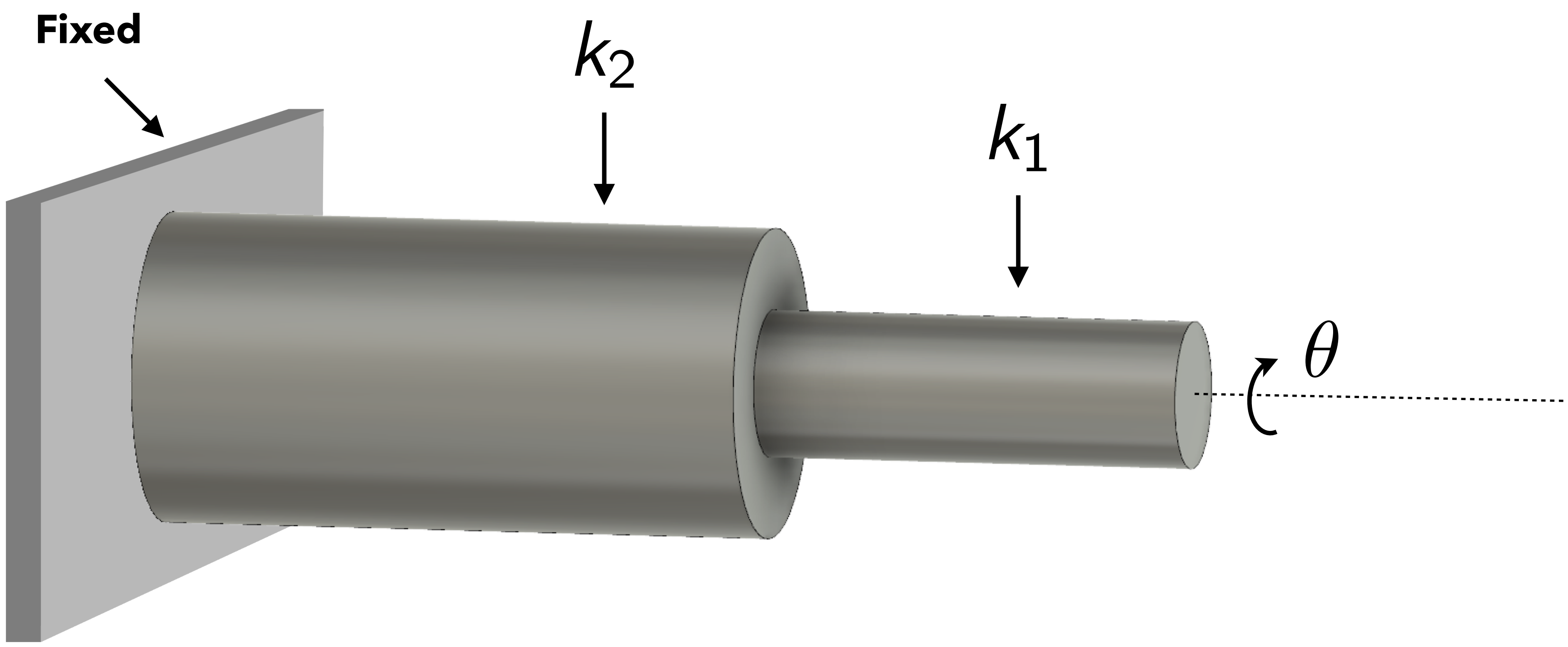

k_1 and k_2 , series or parallel?

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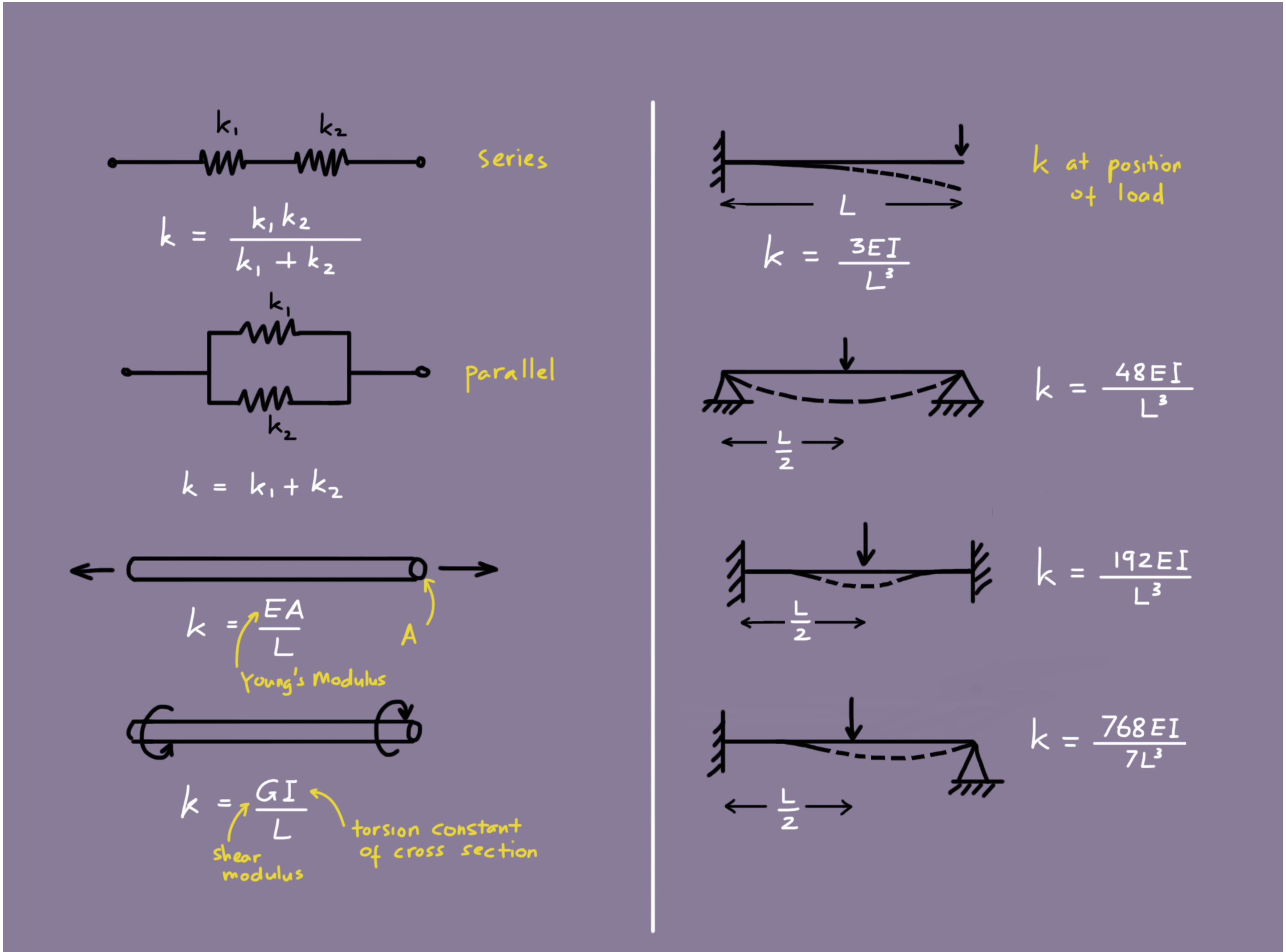
Formulae of stiffness constant are available in stiffness table.



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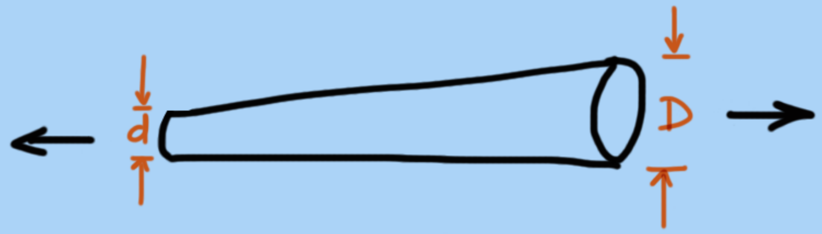
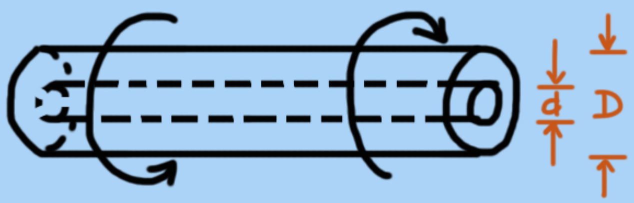
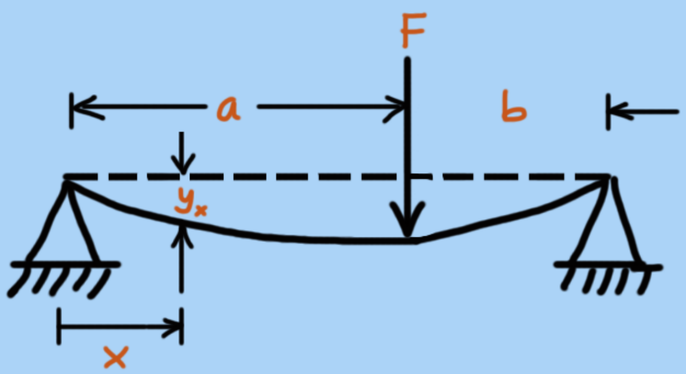


$$k = \frac{\pi E D d}{4L}$$

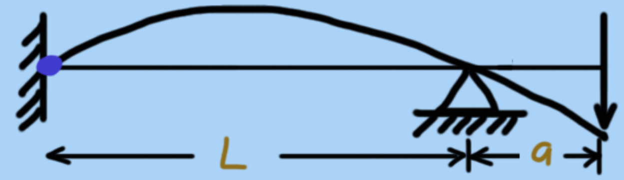


$$k = \frac{\pi G}{32L}(D^4 - d^4)$$

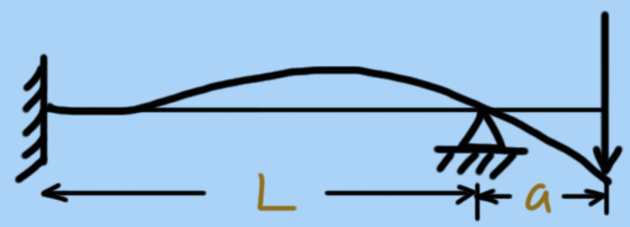


$$k = \frac{3EIL}{a^2 b^2}$$

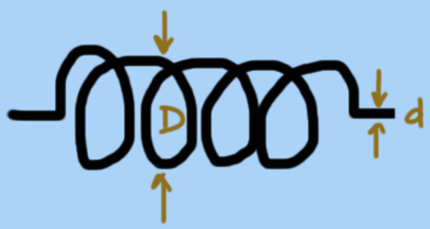
$$y_x = \frac{Fbx}{6EIL}(L^2 - x^2 - b^2)$$



$$k = \frac{3EI}{(L+a)a^2}$$



$$k = \frac{24EI}{a^2(3L+8a)}$$



$$k = \frac{Gd^4}{8nD^3}$$

number of turns

$$k = \frac{EI}{L}$$

total length

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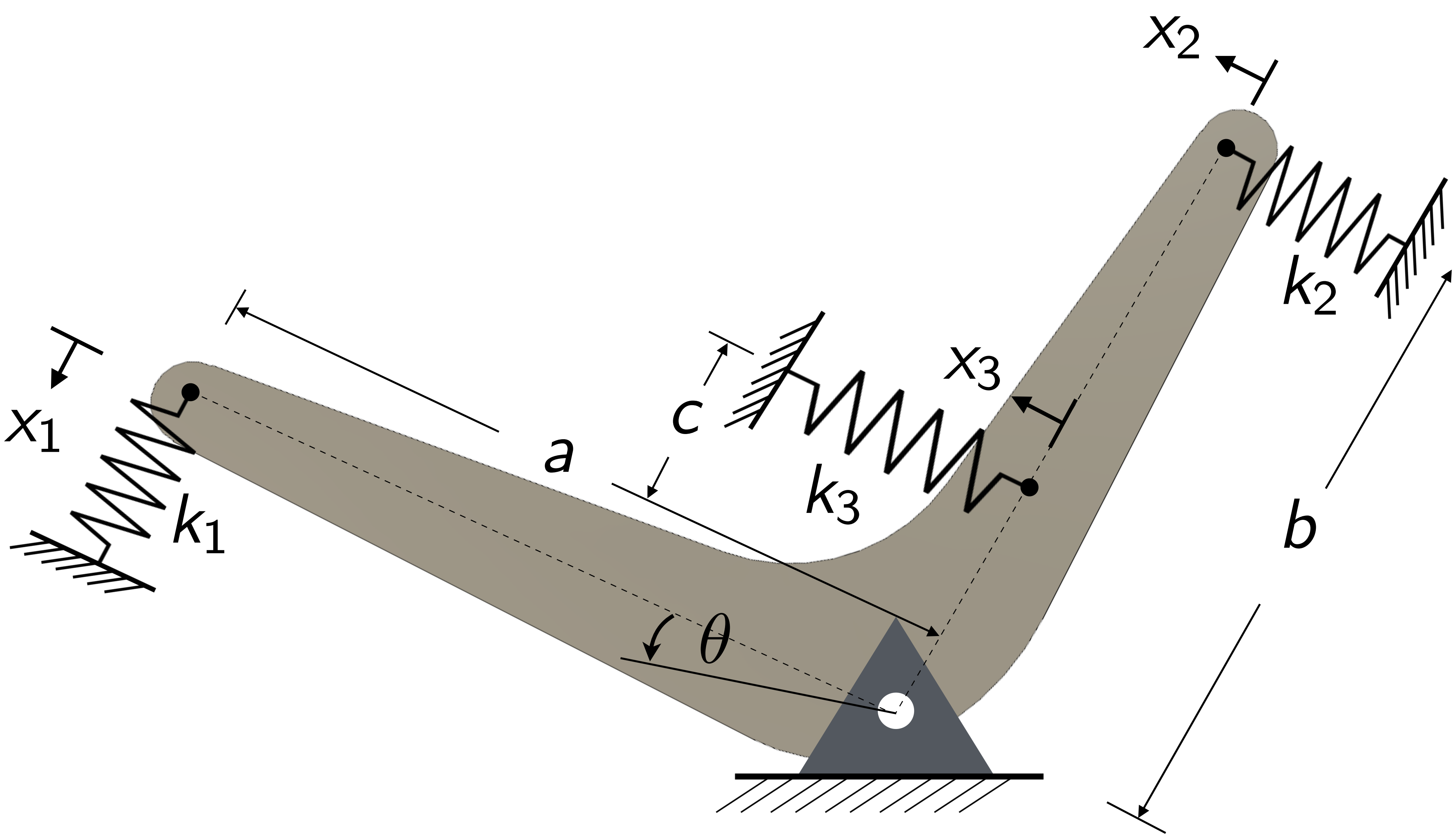
For a more complicated system where no springs are series or parallel, we can use potential energy to find the equivalent stiffness constant of the system.



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Write down the potential energy of each spring:

$$U_1 = \tfrac{1}{2} k_1 x_1^2 \qquad U_2 = \tfrac{1}{2} k_2 x_2^2 \qquad U_3 = \tfrac{1}{2} k_3 x_3^2$$

Choose a **generalised coordinate**.

Say the deflection of spring k_1 as the generalised coordinate.

All potential energies must be written as a function of x_1

Linearisation of displacement:

$$\left.\begin{aligned} x_1 &= a\theta \\ x_2 &= b\theta \\ x_3 &= c\theta \end{aligned}\right\} \rightarrow \frac{x_1}{x_2} = \frac{a}{b} \quad \text{and} \quad \frac{x_1}{x_3} = \frac{a}{c} \;\Big\}\rightarrow\; \boxed{x_2 = \left(\frac{b}{a}\right) x_1} \quad \boxed{x_3 = \left(\frac{c}{a}\right) x_1}$$

Total potential energy:

$$U_{tot} = U_1 + U_2 + U_3 = \frac{1}{2}\underbrace{\left[k_1 + k_2\left(\frac{b}{a}\right)^2 + k_3\left(\frac{c}{a}\right)^2\right]}_{} x_1^2$$

Equivalent stiffness constant, k_{eq}

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Watch the video: “Spring Element”

Scan this QR code

Or click/tap here.

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https://youtu.be/Zdxaod0RRdA

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3.3

MASS/INERTIA ELEMENT

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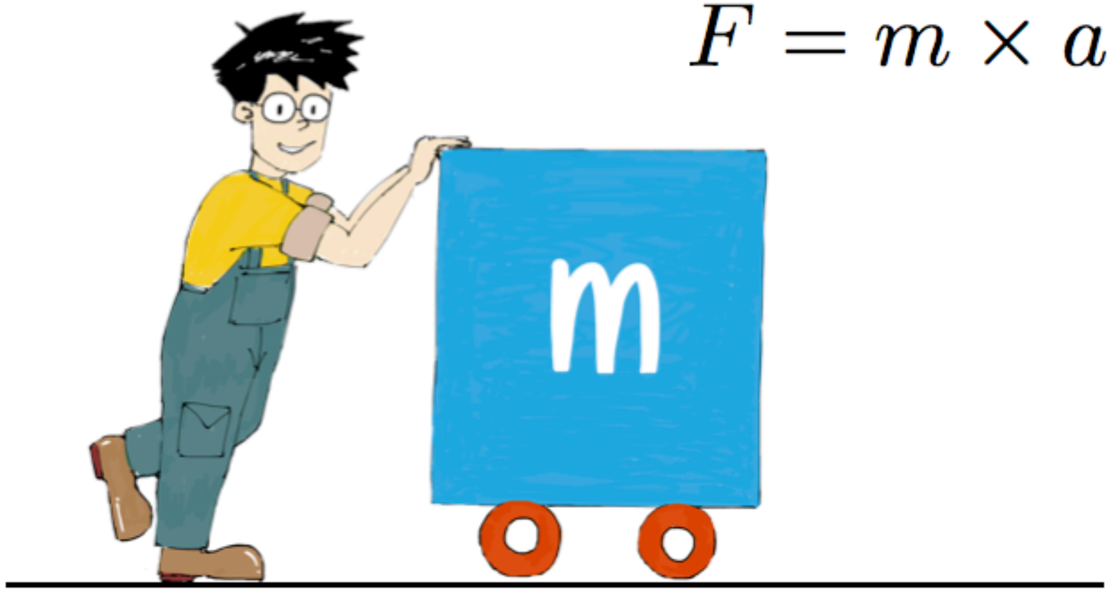
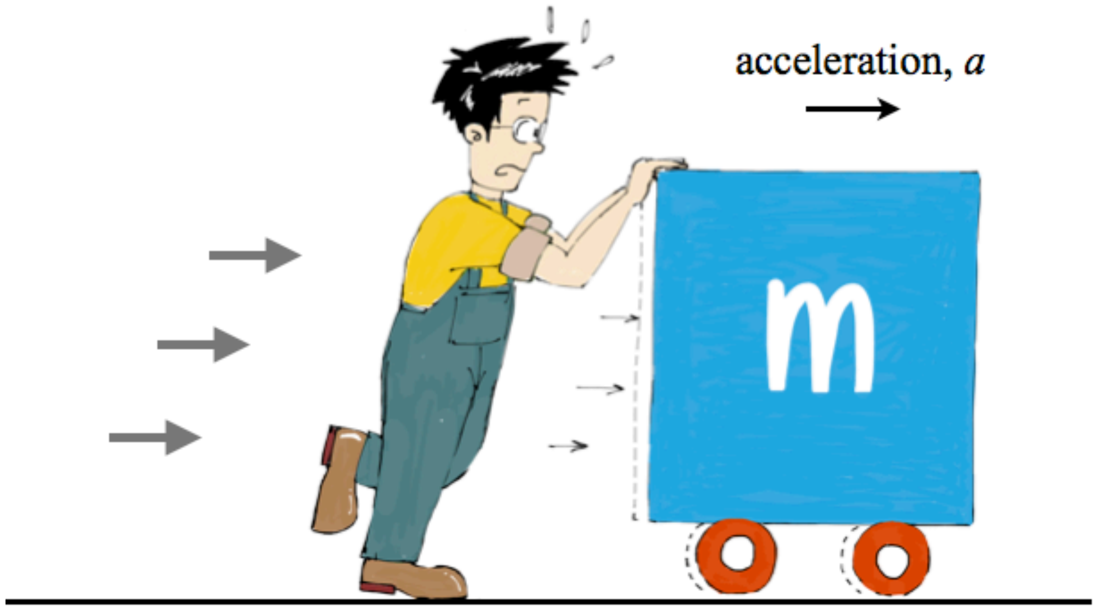
Mass Element

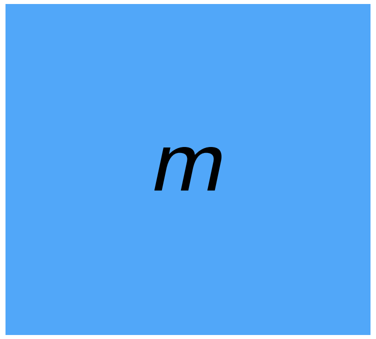
A rigid body. It accelerates when an external force is applied.

$$F = m \times a$$





Mathematical symbol:



Mass [kg]

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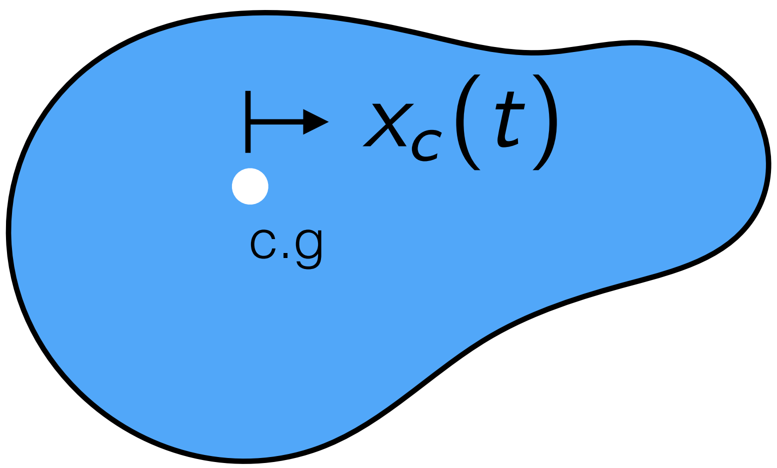
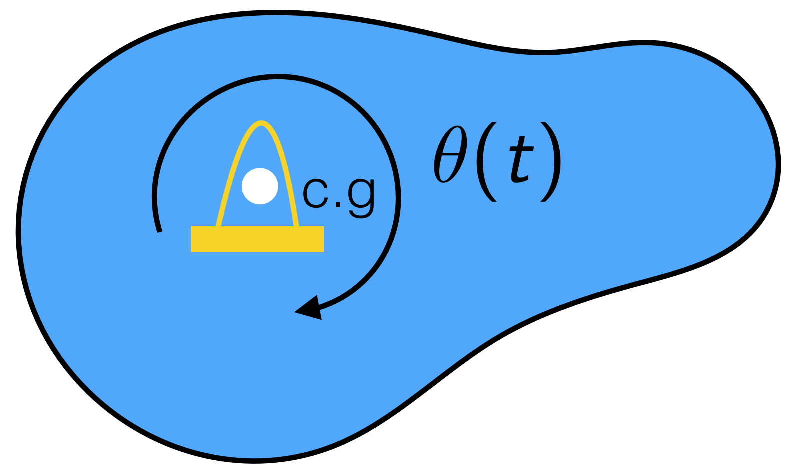
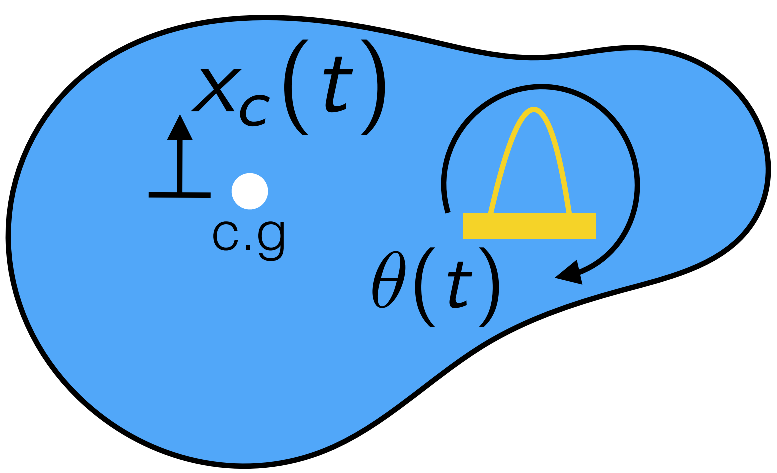
Kinetic Energy



$$T = \frac{1}{2} m \dot{x}_c^2$$



$$T = \frac{1}{2} J \dot{\theta}^2$$



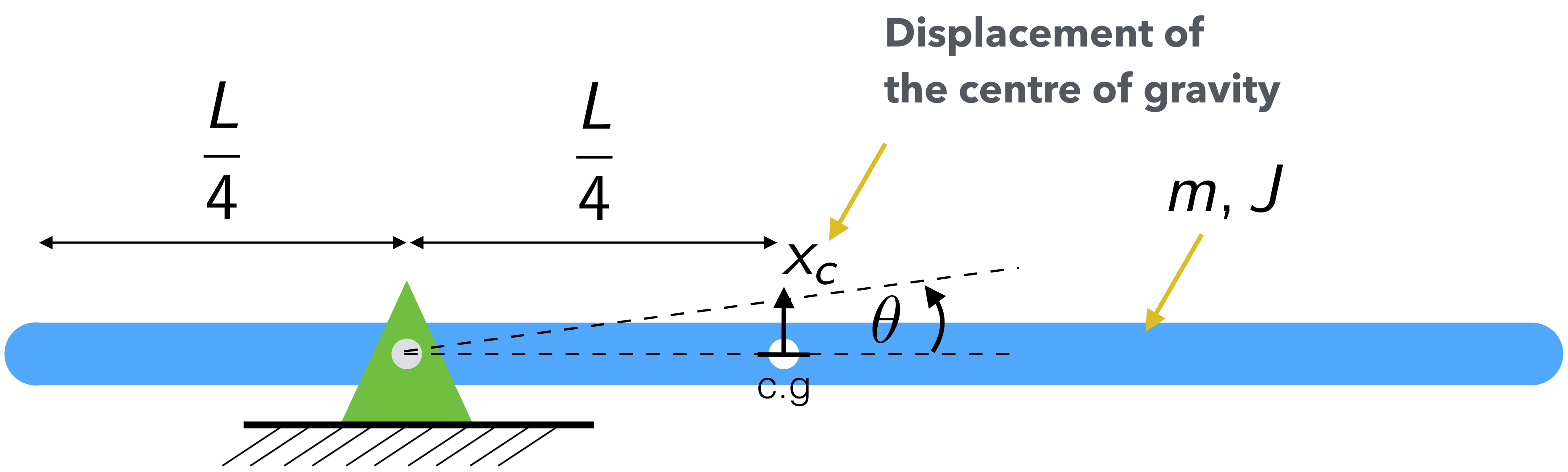
$$T = \frac{1}{2} m \dot{x}_c^2 + \frac{1}{2} J \dot{\theta}^2$$

m = mass

J = second mass moment of inertia

c.g = centre of gravity/centre of mass

A rotating uniform bar.



Kinetic energy: $T = \frac{1}{2} m \dot{x}_c{}^2 + \frac{1}{2} J \dot{\theta}^2$ $\qquad x_c = \left(\frac{L}{4}\right)\theta$

If θ is the generalised coordinate:

$$T = \frac{1}{2} m \left(\frac{L}{4}\right)^2 \dot{\theta}^2 + \frac{1}{2} J \dot{\theta}^2 = \frac{1}{2} \boxed{\left[\frac{mL^2}{16} + J\right]} \dot{\theta}^2$$

Equivalent second mass moment of inertia, J_{eq}

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Additional Resources

My website:
http://www.azmaputra.com

My white-board animation videos:
http://www.youtube.com/c/AzmaPutra-channel

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References

A. Putra, R. Ramlan, A. Y. Ismail, *Mechanical Vibration: Module 9 Teaching and Learning Series*, Penerbit UTeM, 2014

D. J. Inman, *EngineeringVibrations*, Pearson, 4th Ed. 2014

S. S. Rao, *Mechanical Vibrations*, Pearson, 5th Ed. 2011

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