MECHANICAL VIBRATION

BMCG 3233

CHAPTER 2: TERMINOLOGIES AND MODELLING

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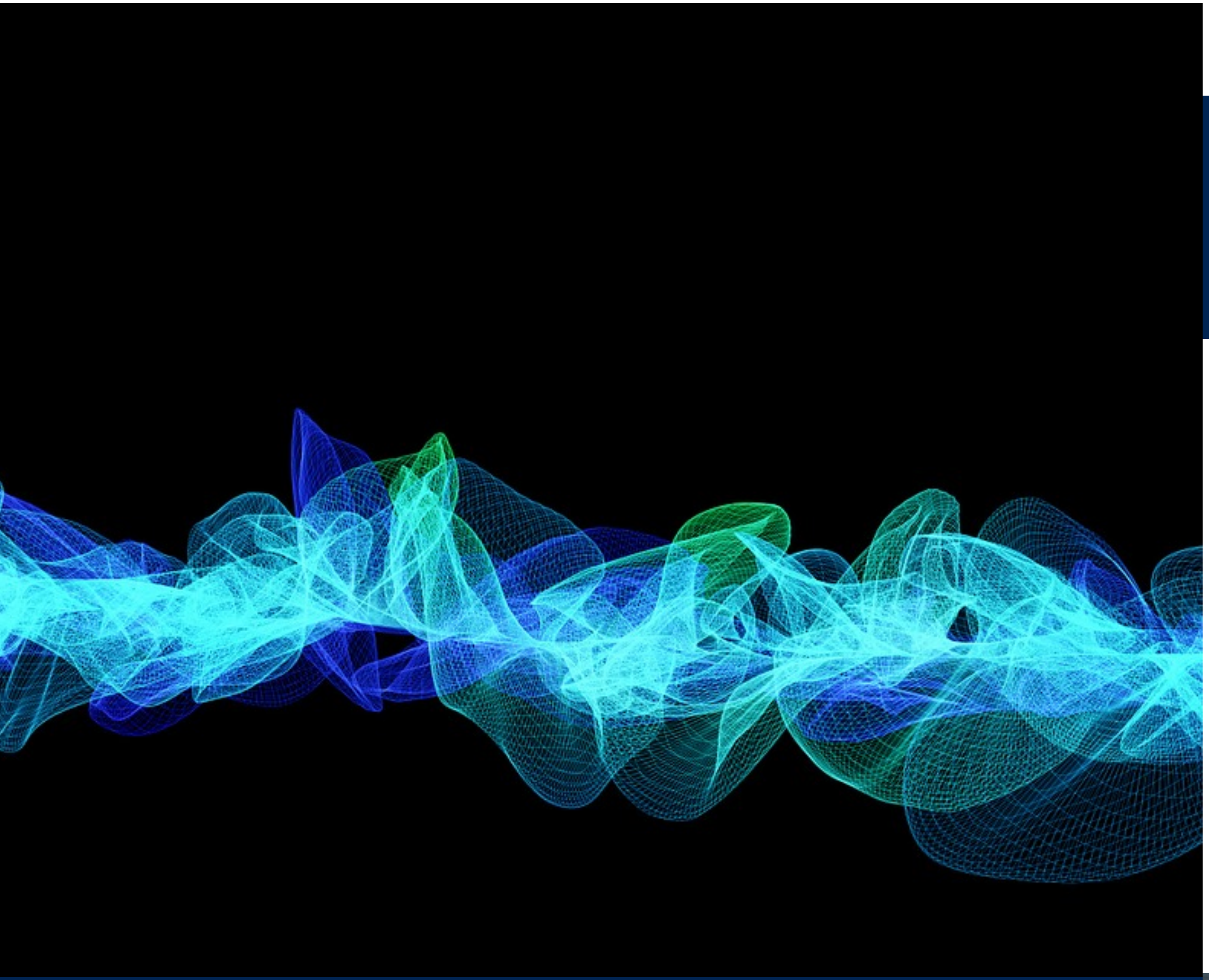


CONTENTS

LEARNING OBJECTIVES

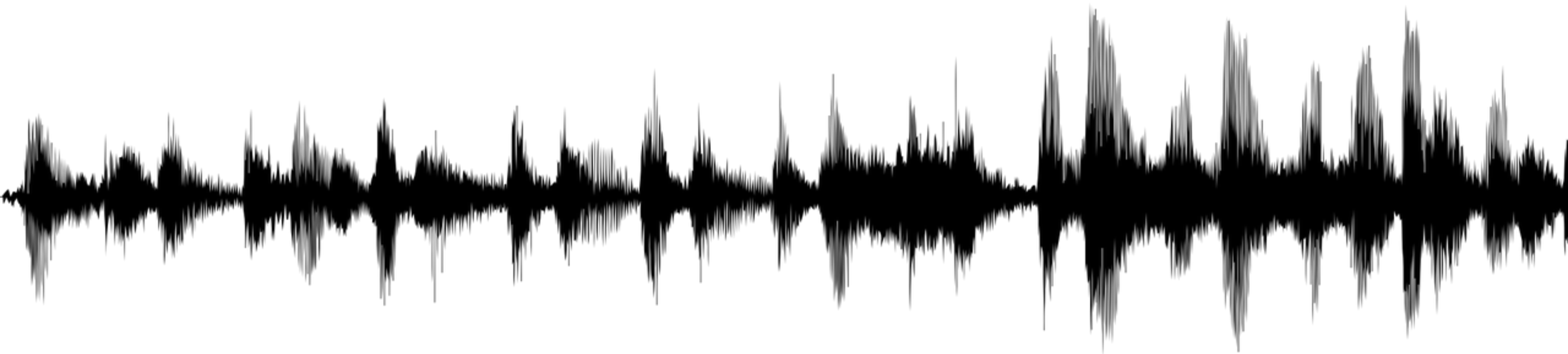
1. Employ the concept of amplitude, frequency and phase
2. Employ the concept of complex exponential notation in amplitude
3. Able to model a vibrating system with mass, spring and damper elements.

2.1

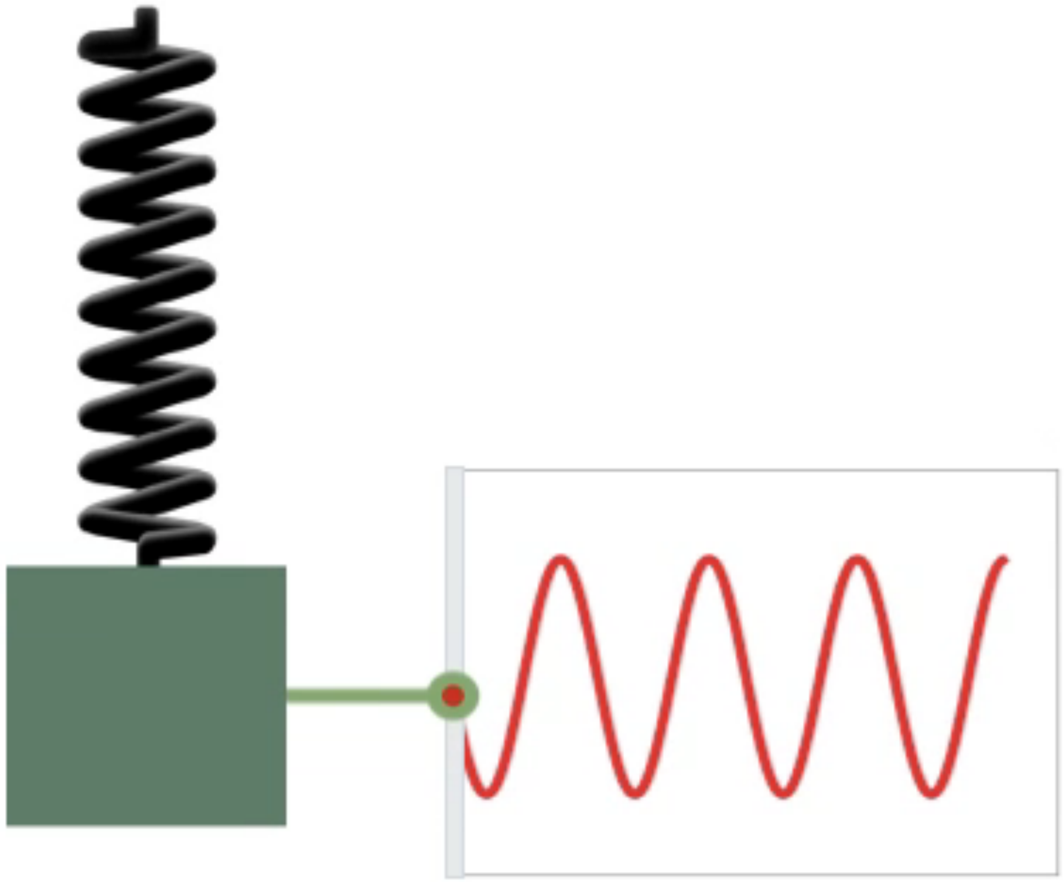
AMPLITUDE, FREQUENCY AND PHASE

Three important variables when analysing the vibration data:

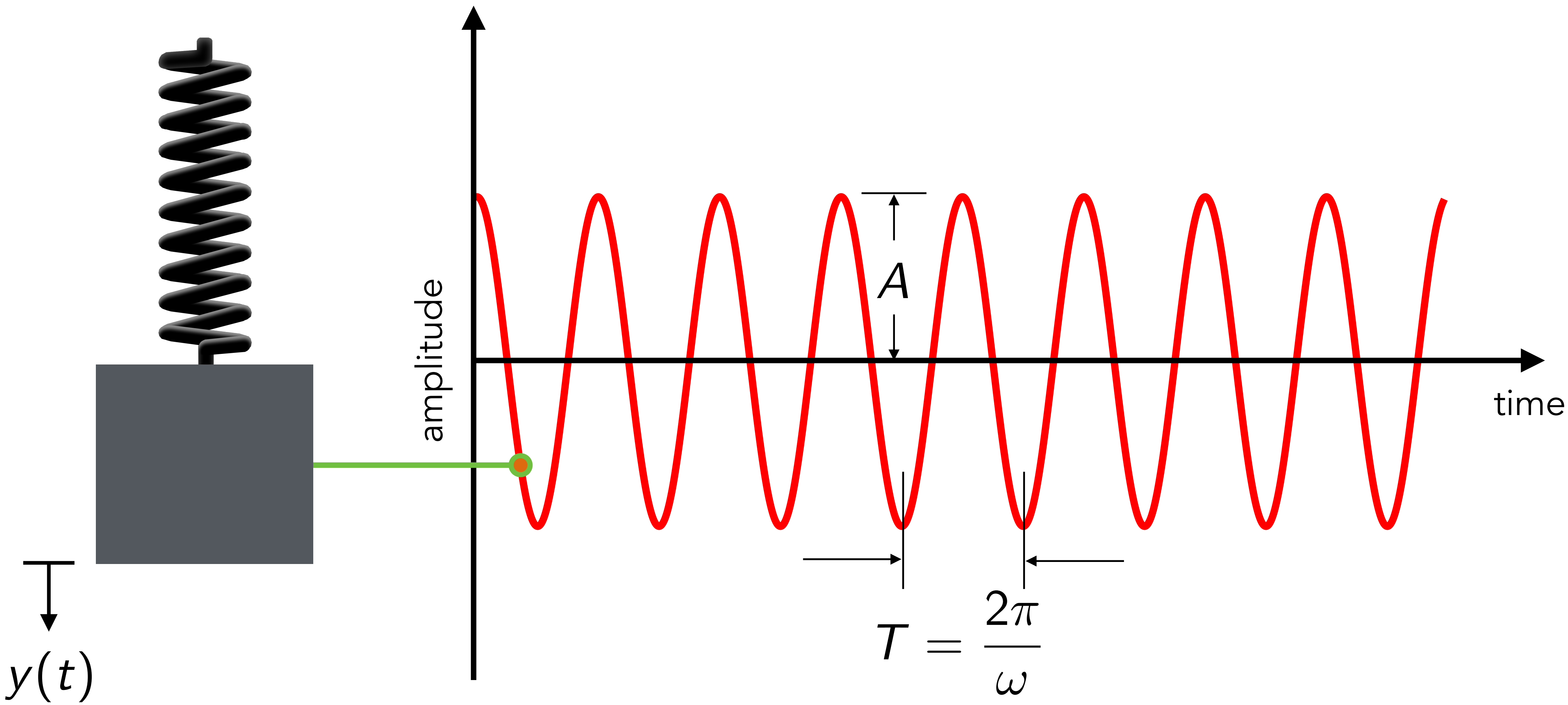
1. Amplitude
2. Frequency
3. Phase



Consider the motion of the mass-spring system.



What mathematical equation can we propose to represent the motion?



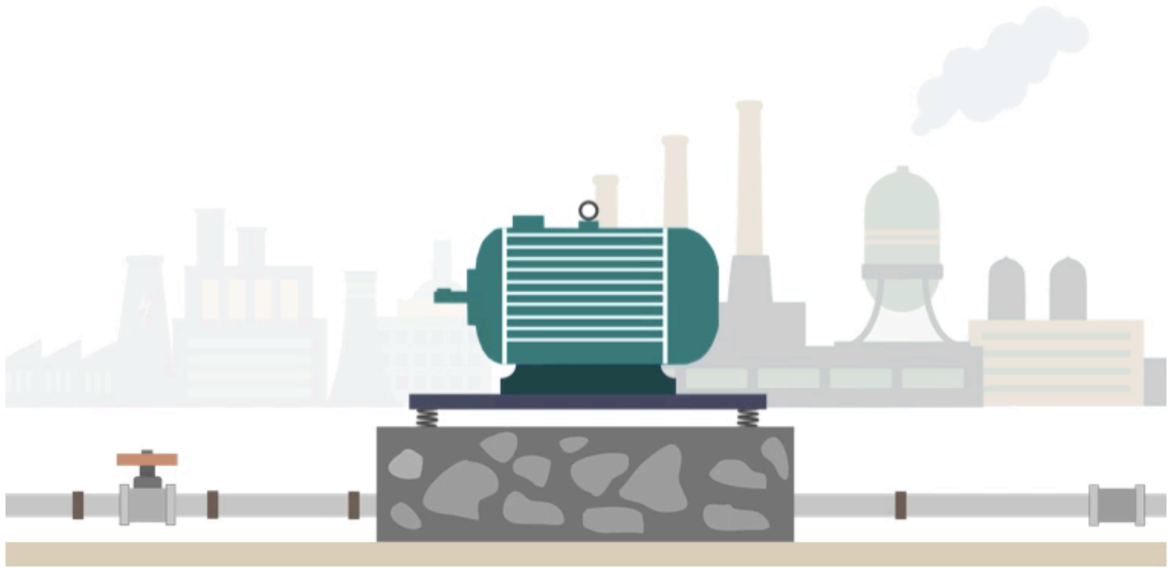
maximum amplitude

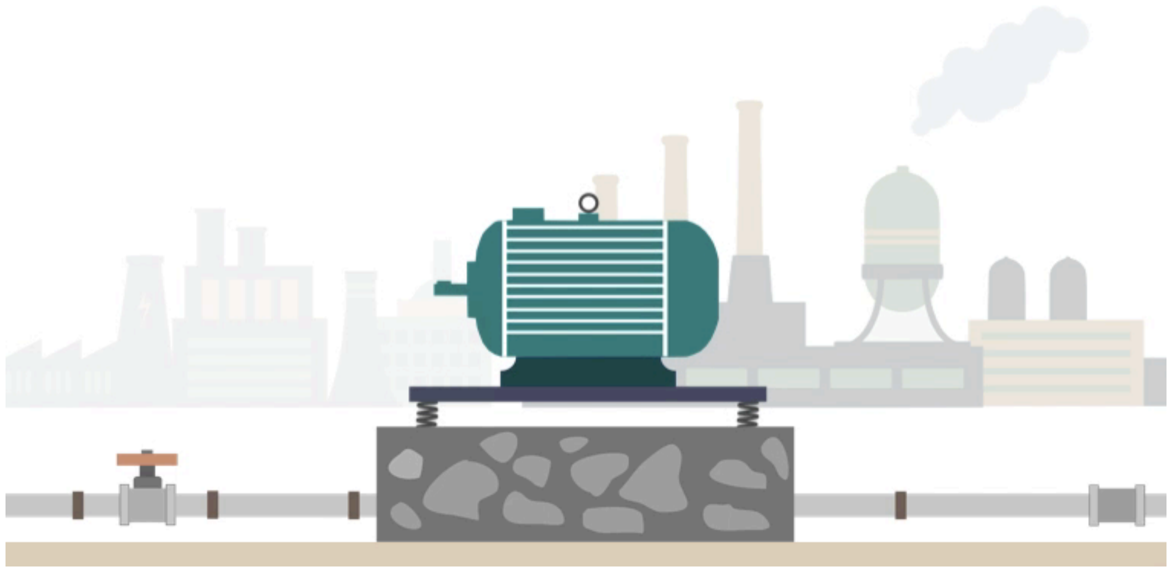
phase

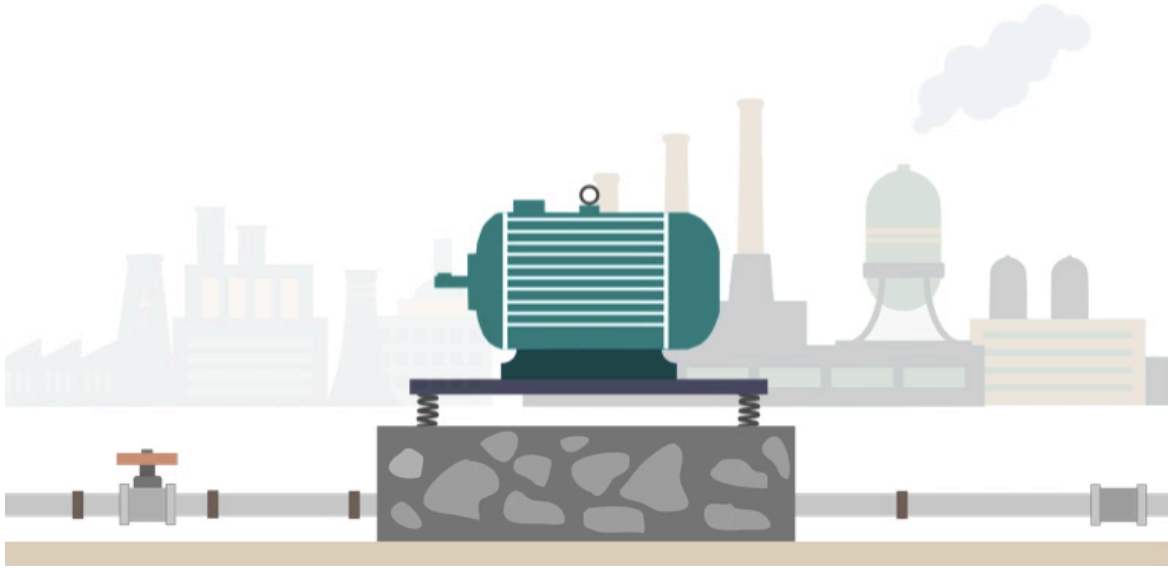
$$\boxed{y(t) = A\sin(\omega t + \phi)}$$

angular frequency

How far the object moves from the equilibrium position.







The amplitude can be stated as:

1. **Displacement**

   Unit: m, mm, in

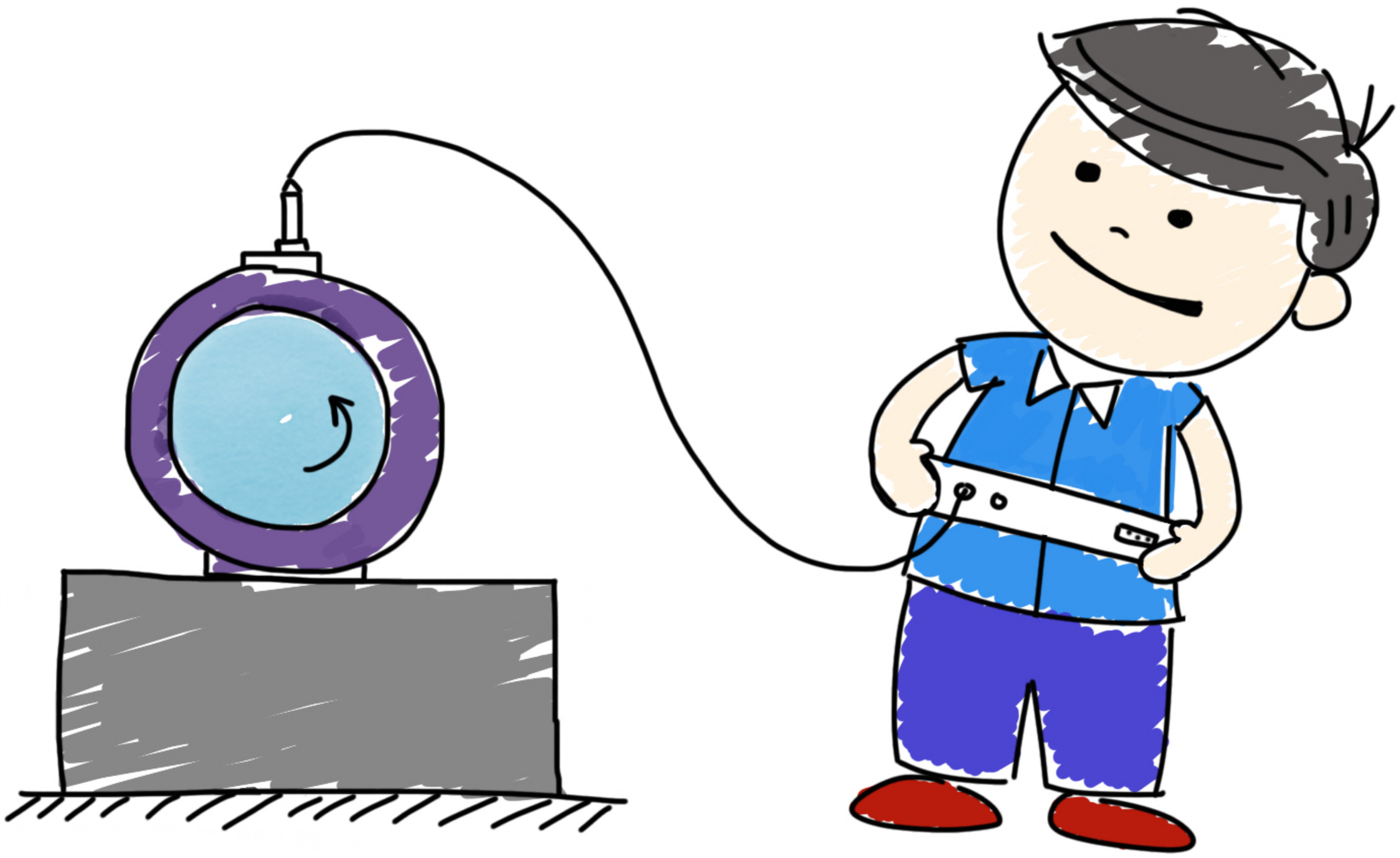
2. **Velocity**

   Unit: m/s, mm/s, in/s

3. **Acceleration**

   Unit: m/s^2, mm/s^2, in/s^2, g

   **1 g = $9.8\ m/s^2$



Since the amplitude is a function of time, we can **convert** a unit to another unit

Displacement: $$y(t) = A\sin(\omega t + \phi)$$

Velocity: $$v(t) = \frac{dy(t)}{dt} = A\omega\cos(\omega t + \phi)$$

Acceleration: $$a(t) = \frac{dv(t)}{dt} = -A\omega^2\sin(\omega t + \phi)$$

Maximum displacement: $$|y_{\text{max}}| = A$$

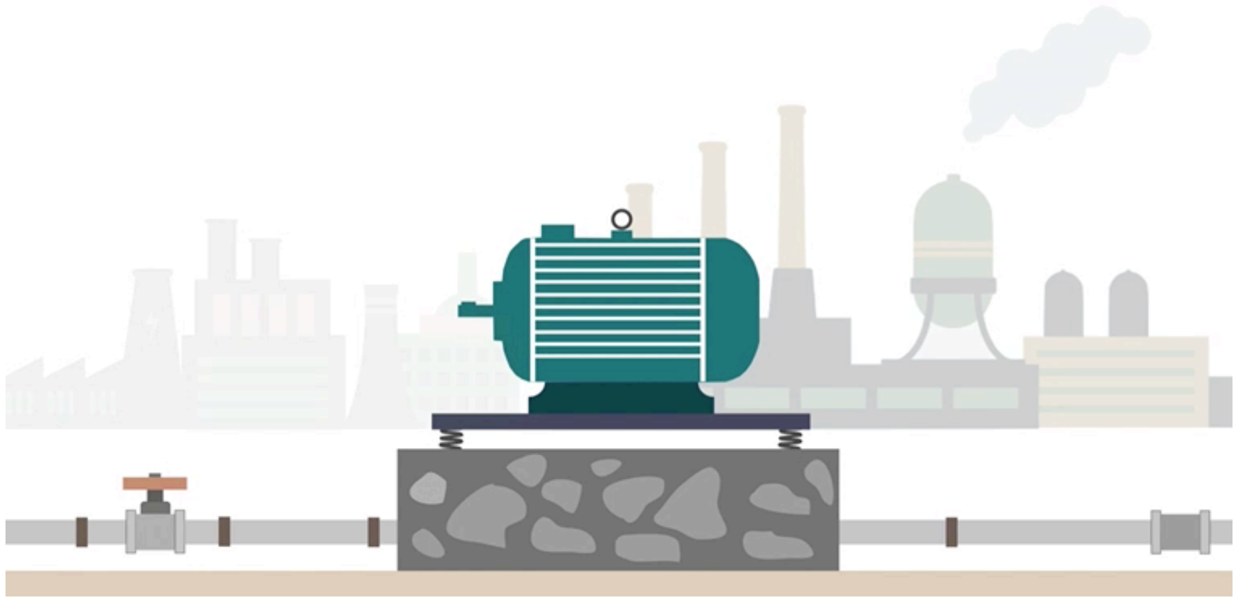
Maximum velocity: $$|v_{\text{max}}| = A\omega$$

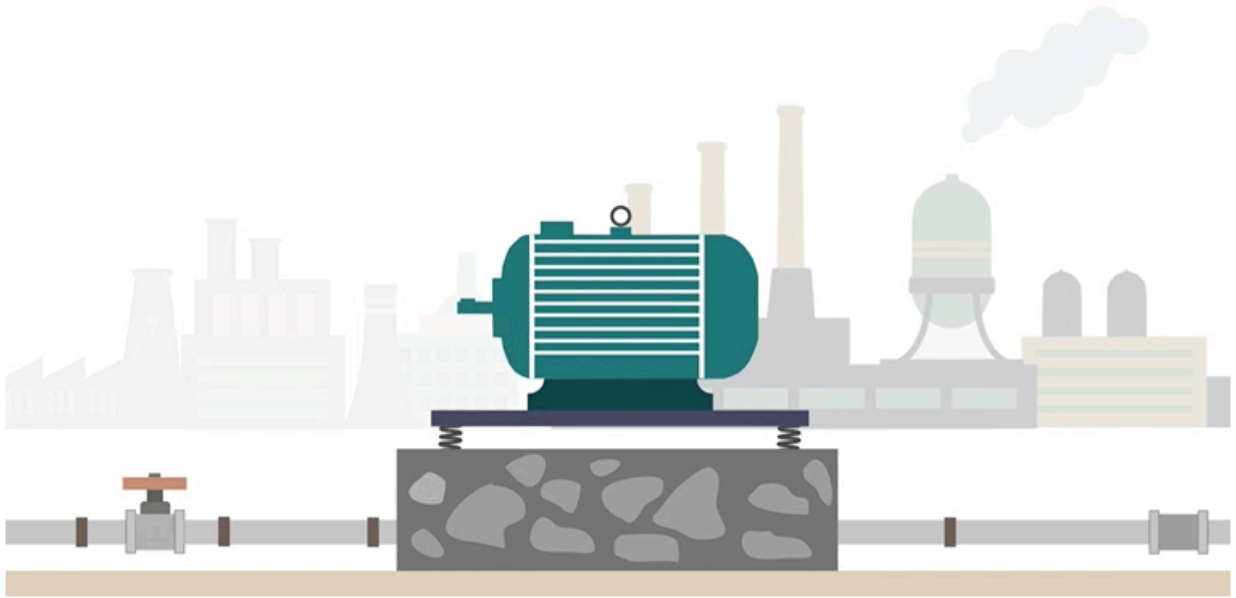
Maximum acceleration: $$|a_{\text{max}}| = A\omega^2$$

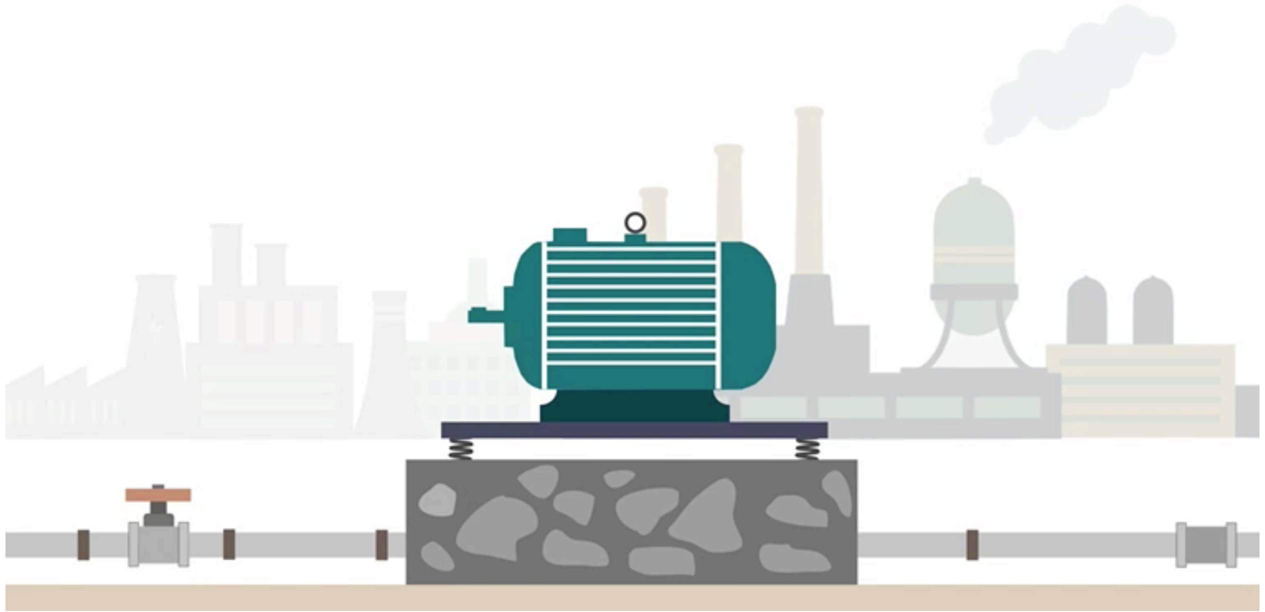
$*\omega = 2\pi f,\ f = \text{frequency}$

Frequency

How fast the object moves around the equilibrium position.





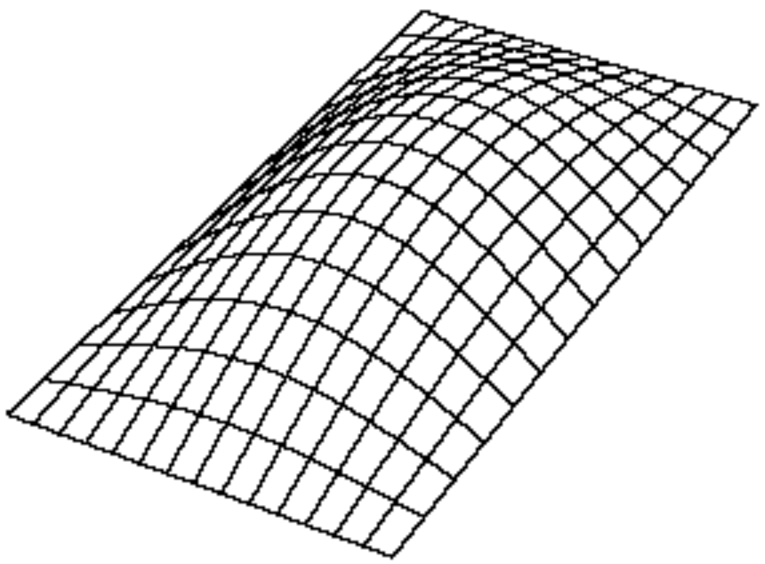


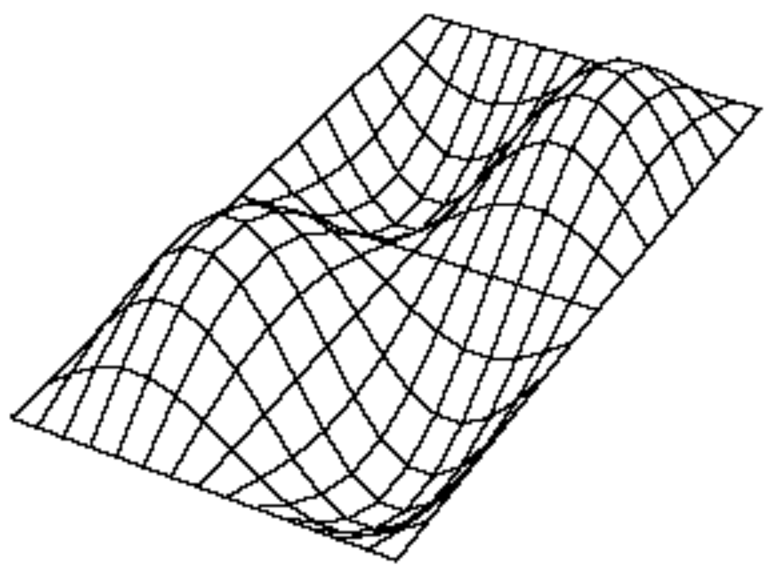
The unit of frequency is **Hertz [Hz]**.

However in practice, the frequency data is also often stated as **Rotation per Minute (RPM)** or **Cycles per Minute (CPM)** to relate the frequency with the speed of the measured vibrating machine.

$$\boxed{1200\ \text{RPM} = \frac{1200}{60\ \text{seconds}} = 20\ \text{Hz}}$$

For a flexible structure, such as beams and plates, frequency of vibration relates to the structural wavelength propagating in the structure.





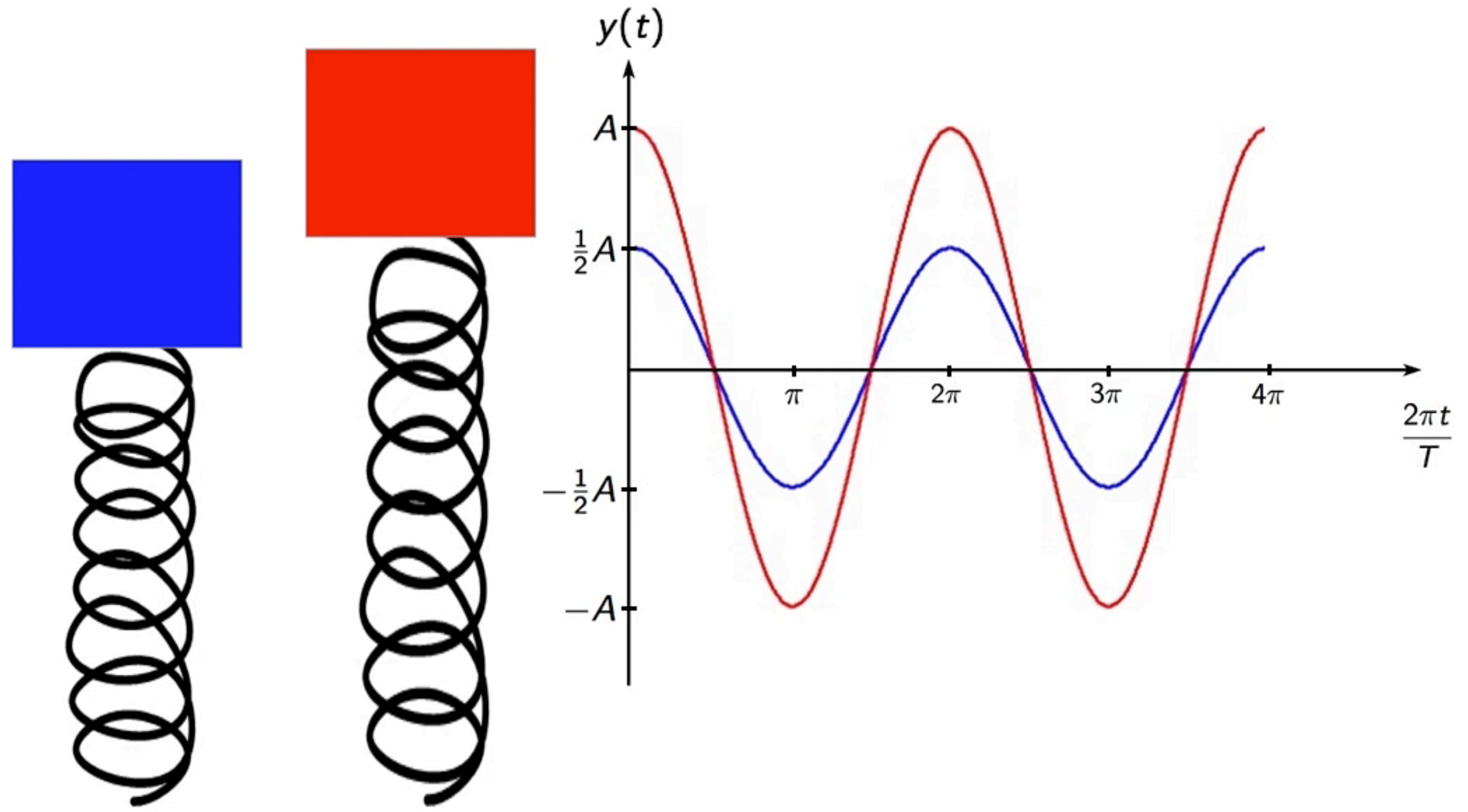
Which one has the higher frequency of vibration?

**Animation courtesy of Dr. Dan Russel

Consider two systems each carrying its absolute phase.

$$y_b(t) = \frac{1}{2}A\sin(\omega t)$$

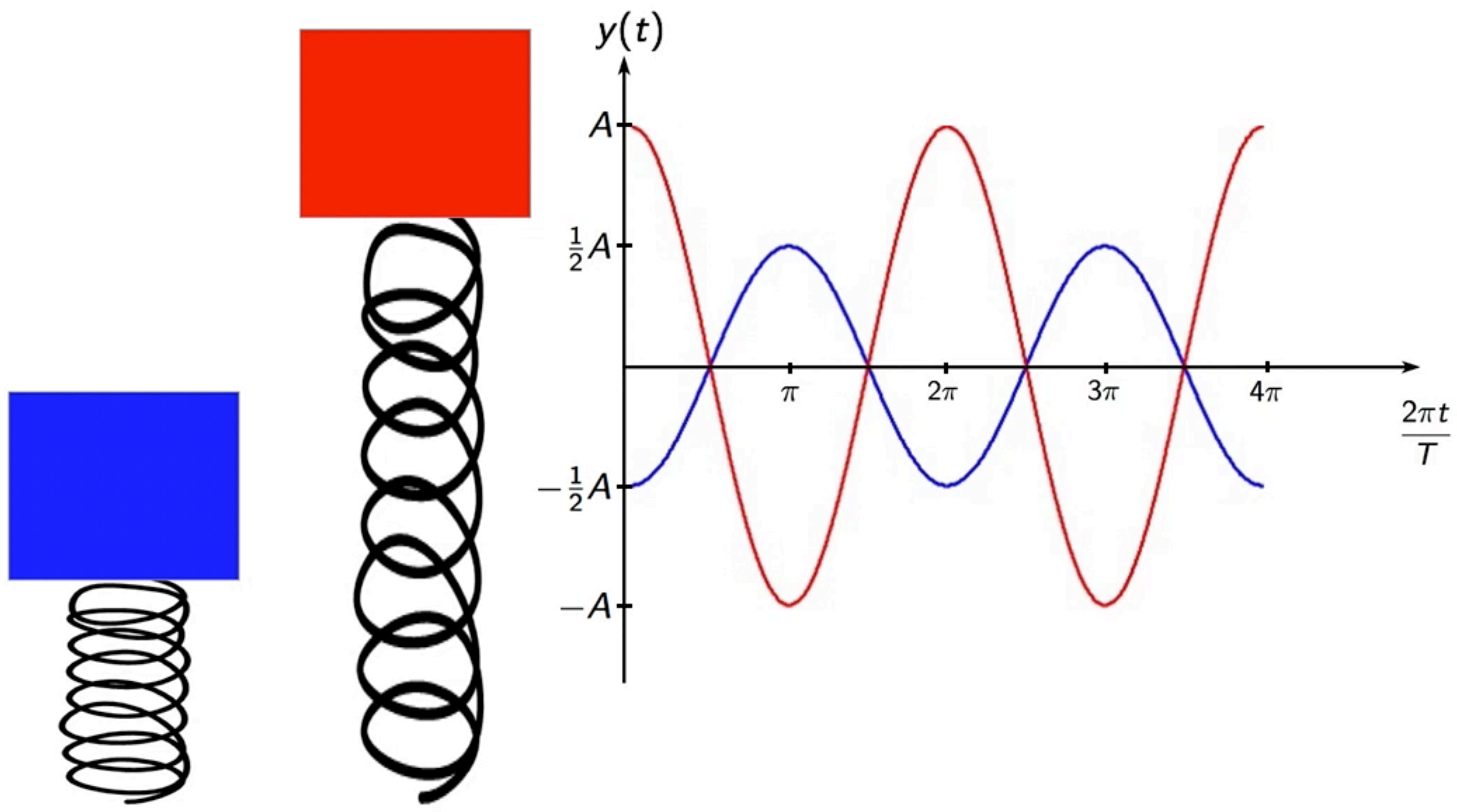
$$y_r(t) = A\sin(\omega t)$$



In-phase: $\Delta\phi = 0$

$$y_b(t) = \frac{1}{2}A\sin(\omega t + \pi)$$

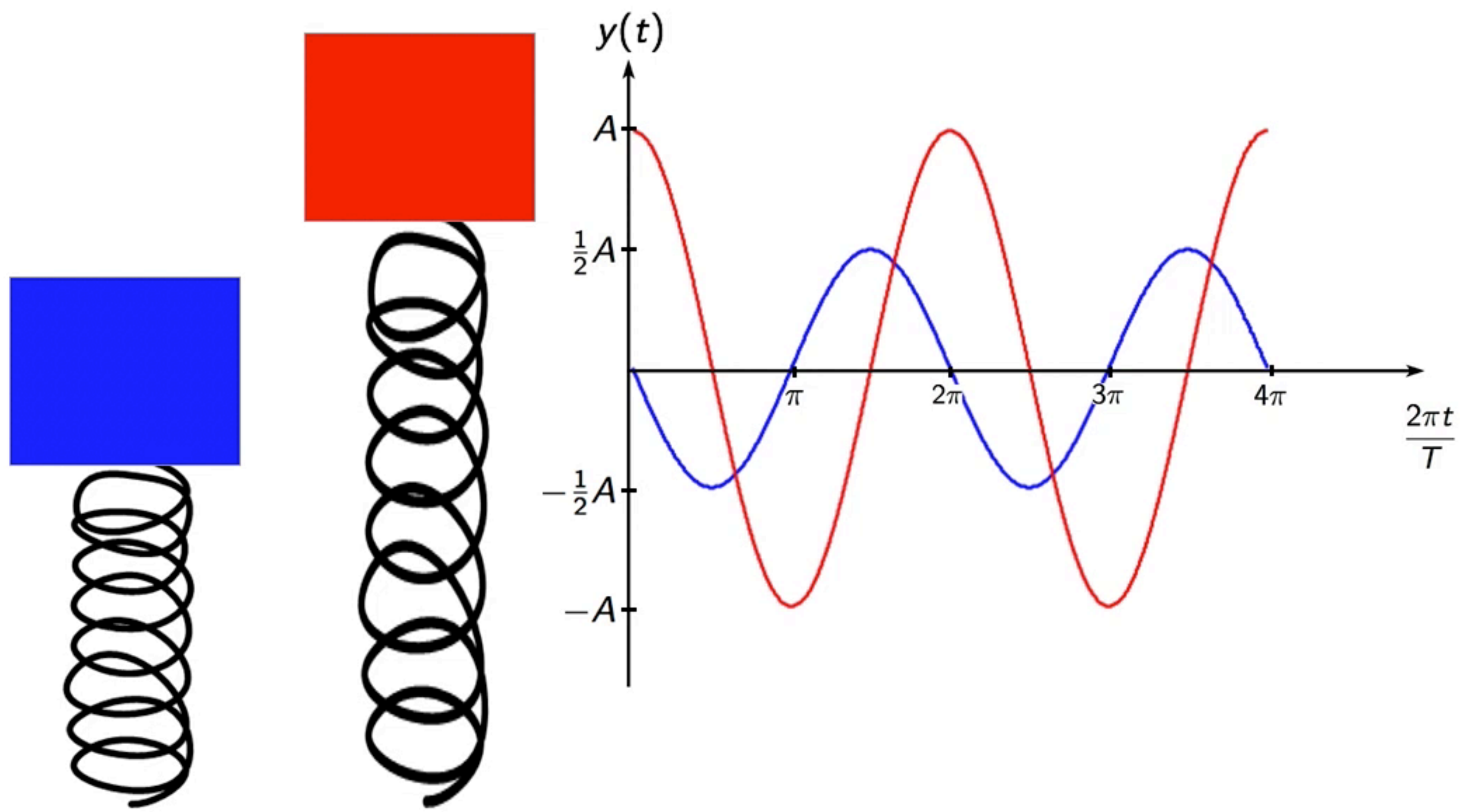
$$y_r(t) = A\sin(\omega t)$$



Out-of–phase: $\Delta\phi = \pi$

$$y_b(t) = \frac{1}{2} A \sin(\omega t + \frac{\pi}{2})$$

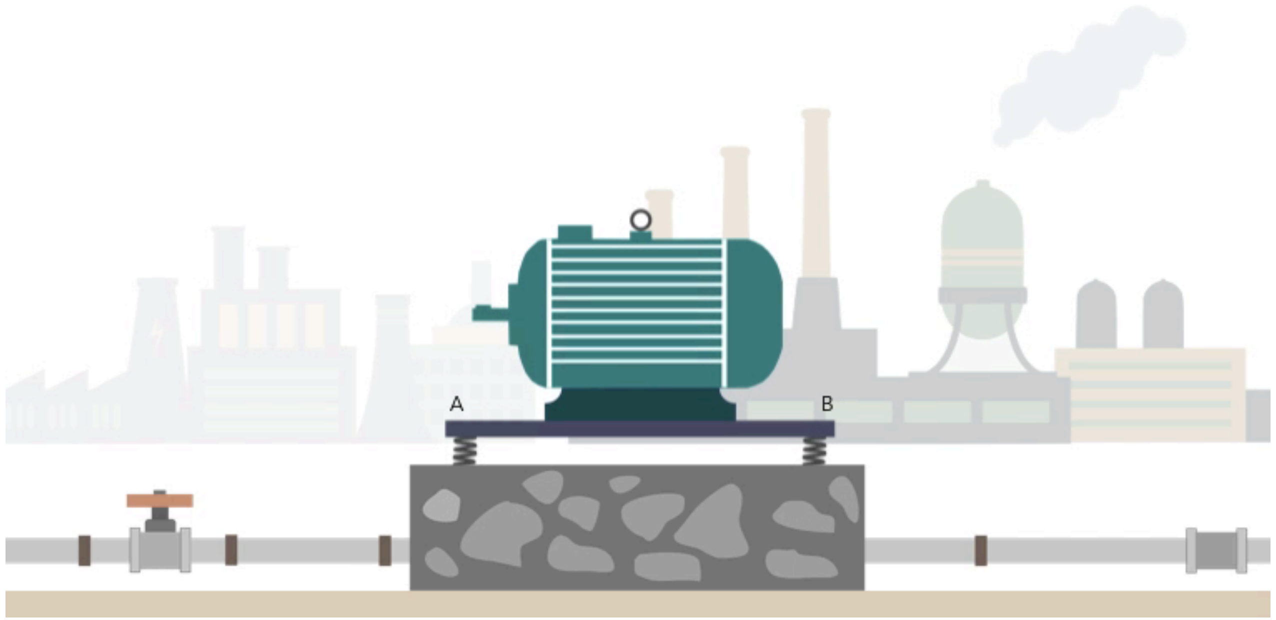
$$y_r(t) = A \sin(\omega t)$$



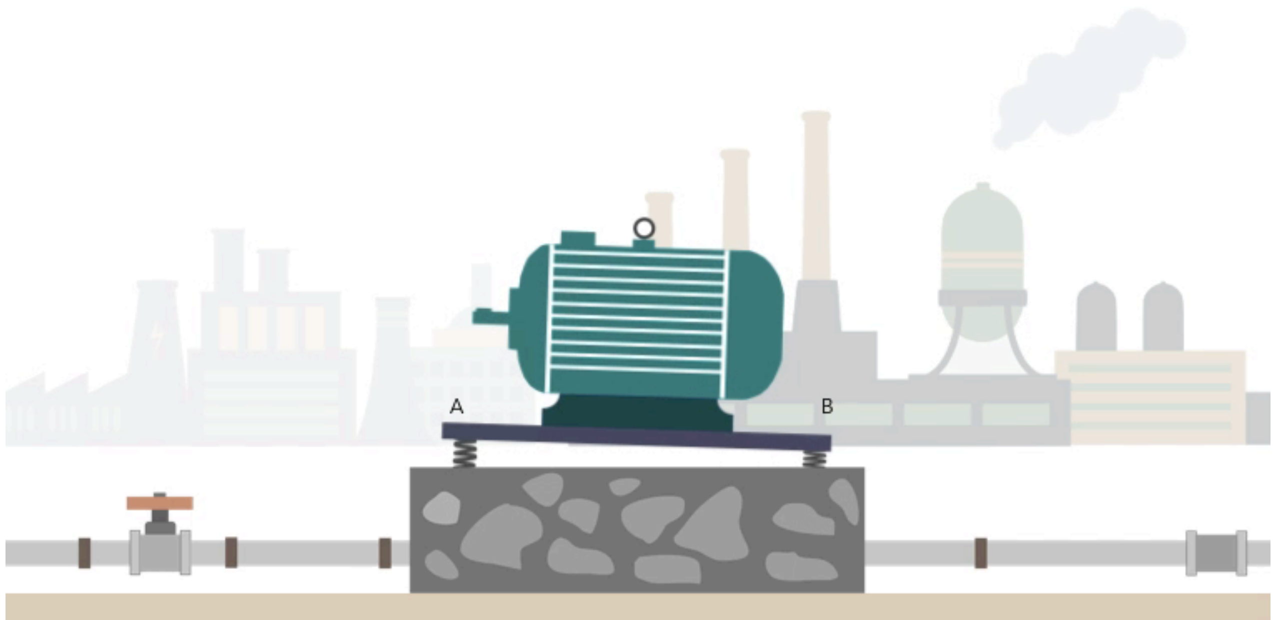
$$\Delta\phi = \frac{\pi}{2}$$

Effect of phase difference between A and B sides on the motion of the motor.

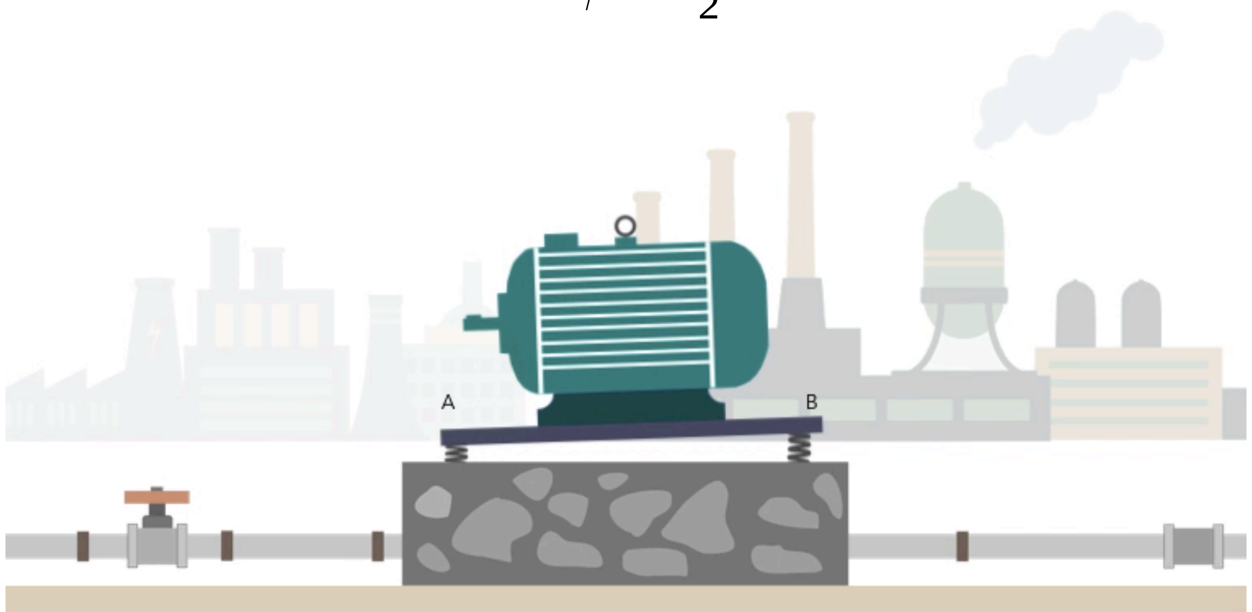
$\Delta\phi = 0$



$\Delta\phi = \pi$



$\Delta\phi = \frac{\pi}{2}$



Example of a sinusoidal vibration response:

$$y(t) = 0.2\sin(150t + 3.14)\ \text{mm}$$

Peak amplitude:	**= 0.2 mm**
Peak to peak amplitude:	**= 0.4 mm**
Frequency:	**= 150 rad/s or 24 Hz**
Period:	**= 0.04 s**
Phase:	**= 3.14 rad or 180°**

Watch the video: “Terminologies and Definitions”

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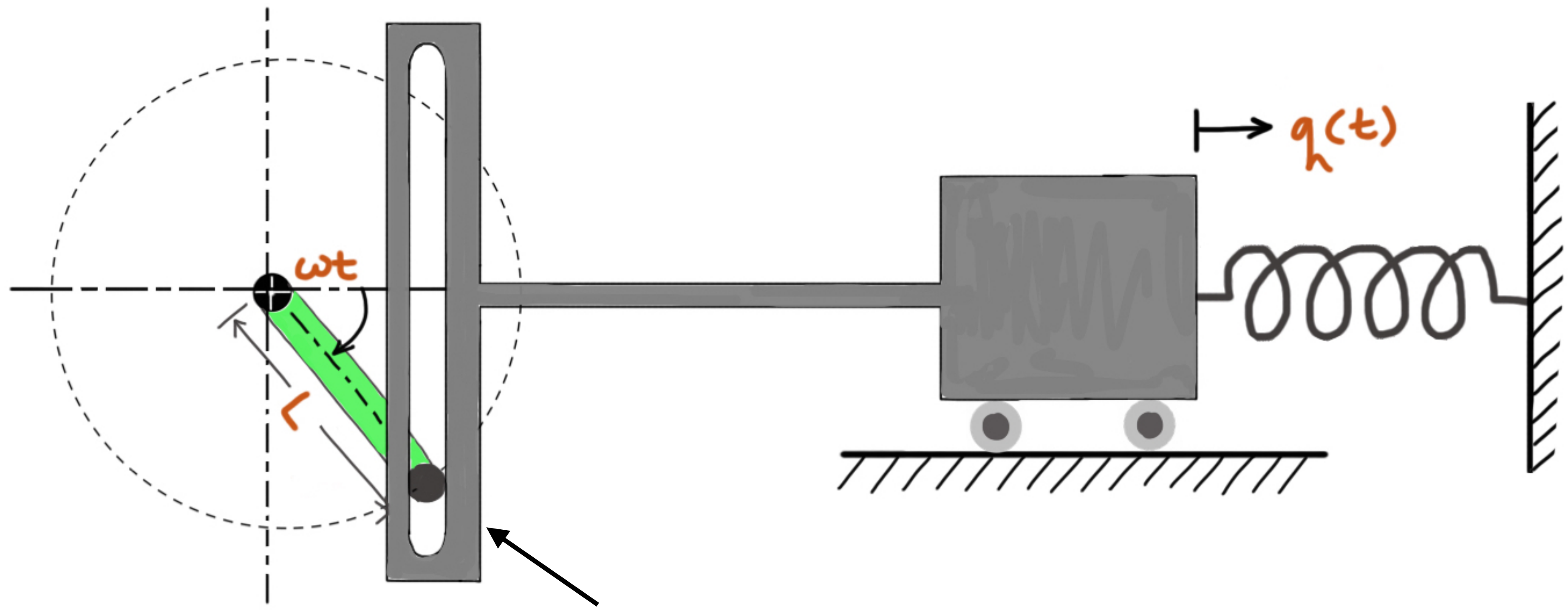
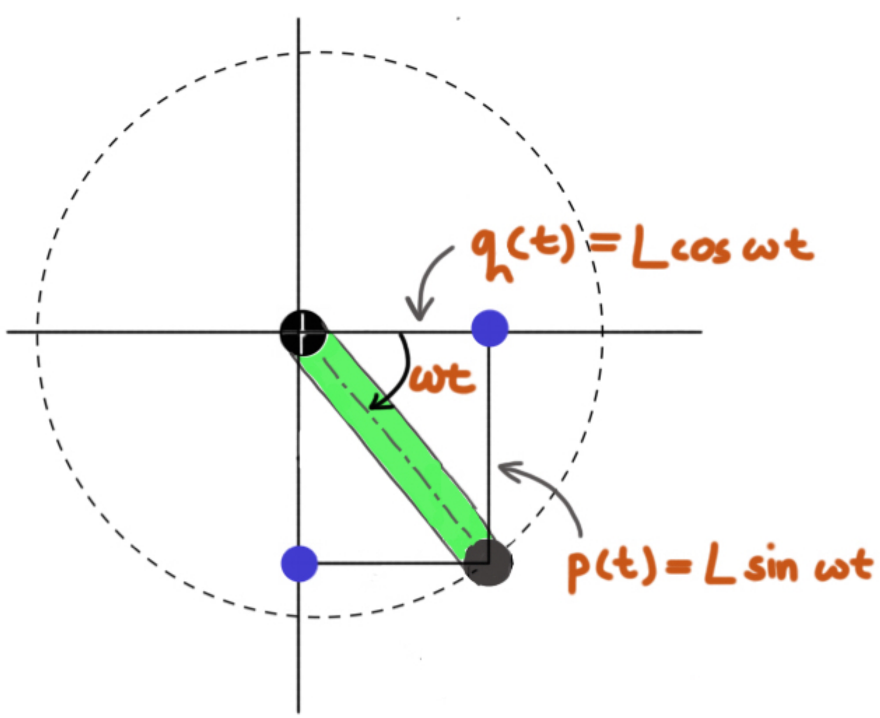
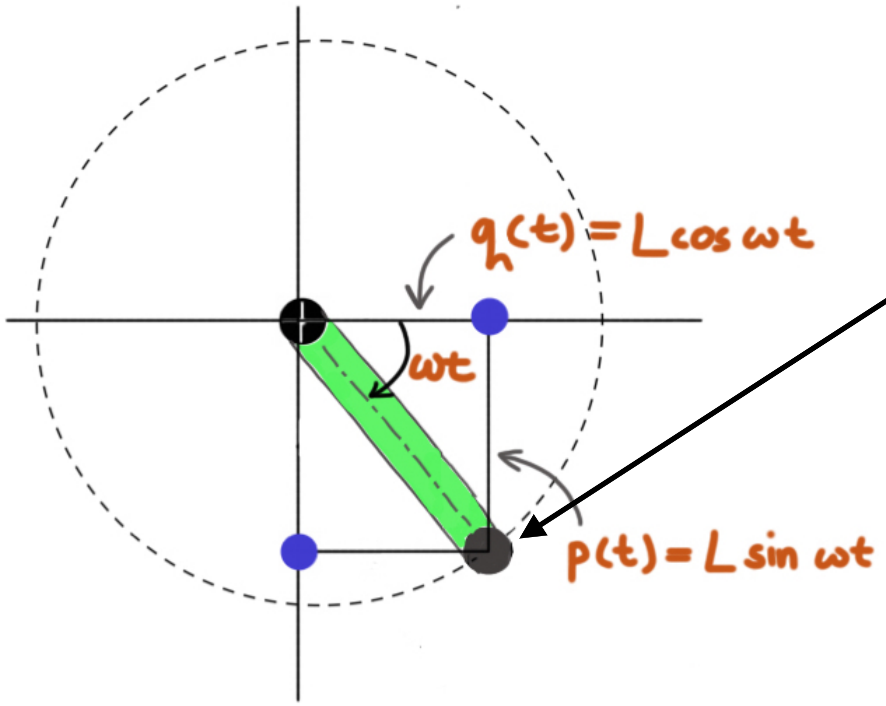
2.2

COMPLEX EXPONENTIAL NOTATION

The ‘Scotch yoke’ mechanism shows the **relation** between the **angular** displacement and the **translational** displacement.

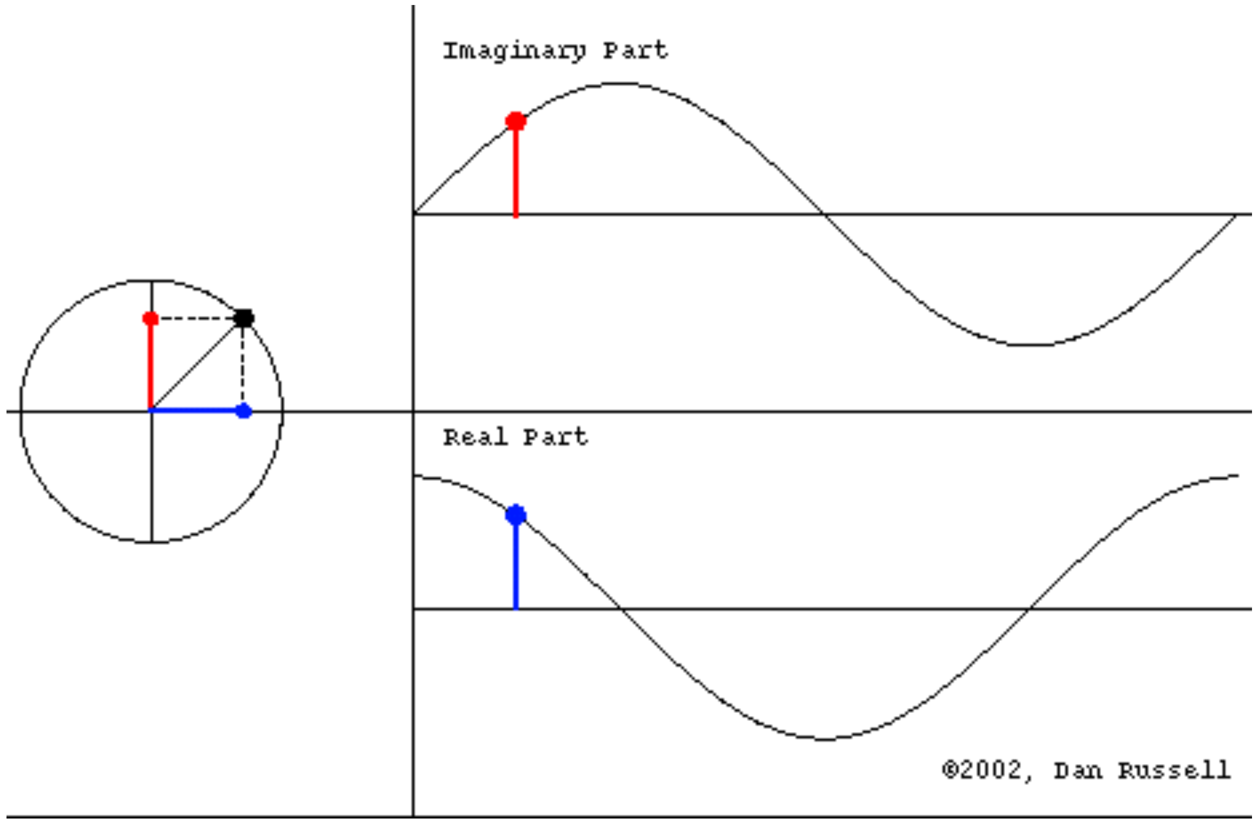






Position of this ball determines the displacement of the slotted rod.

Thus we can represents the ball position in terms of **vector in complex plane**:



$$y(t) = L\cos\omega t + jL\sin\omega t$$

$$j = \sqrt{-1}$$

Because **sin** has 90º phase difference with **cos**

So, another representation of the amplitude of vibration can use **complex number.**

We write down as:

$$\boxed{y(t) = Ye^{j\omega t}}$$

(function of space)

$$Y = a + jb$$

real part

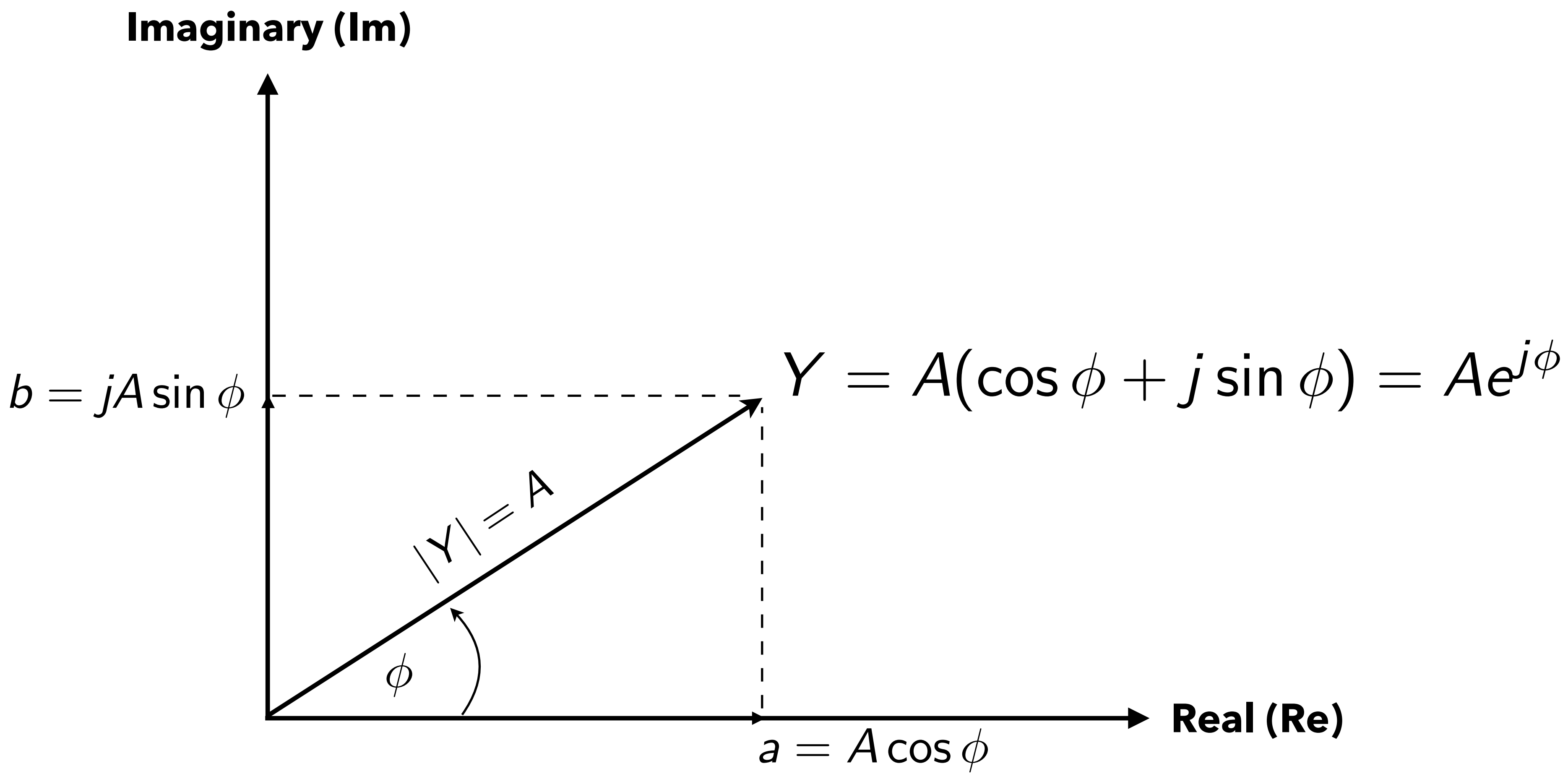
imaginary part

$$j = \sqrt{-1}$$

(function of time)

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

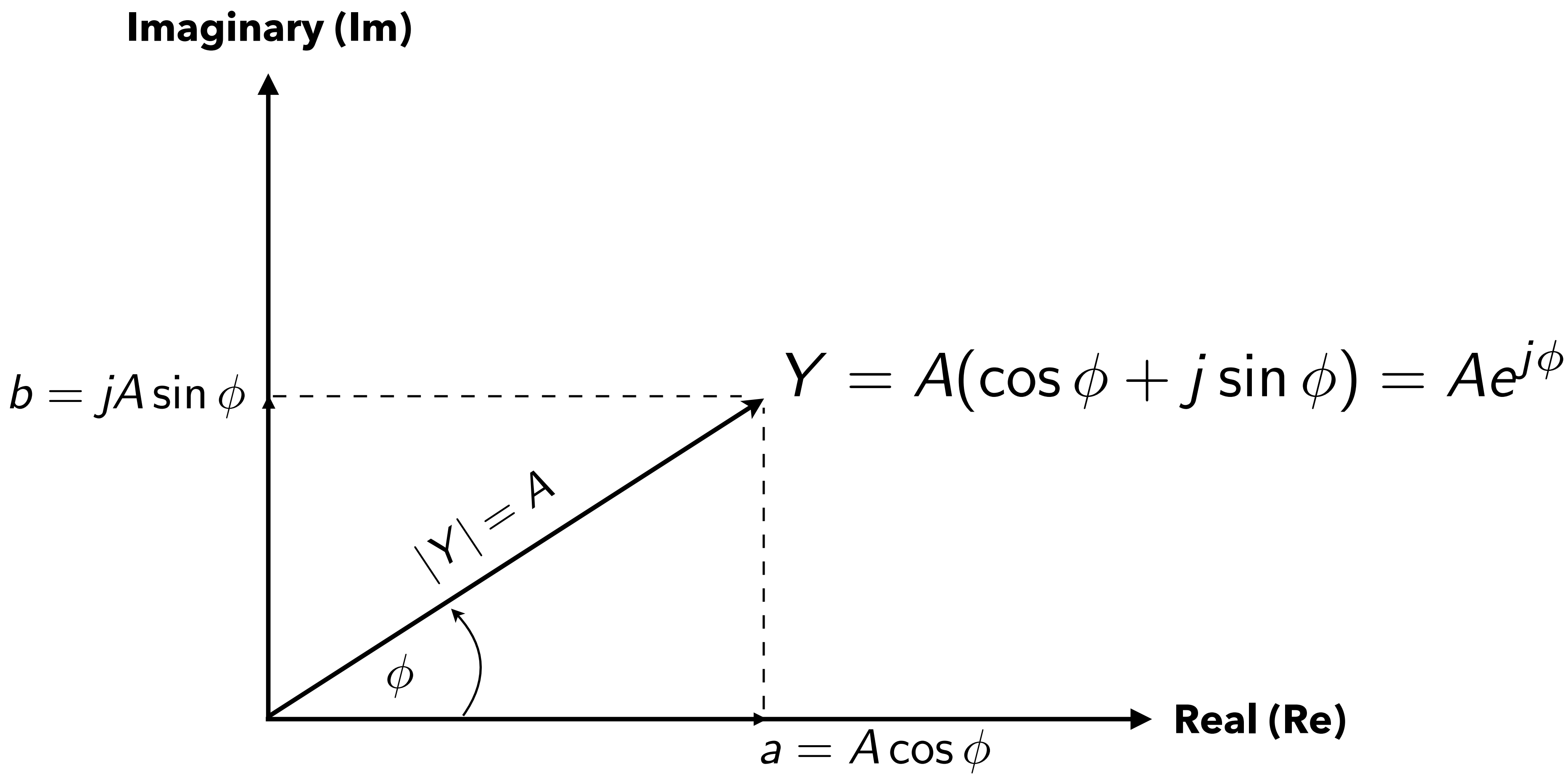
Complex amplitude carries **phase information**



$$y(t) = Ye^{j\omega t} = Ae^{j\phi}Ae^{j\omega t} = Ae^{j(\omega t+\phi)} = A[\cos(\omega t+\phi) + j\sin(\omega t+\phi)]$$

$$\boxed{\mathrm{Re}\{y(t)\} = A\cos(\omega t+\phi)}$$

$$\boxed{\mathrm{Im}\{y(t)\} = A\sin(\omega t+\phi)}$$



Phase angle:

$$\boxed{\phi = \tan^{-1}\left(\frac{\mathrm{Re}}{\mathrm{Im}}\right) = \tan^{-1}\left(\frac{b}{a}\right)}$$

Advantages of using the **complex exponential notation:**

Displacement: $$y(t) = Ye^{j\omega t}$$

Velocity: $$v(t) = \frac{dy(t)}{dt} = j\omega Ye^{j\omega t}$$

Accleration: $$a(t) = \frac{dv(t)}{dt} = \frac{d^2y(t)}{dt^2} = -\omega^2 Ye^{j\omega t}$$

Maximum displacement: $$y_{\mathrm{max}} = |y(t)| = |Y|$$

Maximum velocity: $$v_{\mathrm{max}} = |v(t)| = \omega|Y|$$

Maximum acceleration: $$a_{\mathrm{max}} = |a(t)| = \omega^2|Y|$$

$$**\,|e^{j\omega t}| = \sqrt{\cos^2 \omega t + \sin^2 \omega t} = 1$$

Example 1.1

A time harmonic response is given by $x(t) = (2 - j)e^{j\omega t}$ mm.

a. What is the maximum velocity at frequency 5 Hz?

b. What is the phase carried by the response?

Solution:

a). Displacement $\rightarrow$ $x(t) = \overbrace{(2 - j)}^{X} e^{j\omega t}$ mm

Velocity $\rightarrow$ $v(t) = \dfrac{dx(t)}{dt} = j\omega X e^{j\omega t}$

Magnitude of $v(t)$ $\rightarrow$ $V_{max} = |j\omega X| \, \overbrace{|e^{j\omega t}|}^{=1}$

$2 - 1j$

Re Im

$$V_{max} = \omega|X| = \omega\sqrt{2^2 + (-1)^2}$$
$$= 2\pi f \sqrt{5}$$
$$\approx 70 \text{ mm/s}$$

b). Phase : $\phi = \tan^{-1}\left(\dfrac{Im}{Re}\right)$

$$= \tan^{-1}\left(\frac{-1}{2}\right) = \tan^{-1}(-0.5) = -0.46 \text{ rad} = -27^\circ$$

Example 1.2

A time harmonic motion at 1 kHz has a peak acceleration of 100g (1 g = 9.8 ms⁻²). What is the peak displacement?

Solution:

Frequency → $f = 1\ \text{kHz}$

Angular frequency → $\omega = 2\pi f = 2\pi(1000)$

$= 6280\ \text{rad/s}$

Acceleration : $a_{max} = \omega^2 |Y|$

peak acceleration

peak displacement

$$100(9.8) = (6280)^2 |Y|$$

$$|Y| = 2.48 \times 10^{-5}\ \text{m}$$

$$= 0.025\ \text{mm}$$

Watch the video: "Complex Exponential Notation"

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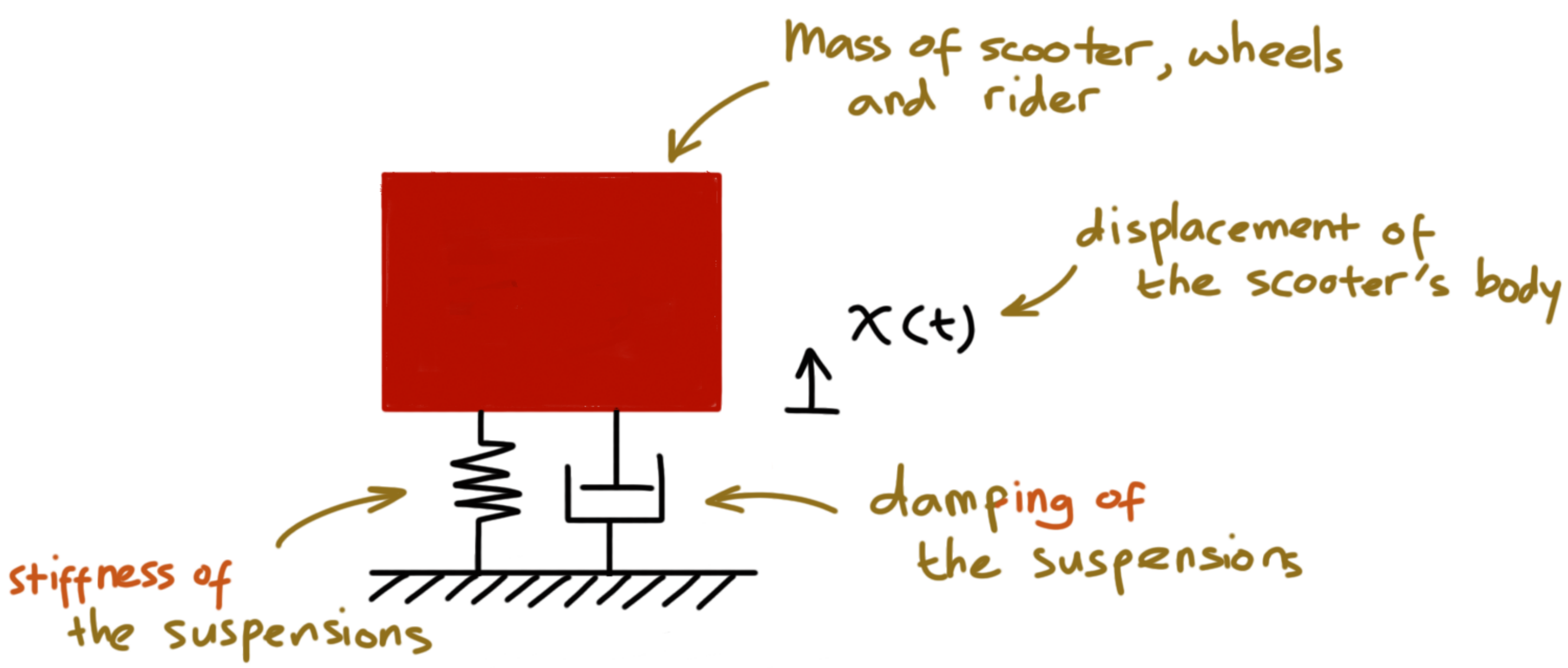
2.3

MODELLING A VIBRATING SYSTEM

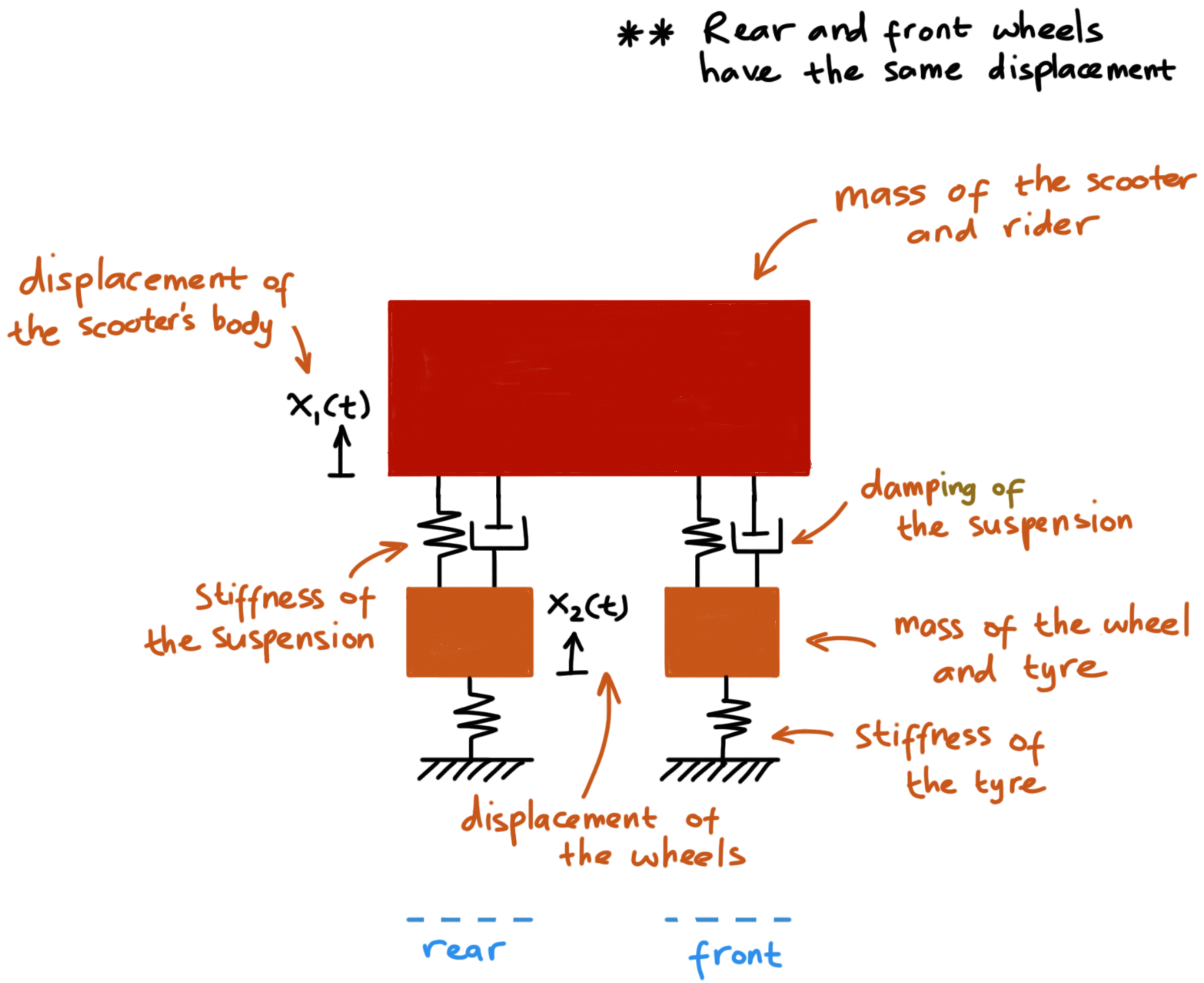
Only the body of the scooter together with the rider are moving vertically.

Single-Degree-of-Freedom System (1 DOF)



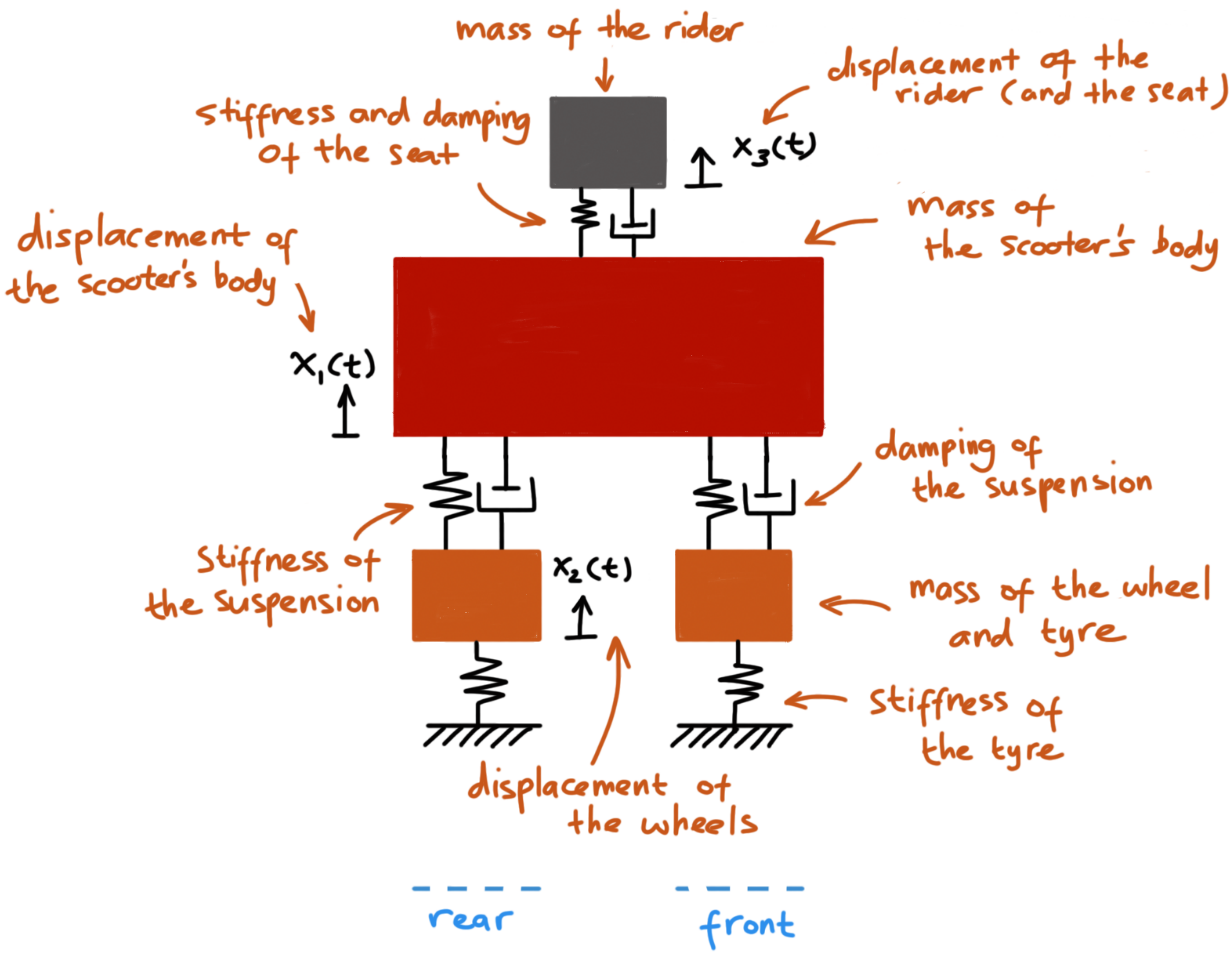
The wheels also have vertical displacement relative to the scooter's body. Both are assumed to have the same displacement and move in-phase.

Two-Degree-of-Freedom System (2 DOF)



The rider also has vertical displacement relative to the scooter's body and the wheels.

Three-Degree-of-Freedom System (3 DOF)



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http://www.azmaputra.com/animations/

My white-board animation videos:
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References

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