

SOLID MECHANICS

BETM 2303

MECHANICAL PROPERTIES OF MATERIAL (STRAIN ENERGY AND POISSON'S RATIO)

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LESSON OUTCOME

1. To show how to determine the strain energy internally stored in a material.
2. To understand how to compute the Poisson's ratio of a material.
3. To correlate the properties of some engineering materials to the shear stress-shear strain diagram

Hooke's Law

Hooke's law states that stress is proportional to strain, i.e., increase stress, proportionate increase strain

Material is said to obey Hooke's law, if the stress is below the yield stress i.e., *deform elastically*

$$\sigma \propto \varepsilon$$

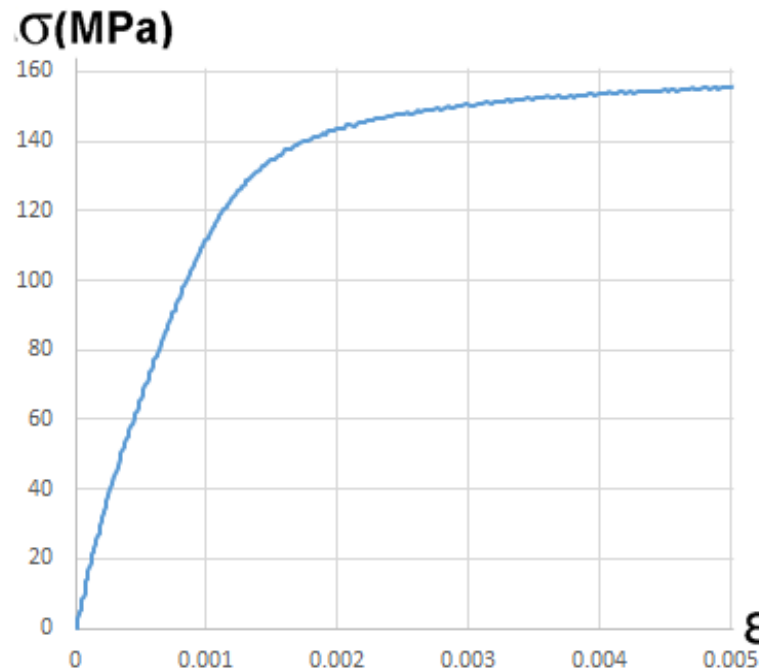
$$\sigma = E\varepsilon$$

Where, E = constant of proportionality (Modulus of Elasticity or Young's Modulus)

From the stress-strain diagram, the modulus of elasticity, E, represents the slope of the straight elastic line from the stress strain curve

Example 1

The figure below depicts the stress–strain curve for 1 mm mild steel flat tensile sample. Determine the Young's Modulus of the material.

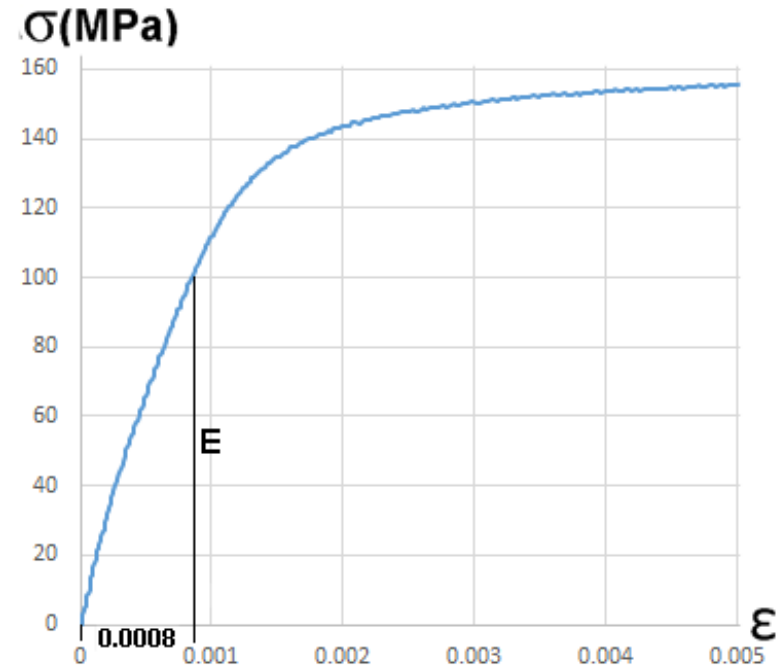


Example 1 (SOLUTION)

- The Young's Modulus of the material is the slope of the straight line,

$$E = \frac{100 \text{ MPa}}{0.0008}$$

$$= 125 \text{ GPa}$$



Hooke's Law

– Strain Hardening

Below the elastic limit, the material behaves elastically, i.e., if the stress is removed, then the strain will disappear. Hence the material will go back to its original position.

Beyond the elastic limit, the material behaves plastically, i.e., the material goes into yielding (yield stress) when the strain does not return to the original position after the stress is removed.

In the plastic region, as the stress increases, the material tends to strengthen itself against the stress. Then more and more stress is needed to produce more plastic deformation.

Strain hardening implies that the material becomes stronger as the strain increases.

Recoverable elastic strain and plastic strain

- Strain in a stress-strain diagram can be describe in two ways:

- i) Recoverable elastic strain: is the temporary strain on the material that can be recovered after the stress is removed.

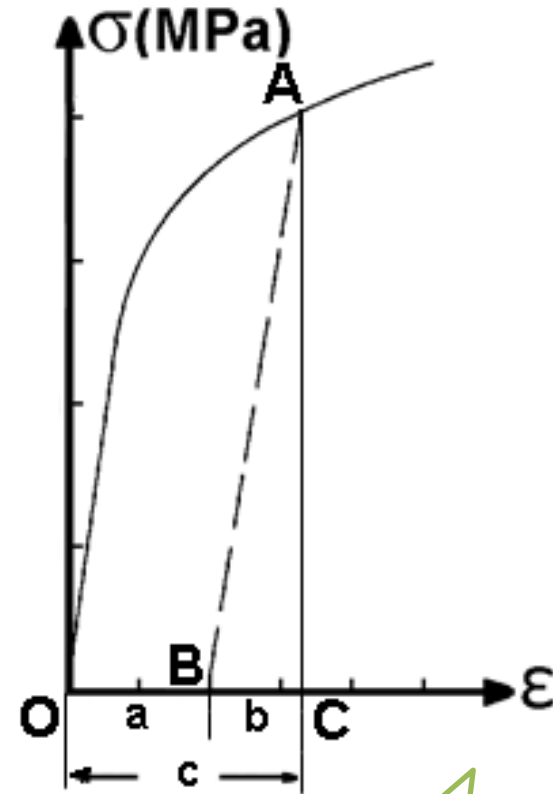
- ii) Permanent plastic strain: is a permanent strain on the material that cannot be recovered after the stress is removed.

Recoverable elastic strain and plastic strain

- For example, loading of material above the yield point to **A** and then unloading to point B. Then
- The total strain at point A is **c** (OAC)
- The recoverable elastic strain is **b** (BAC) on unloading is given by

$$b = \frac{\sigma}{E}$$

- The permanent plastic strain is **a** (OAB)



Understand
carefully!

Strain Energy

Strain energy is energy internally stored throughout volume as a material deformed by external load.

Strain energy can be obtained from the stress-strain curve as the area under the curve.

Strain energy can be expressed in two ways, namely:

- (i) Modulus of resilience
- (ii) Modulus of toughness

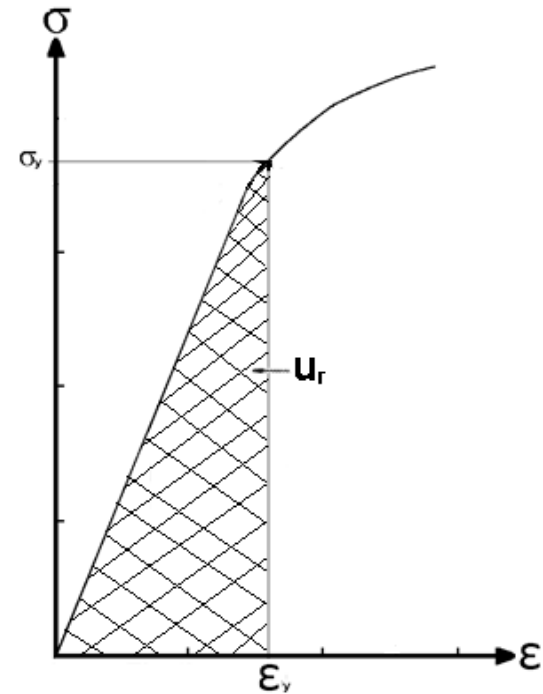
Unit: J/m^3

Strain Energy – Modulus of Resilience

Modulus of resilience: is ability to absorb energy without any permanent damages.

This can be simply expressed as the area under the curve within the elastic region.

Symbol is u_r



Strain Energy – Modulus of Resilience

From the Figure, the area under the graph is equal to the area of triangle.

$$\text{Area of triangle} = \frac{1}{2} (b \times h)$$

Then,

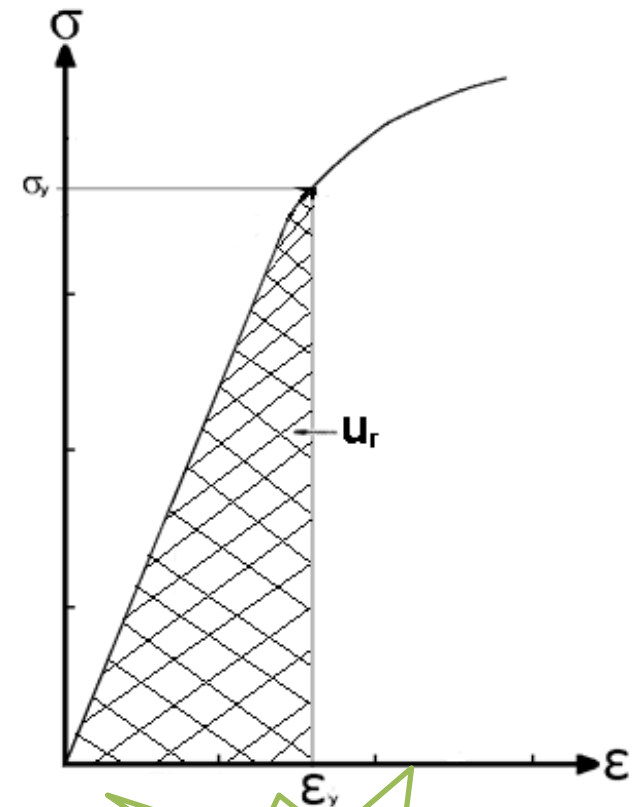
$$u_r = \frac{1}{2} \sigma_y \epsilon_y$$

For linear elastic, Hooke's law $\sigma = E\epsilon$ applied

$$\epsilon_y = \frac{\sigma_y}{E}$$

Substituting for ϵ_y , then we have

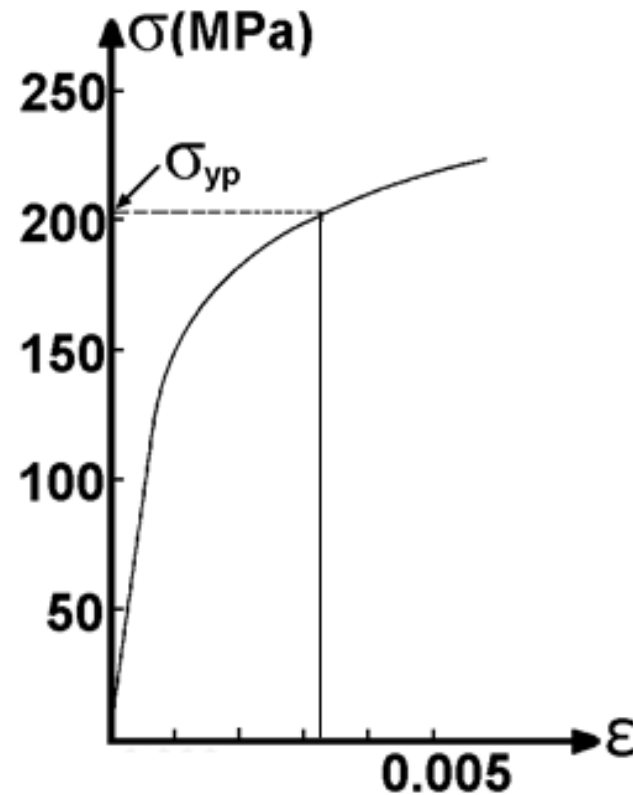
$$u_r = \frac{1}{2} \sigma_y \epsilon_y = \frac{1}{2} \frac{\sigma_y^2}{E}$$



Understand
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Example 2

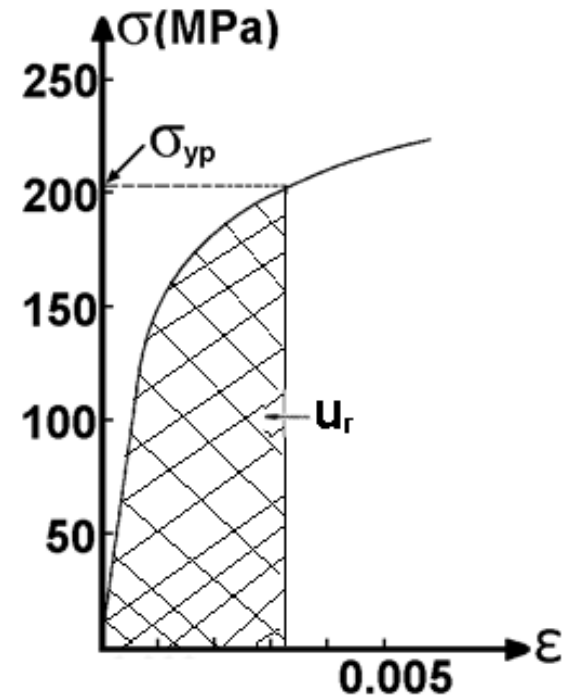
The stress–strain diagram for mild steel tensile sample is shown **below**. Determine the modulus of resilience of the material.



Example 2 (SOLUTION)

- The modulus of resilience represent, the area under the curve.
- Computing the modulus of resilience,

$$\begin{aligned}(u_r) &= \frac{1}{2} \sigma_y \varepsilon_y \\ &= \frac{1}{2} (200)(0.0033) \\ &= 0.33 \text{ MJ/m}^3 \text{ (Ans)}\end{aligned}$$

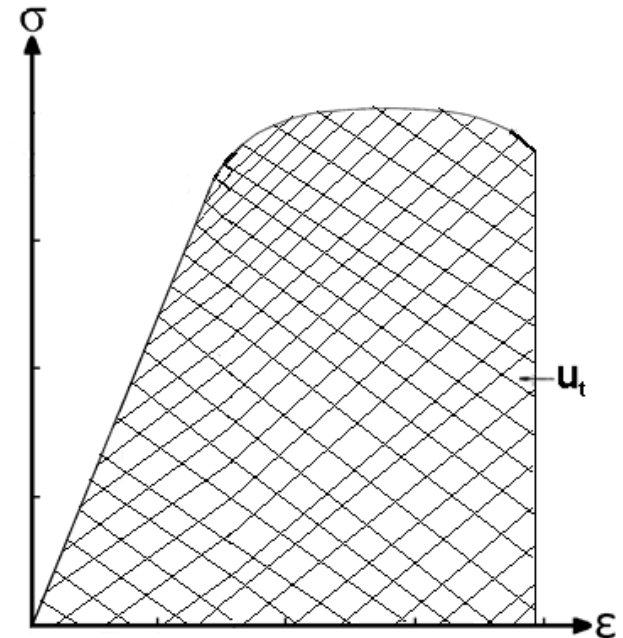


Strain Energy – Modulus of Toughness

Modulus of toughness: is ability of the material to absorb energy with large strain hardening (permanent damages).

This can be simply expressed as the total area under the stress-strain curve.

Symbol is u_t



Understand
carefully!

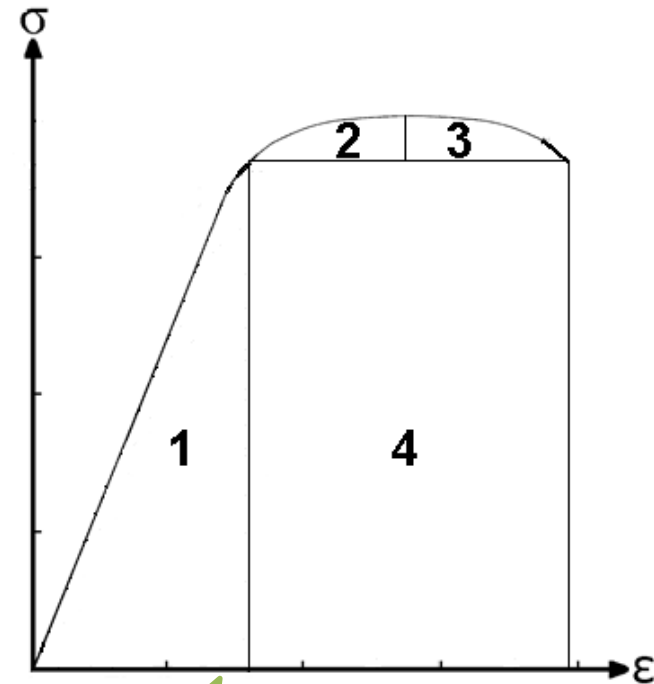
Strain Energy – Modulus of Toughness

From the Figure, the area under the graph does not have a well defined shape.

Divide the curve into different segment with a well defined shape.

Then, calculate the area of each shape and sum together.

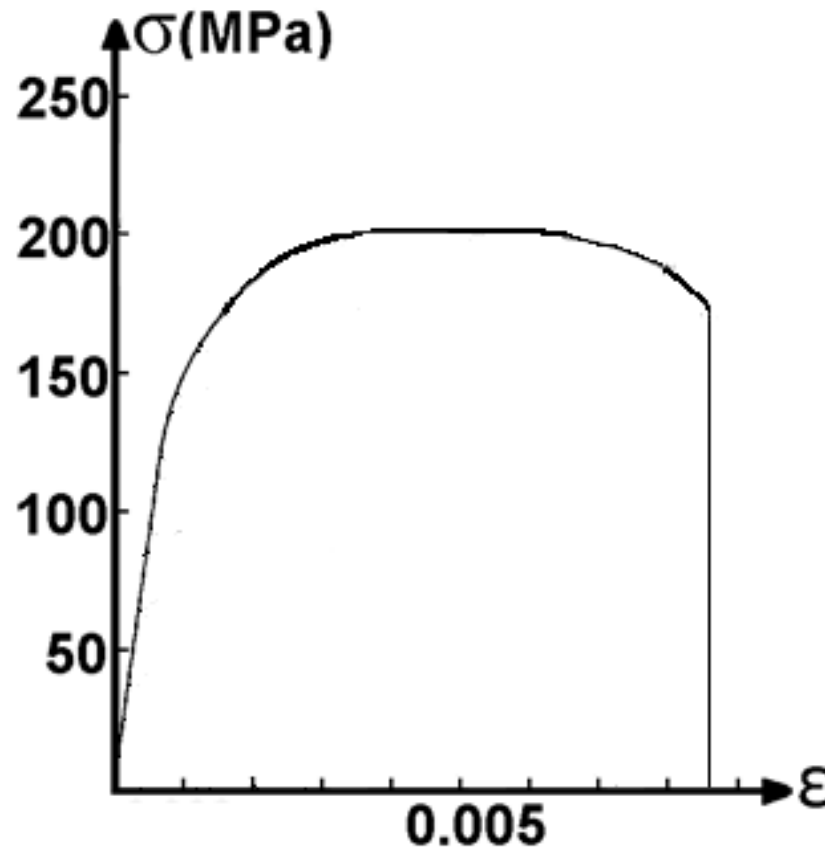
$$u_t = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4$$



Understand carefully!

Example 3

The stress–strain diagram for mild steel tensile sample is shown below. Determine the modulus of toughness of the material.



Example 3 (SOLUTION)

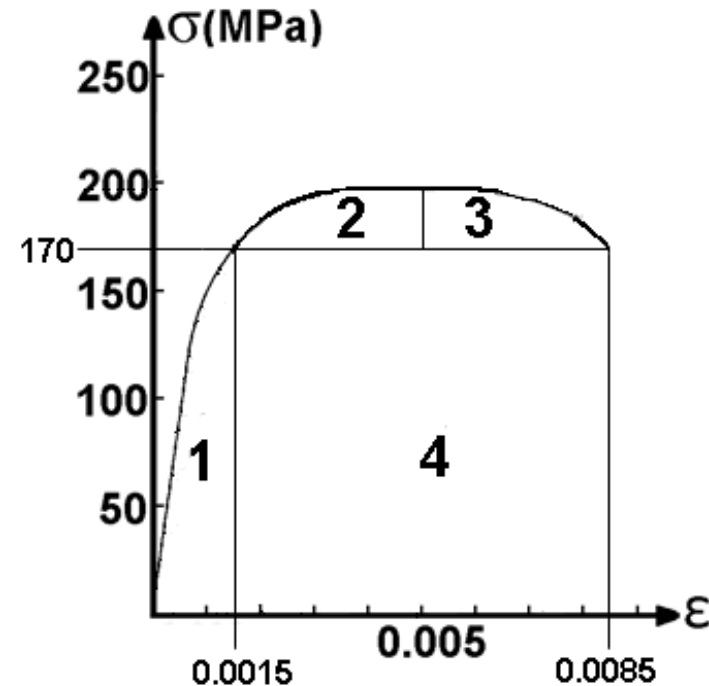
- The modulus of toughness represent, the entire area under the curve.
- Computing the area of each segments,

$$Area1 = \frac{1}{2}bh = \frac{1}{2}(0.0015 \times 170) = 0.1275 \text{ MJ/m}^3$$

$$Area2 = \frac{1}{2}bh = \frac{1}{2}(0.0035 \times 30) = 0.0525 \text{ MJ/m}^3$$

$$Area3 = \frac{1}{2}bh = \frac{1}{2}(0.0035 \times 30) = 0.0525 \text{ MJ/m}^3$$

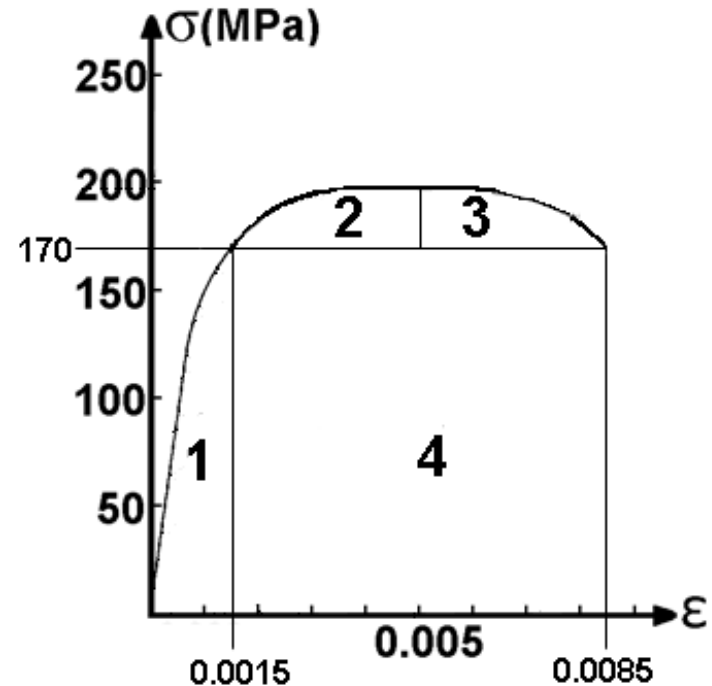
$$Area4 = lxb = (0.007 \times 170) = 1.19 \text{ MJ/m}^3$$



Example 3 (SOLUTION)

- Computing the modulus of toughness, by summing up the area of all segments

$$\begin{aligned}
 u_t &= Area_1 + Area_2 + Area_3 + Area_4 \\
 &= 0.1275 + 0.0525 + 0.0525 + 1.19 \\
 &= 1.4225 \text{ MJ/m}^3 \text{ (Ans)}
 \end{aligned}$$



Poisson's Ratio

When a deformable body is subjected to an axial tensile force, not only does it elongate but it also contracts laterally.

$$\text{Poisson's ratio } (\nu) = - \frac{\text{lateral strain } (\epsilon_x)}{\text{axial strain } (\epsilon_y)}$$

Where,

$$\epsilon_x = - \frac{\Delta d}{d_o}$$

$$\epsilon_y = \frac{\Delta l}{l_o}$$

$$\text{Poisson's ratio } (\nu) = - \frac{\Delta d/d_o}{\Delta l/l_o}$$

$$\epsilon_x = -\nu \epsilon_y$$

The negative sign is added to yield a positive value for ν

Example 4

A specimen has an original length of 50 mm and a diameter of 10 mm. When subjected to axial load, the specimen length elongates 2.0 mm and its diameter contracts 0.15 mm. If the material behaves elastically. Determine the Poisson's ratio of the material from which the specimen is made.

Example 4 (SOLUTION)

$$\begin{aligned}
 \text{Poisson's ratio } (\nu) &= -\frac{\Delta d/d_o}{\Delta l/l_o} \\
 &= -\frac{(-0.15)/10}{2/50} \\
 &= -\frac{(-0.015)}{0.04} \\
 &= 0.375 \text{ (Ans)}
 \end{aligned}$$

Shear Stress-Strain Diagram

The shear stress-shear strain diagram is similar to the stress-strain diagram discussed in previous slide.

For stress-strain diagram, the use of normal stress and normal strain obtained from tensile/compression test is employed.

While in shear stress-shear strain diagram, the use of shear stress and shear strain obtained through torsion test is adopted.

Shear Stress-Strain Diagram

One of the most important test to be carried out in order to plot the τ - γ diagram is the torsion test.

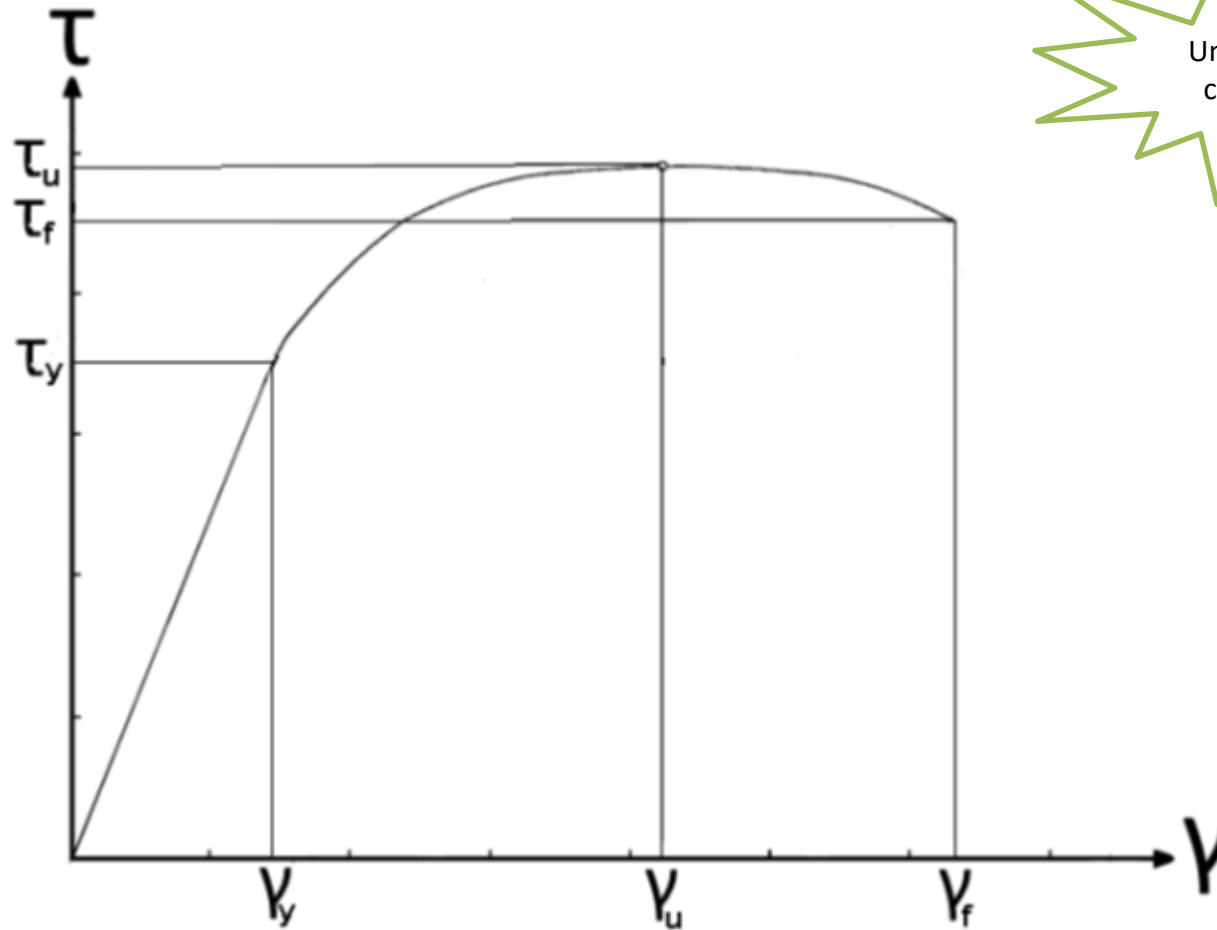
During the test, the applied torque on the sample and its corresponding angle of twist are recorded.

Then, the measured torque can be converted to shear stress, using, $\tau = Tc/J$ (where c is the radius of the tube and J is the polar moment of inertia of the cross sectional area).

And, the measured angle of twist can be converted to shear strain using, $\gamma = cd\phi/dx$ (where dx is elemental length of the tube).

Once the shear stress and shear strain has been computed. Then, you can plot the τ - γ diagram.

Shear Stress-Strain Diagram



Understand
carefully!

Typical shear stress-shear strain profile

Shear Stress-Strain Diagram

- Similar to the conventional stress-strain diagram, below the yield, the material obey Hooke's law, i.e., deform elastically, then

$$\tau \propto \gamma$$

$$\tau = G\gamma$$

- Where, G = constant of proportionality (modulus of rigidity or shear modulus)
- From the $\tau - \gamma$ diagram, the modulus of rigidity, G , represents the slope of the straight elastic line

Shear Stress-Strain Diagram

- Modulus of rigidity, G is related to the modulus of elasticity E and Poisson's ratio ν .

Elastic behavior :

$$\tau = G \gamma$$

$$G = E / 2(1 + \nu)$$

Self-review Questions

1. In which two ways can strain energy can be expressed:
- (a) modulus of elasticity and modulus of rigidity
 - (b) modulus of resilience and modulus of rigidity
 - (c) modulus of rigidity and modulus of toughness
 - (d) modulus of resilience and modulus of toughness
 - (e) all of the above

Self-review Questions

2. Poisson's ratio is defined as:

- (a)-lateral strain/axial strain
- (b)lateral strain/axial strain
- (c) axial strain/lateral strain
- (d)-axial strain/lateral strain
- (e)none of the above

Self-review Questions

3. An aluminum material has a modulus of rigidity, $G = 26 \text{ GPa}$ and young's modulus, $E = 70 \text{ GPa}$. Determine the Poisson's ratio, ν , of the material.

- (a) 2.692
- (b) 1.346
- (c) 0.346
- (d) 1.371
- (e) 0.371

Self-review Questions

4. The measure of the entire area under the stress-strain diagram is known as:

- (a) modulus of rigidity
- (b) modulus of toughness
- (c) modulus of resilience
- (d) modulus of elasticity
- (e) modulus of plasticity

Self-review Answers

1. d

2. a

3. c

4. b

Summary

- Hooke's law
 - Strain hardening
 - Recoverable elastic strain and plastic strain
- Strain energy
 - Modulus of resilience
 - Modulus of toughness
- Poisson's Ratio
- Shear stress-strain diagram
 - Modulus of rigidity