SOLID MECHANICS
BETM 2303
STRESS

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LESSON OUTCOME

1. To show how to calculate the internal loadings in a member subjected to various loads.
2. To introduce and apply the concepts of normal stress.

Introduction to Solid Mechanics

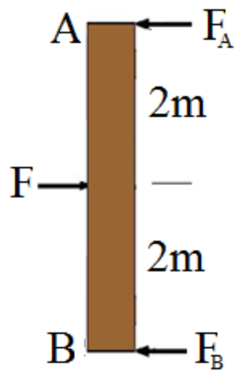
- Solid mechanics also known as Mechanics of material is the studies of relationship between force applied to a solid object and the deformation that occurs on such object as a result of the force applied.
- Solid mechanics can be simply expressed as the relationship between stress and strain on a solid object.
- For better understanding of solid mechanics, the knowledge of the fundamental of statics is important to determine the resultant loading acting on the body.

Conditions for Equation of Equilibrium

- A solid body is said to be in equilibrium if the solid body is in a stable state. That is, both the forces and moments acting on the solid body are equal and balanced.
- Equation of equilibrium: is an equation that describe a body in an equilibrium state.
- Therefore, for a body to be in equilibrium, the total force acting at the top of the solid body must be equal to the total force acting at the bottom in an opposite direction.
- For 2-dimensional loading, there are three conditions for equation of equilibrium.

Conditions for Equation of Equilibrium

Condition 1: Considering forces acting in x-direction



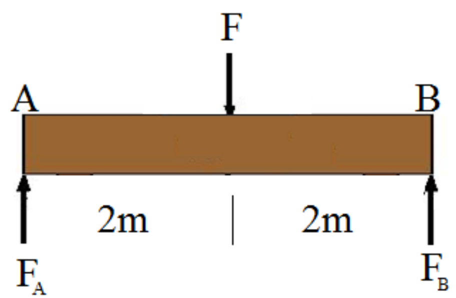
For the body to be in equilibrium, sum of force, F must be equal to sum of forces, F_A and F_B.

i.e.,

$$\Sigma F_X = 0$$

Conditions for Equation of Equilibrium

Condition 2: Considering forces acting in y-direction



For the body to be in equilibrium, sum of force, F must be

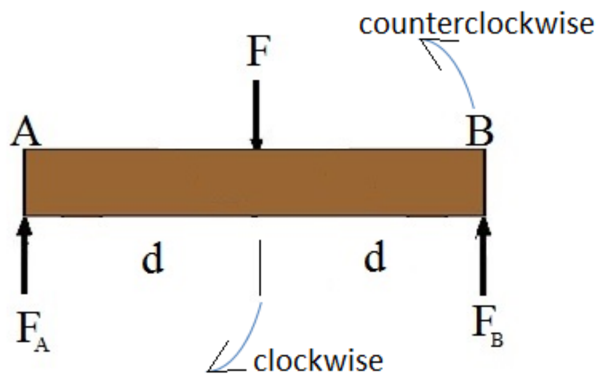
equal to sum of forces, F_A and F_B.

i.e.,

$$\Sigma F_y = 0$$

Conditions for Equation of Equilibrium

Condition 3: Considering moment about a point, say point A



For the body to clockwise moments (= d x F), must be equal to sum of counterclockwise moments (= (d+d) x F_B)

i.e. M M

$\Sigma M_A = 0$

Conditions for Equation of Equilibrium

- For 2-D loadings, the equation of equilibrium (EoE) are:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

Free-body Diagrams

Free body diagram (FBD): is a graphic representation and specification of all the external forces, internal forces (support reaction) and couple moments acting on a body.

Support Reactions: are the forces and moments that develop at the support or point of contact between bodies. Support reactions exist in various form, such as:

- Roller Support
- Pin Support
- Fixed Support

Free-body Diagrams

General rules for support reaction for a 2-D force system:

1. If a support restrict the movement in x-direction, then a force is developed in x-direction.
2. If a support restrict the movement in y-direction, then a force is developed in y-direction.
3. If rotation is resticted, a couple moment is developed.

Free-body Diagrams

Roller Support

x-direction – not restricted (no force developed in x-direction)

y-direction – restricted (a force is developed in y-direction)

Rotation is not restricted (no couple moment)

FBD

Free-body Diagrams

Pin Support

x-direction – restricted (a force is developed in x-direction)
y-direction – restricted (a force is developed in y-direction)
Rotation is not restricted (no couple moment)

FBD

Free-body Diagrams

Fixed Support

x-direction – restricted (a force is developed in x-direction)

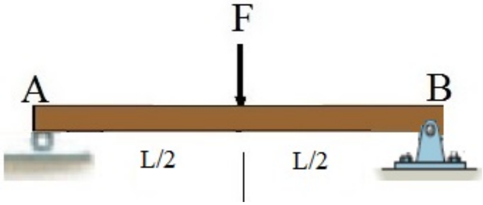
y-direction – restricted (a force is developed in y-direction)

Rotation - restricted (a couple moment is developed)

FBD

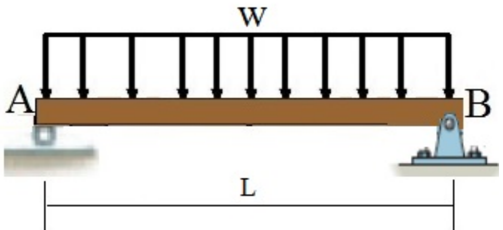
Resultant Internal Loadings

Concentrated Load: is a force which is applied/focused at a single point on the structure.



Concentrated loading

Distributed Load: is forces which are applied to a **considerable length** of the structure. Distributed load is measured as **force per unit length.**

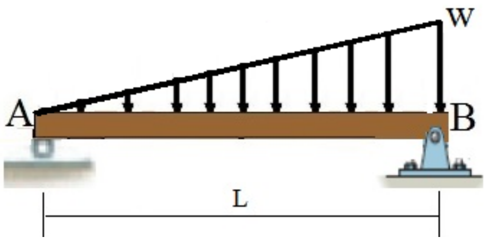


Uniformly distributed load

Distributed load can be classified as:

i) Uniformly distributed

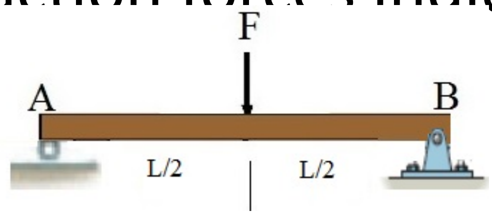
ii) Non-uniformly distributed

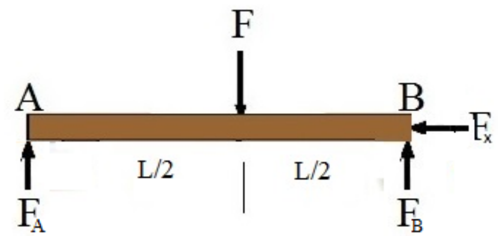


Non-uniformly distributed load

Steps for analysis for Concentrated Load

1) Draw the free body diagram with the support reaction forces indicated





2) Use equation of equilibrium (EoE) to determine the internal loading

$\sum F_x = 0$

i.e., $F_x = 0$

$\sum F_y = 0$

i.e., $F_A + F_B - F = 0$

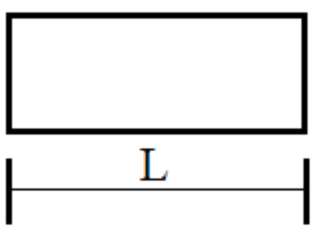
Resultant Internal Loadings

Steps for analysis for Distributed load Load

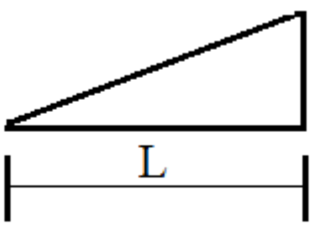
1) Change the distributed load to concentrated load and determine the magnitude of concentrated load

Magnitude of concentrated load = area under consideration

2) Determine the centroid/center of gravity of the body



= 1/2 x L



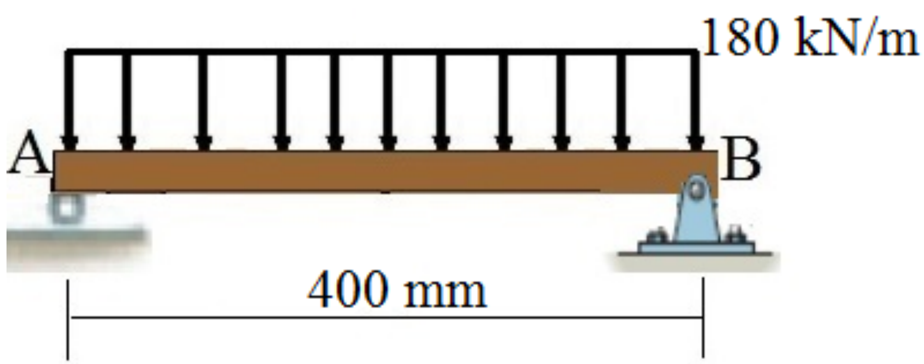
= 1/3 x L

3) Draw the free body diagram with the support reaction forces indicated

4) Use equation of equilibrium (EoE) to determine the internal loading

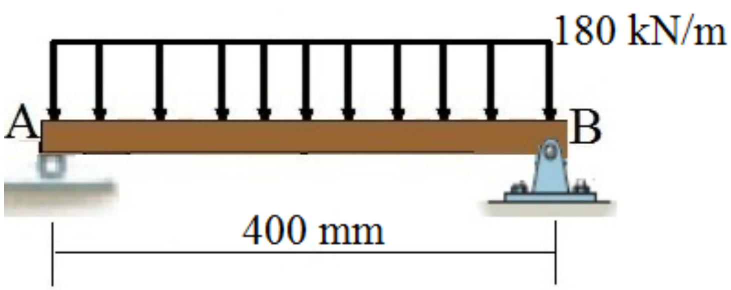
Example I

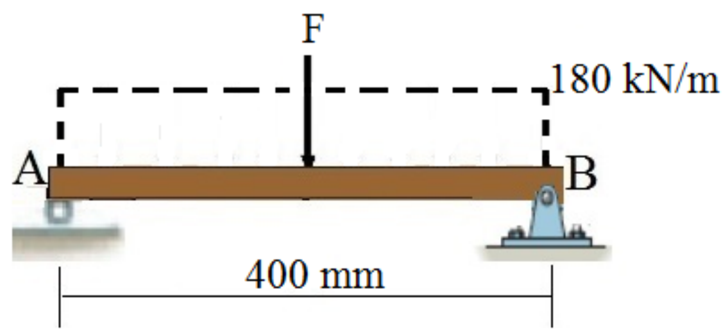
Calculate the resultant internal loadings acting on the beam shown below



Example I (SOLUTION)

Step 1. Change the distributed load to concentrated load and determine the magnitude of concentrated load





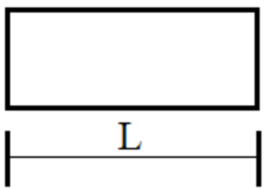
F = area under consideration (i.e., L x b)

= 180 kN/m x 0.4 m

= 72 kN

Example I (SOLUTION)

Step 2. Determine the centroid/center of gravity of the body. For rectangle, centroid

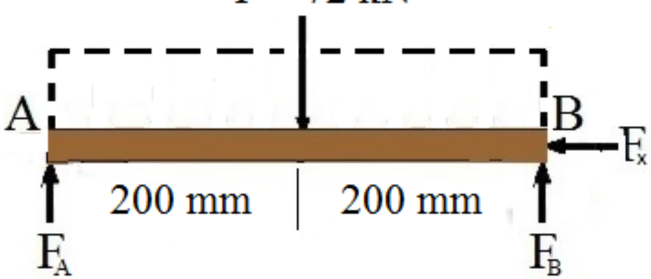


= 1/2 x L

= 1/2 x 400

= 200 mm

Step 3. Draw the free body diagram with the support reaction forces indicated



Example I (SOLUTION)

Step 4. Applying Equation of Equilibrium to determine the internal loadings

$\sum F_x = 0$

$F_x = 0$

$\sum F_y = 0$

$F_A + F_B - 72 \text{ kN} = 0$

Since, F_A and F_B *are equal, then,* $F_A + F_B = 2F_B$

Hence,

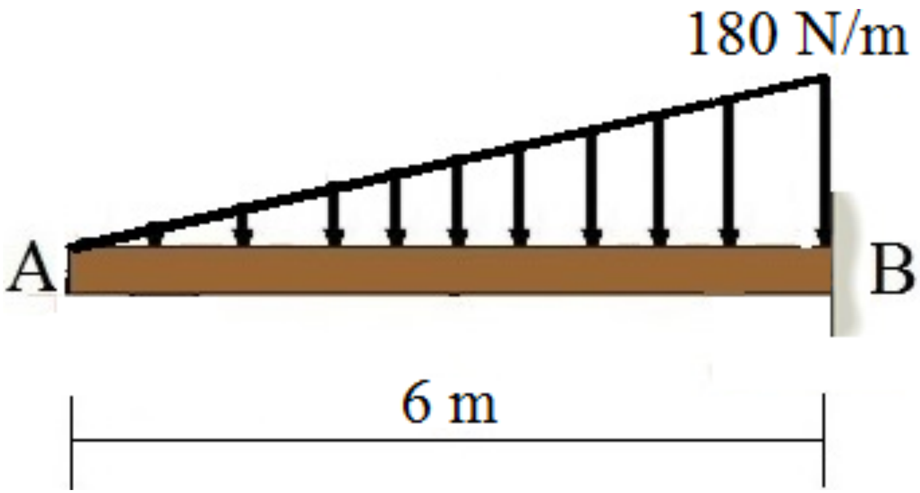
$2F_B - 72 \text{ kN} = 0$

$F_B = 72 \text{ kN}/ 2$

$F_B = 36 \text{ kN}$: $F_A = 36 \text{ kN}$

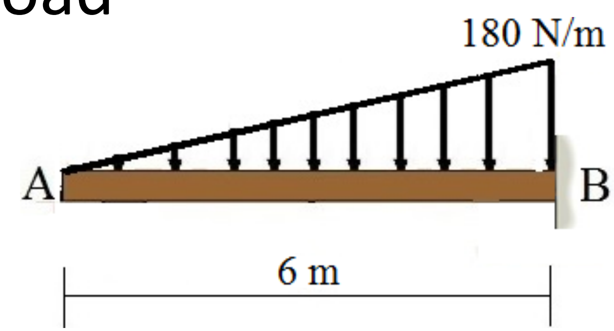
Example II

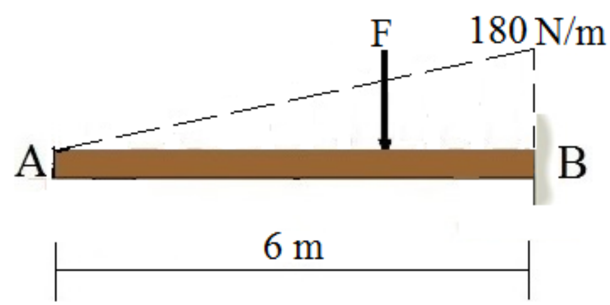
Calculate the resultant internal loadings acting on the beam shown below



Example II (SOLUTION)

Step 1. Change the distributed load to concentrated load and determine the magnitude of concentrated load





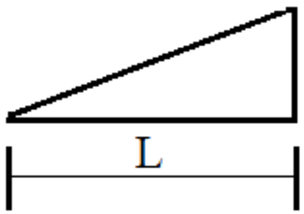
F = area under consideration (i.e., ½ x b x h)

= 1/2 x 180 kN/m x 6 m

= 540 N

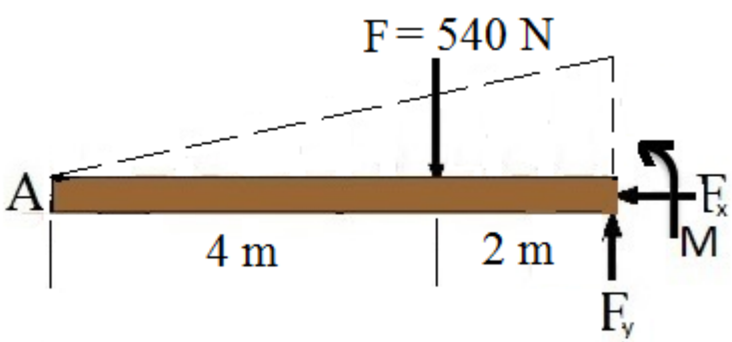
Example II (SOLUTION)

Step 2. Determine the centroid/center of gravity of the body. For triangle, centroid



= 1/3 x L
= 1/3 x 6
= 2 m (from B)

Step 3. Draw the free body diagram with the support reaction forces indicated



Example II (SOLUTION)

Step 4. Applying Equation of Equilibrium to determine the internal loadings

$\sum F_x = 0$

$F_x = 0$

$\sum F_y = 0$

$$F_y - 540\ \text{N} = 0$$

$$F_y = 540\ \text{N}$$

Taking moment about point B

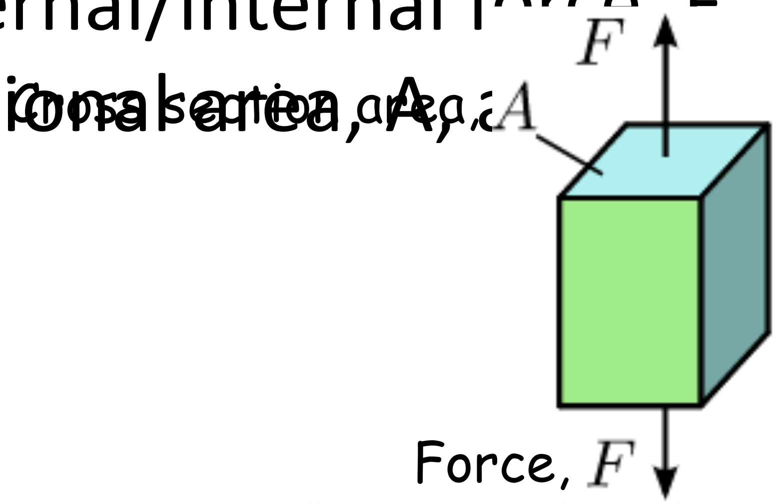
$\sum M_B = 0$

$M - 540\ \text{N}\ (2\text{m}) = 0$

$M = 1080\ \text{Nm}$

Stress

- Consider, a body subjected to external/internal force, F acting on a cross-sectional area, A, a… below



https://upload.wikimedia.org/wikipedia/commons/0/0f/Axial_stress_noavg.svg (CC-BY-SA-3.0)

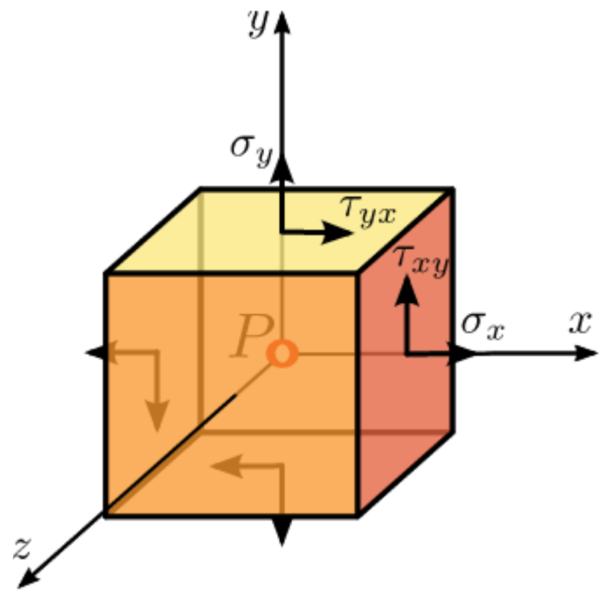
$$\text{Stress} = \frac{\text{Force}}{\text{Cross section area}} = \frac{F}{A}$$

Stress can simply be defined as force divided by the cross section area. Unit: Pascal, Pa @ N/m^2

Stress

- **Normal Stress**
  - The magnitude of the force per unit cross-sectional area, acting normal to ΔA
  - Symbol is **σ (sigma)**
  - If normal force “pulls” on ΔA -> tensile stress
  - If normal force “pushes” on ΔA -> compressive stress

$$\sigma = \frac{F}{A}$$

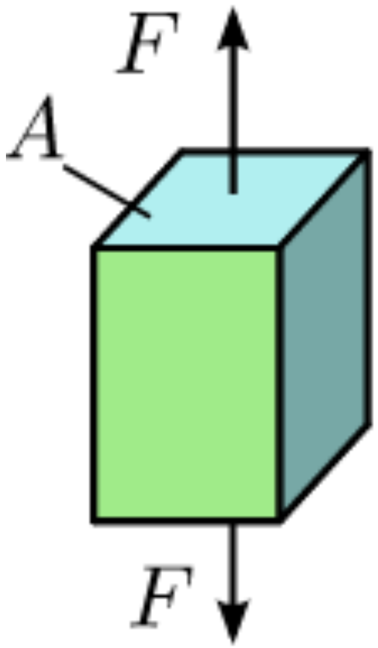


https://upload.wikimedia.org/wikipedia/commons/7/78/Stress_transformation_2D.svg (CC-BY-SA-3.0)

- **Shear Stress**
  - The magnitude of the force per unit cross sectional area, acting tangent to ΔA
  - Symbol is **τ (tau)**

$$\tau = \frac{V}{A}$$

Average Normal Stress



$$\sigma = \frac{F}{A}$$

→ F is resultant of the internal forces distributed over the cross section area A

→ σ is average value of stress

https://upload.wikimedia.org/wikipedia/commons/0/0f/Axial_stress_noavg.svg (CC-BY-SA-3.0)

Internal load F is normal to the plane of the area

Average Normal Stress

Procedure for Analysis

- For members subjected to axial load:
  i. Obtaining internal Loading:
     - Cut the member *perpendicular* to its longitudinal axis at point where normal stress is to be determined
     - Draw the free-body diagram
     - Use equation of force equilibrium (EoE) to obtain internal axial force **F** at the cut section

Average Normal Stress

Procedure for Analysis

ii. Obtaining the cross-section area:
   - Determine the cross-sectional area at the section member's

iii. Obtaining the average normal stress:
   - Compute the average normal stress $\sigma = F/A$

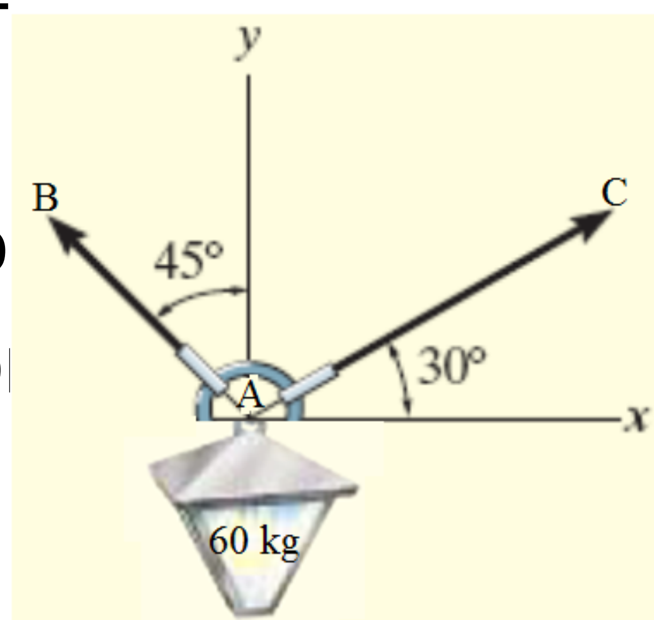
Example III

A lamp holder is made of two rod AB and AC. Rod AB has a

diameter of 8mm and rod AC has a diameter of 10mm. If

the lamp has ... g, determine average no...

stress acting o...



Example III (SOLUTION)

Internal loading

Determine the axial force in each rod using EoE:

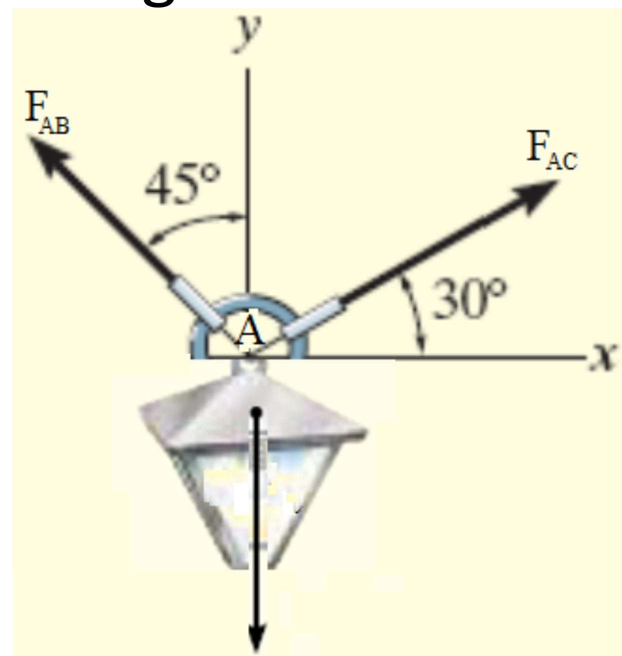
$$\Sigma F_x = 0:$$

$$F_{AC}\cos 30^\circ - F_{AB}\sin 45^\circ = 0$$

$$\Sigma F_y = 0:$$

$$F_{AC}\sin 30^\circ + F_{AB}\cos 45^\circ - 588.6N = 0$$

$$F_{AC} = 430.89N, F_{AB} = 527.798N$$



W = mg = 60(9.81) = 588.6 N

Example III (SOLUTION)

Average normal stress

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{527.8N}{\pi(0.004m)^2} = 10.5MPa$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{430.89N}{\pi(0.005m)^2} = 5.49MPa$$

Done!!

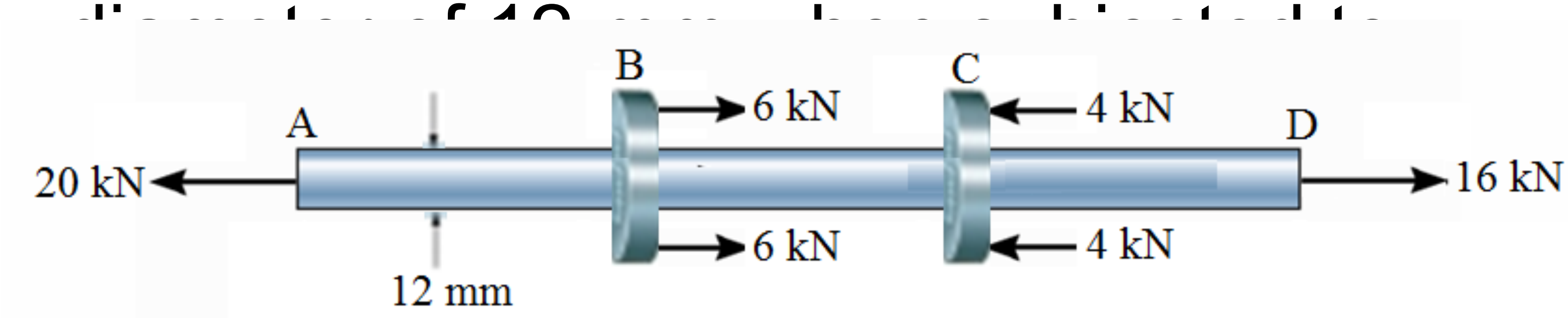
Maximum Normal Stress

Maximum normal stress

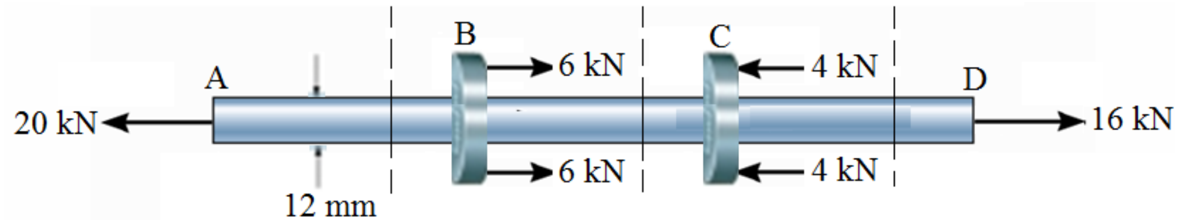
- In the previous example, both the axial force F, and cross-sectional area A, were constant.
- For such problems, normal stress $\sigma = F/A$ is also *constant*
- However, there are few occasion where the body is subjected to several axial force, or change in the area changes.
- Then, it is necessary to determine the *location* where ratio *F/A* is a maximum (*maximum* normal stress)

Example IV

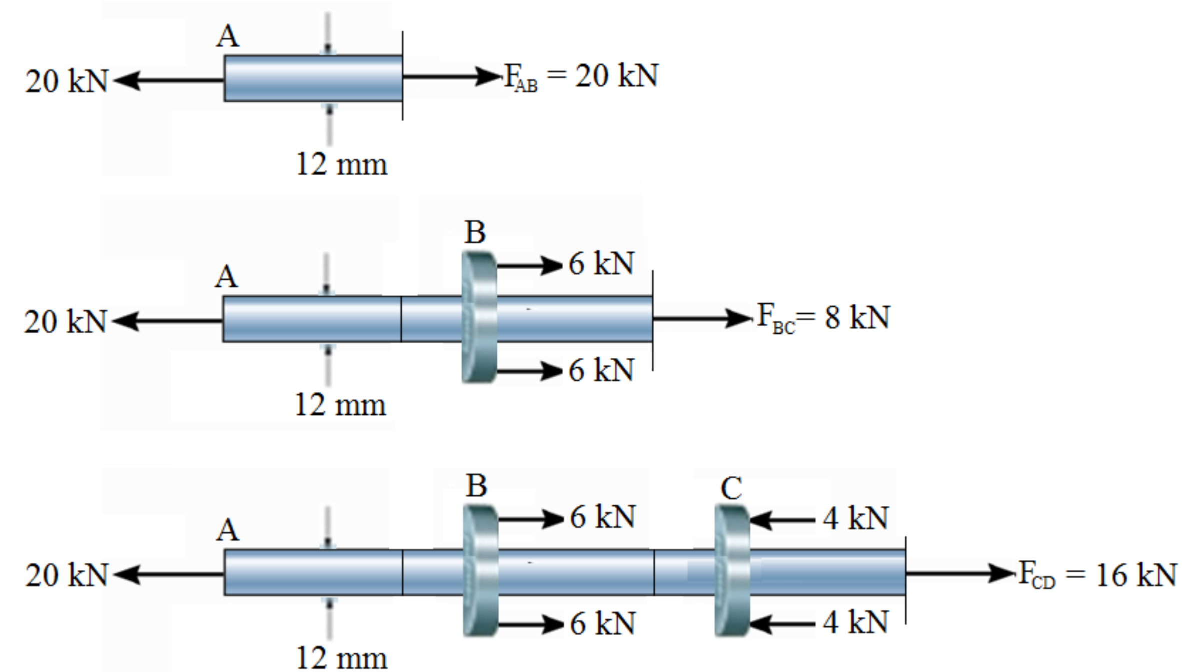
Determine maximum normal stress in rod having a



Example IV (SOLUTION)



Internal loading (method of section)



Example IV (SOLUTION)

Maximum normal stress

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{20(10^3)\ \text{N}}{\pi(0.006)^2} = \mathbf{176.84\ MPa}$$

Done!!

Self-review Questions

1. Which of the following are true for condition of equilibrium for a rigid body

(i) $\sum F_x = 0$

(ii) $\sum F_x \neq 0$

(iii) $\sum F_y = 0$

(iv) $\sum F_y \neq 0$

(v) $\sum M \neq 0$

(vi) $\sum M = 0$

(a) i, iii, vi

(b) ii, iv, v

(c) i, ii, iii, iv, v, vi

(d) all of the above

(e) none of the above

Self-review Questions

2. What is/are the support reaction develop at the support in figure below

(a) force in y direction only

(b) forces in x and y direction only

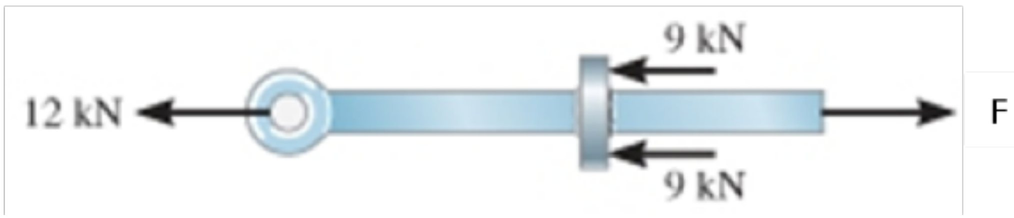
(c) forces in x and y direction and moments about the fixed point

(d) all of the above

(e) none of the above

Self-review Questions

3. What is the magnitude of force F, in figure below



(a) 18 kN

(b) 12 kN

(c) 6 kN

(d) -6 kN

(e) 30 kN

Self-review Questions

4. In solid mechanics, stress can be defined as

(a) change in length divided by original length

(b) when someone is under mental, spiritual and financial emotion

(c) force divided by the cross sectional area

(d) change in angle divided by area

(e) change in length divided by area

Self-review Questions

5. Determine the normal stress on a rigid body with cross sectional area of 10 m^2 subjected to a normal force of 5 kN

(a) 0.5 Pa

(b) 50 Pa

(c) 2 Pa

(d) 500 Pa

(e) 0.005 Pa

Self-review Answers

1. a

2. b

3. e

4. c

5. d

Summary

- Introduction
  - Mechanics of material
  - Condition for equilibrium
  - Free body diagram
- Internal Resultant Loading
- Stress
  - Normal Stress
  - Shear Stress
- Average Normal Stress in an Axially Loaded member
- Maximum Normal Stress in an Axially Loaded member