Solid Mechanics
BETM 2303
Axial Load

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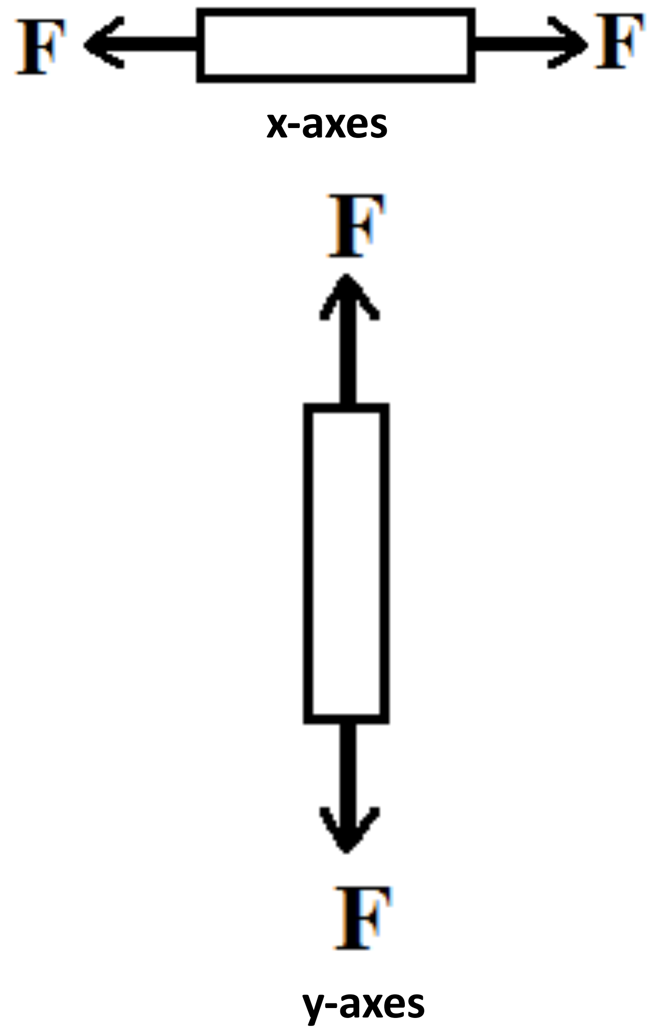
Lesson Outcome

- ✓ To establish the deformation of axially loaded members.
- ✓ To solve the problem on support reactions when it cannot be solved by the equilibrium's equations only.

Definition

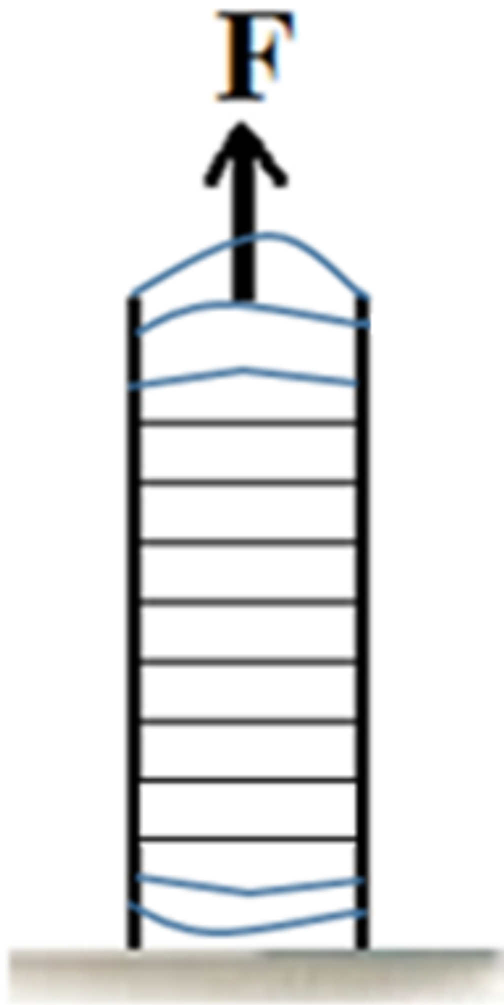
Axial load: is a force/load acting normal to the lines of the axes (x, y or z axes).

- ✓ Deformation occurs when a body is subjected to axial load.
- ✓ Closely related to stress and strain in a body
- ✓ Connection between stress and strain depends upon material used for the body
- ✓ If material behavior is linear elastic, then Hooke's law is obeyed



Saint Venant's Principle

- ✓ Saint Venant's principle states that both the localized stress and deformation which occur within the region of load application or region of support reaction tends to disappear at a distance removed from these regions.
- ✓ If the location of cross-sectional area is away from given load and support, stress and deformation distributions will tend to disappear



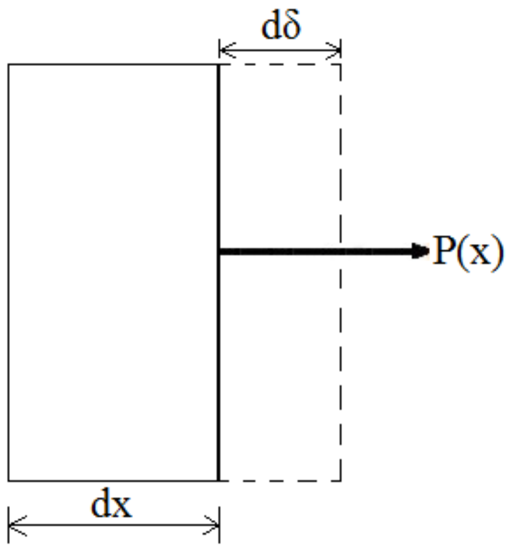
Axially Loaded Body - Elastic Deformation

- Recall from previous class;

$$\sigma = \frac{P}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta L}{L}$$

- If stress and strain did not go beyond elastic limit, hooke's law can be used to determine the deformation of the member.

Hooke's Law, i.e. $\sigma = E\,\varepsilon$



$$\frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)dx}{A(x)E}$$

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

Axially Loaded Body - Elastic Deformation

- When **elastic modulus, load** and **area of cross section** are constant,

$$\delta = \frac{PL}{AE}$$

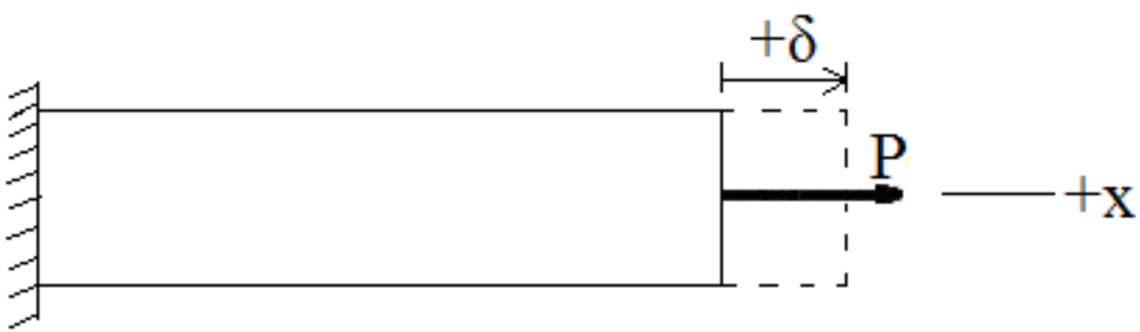
- If
  i. different axial forces is applied to the bar
  ii. cross section area changes from one region to another
  iii. modulus of elasticity changes from one region to another,

  The above equation can be applied on each segment and the solution can be obtained from addition of every elongation.

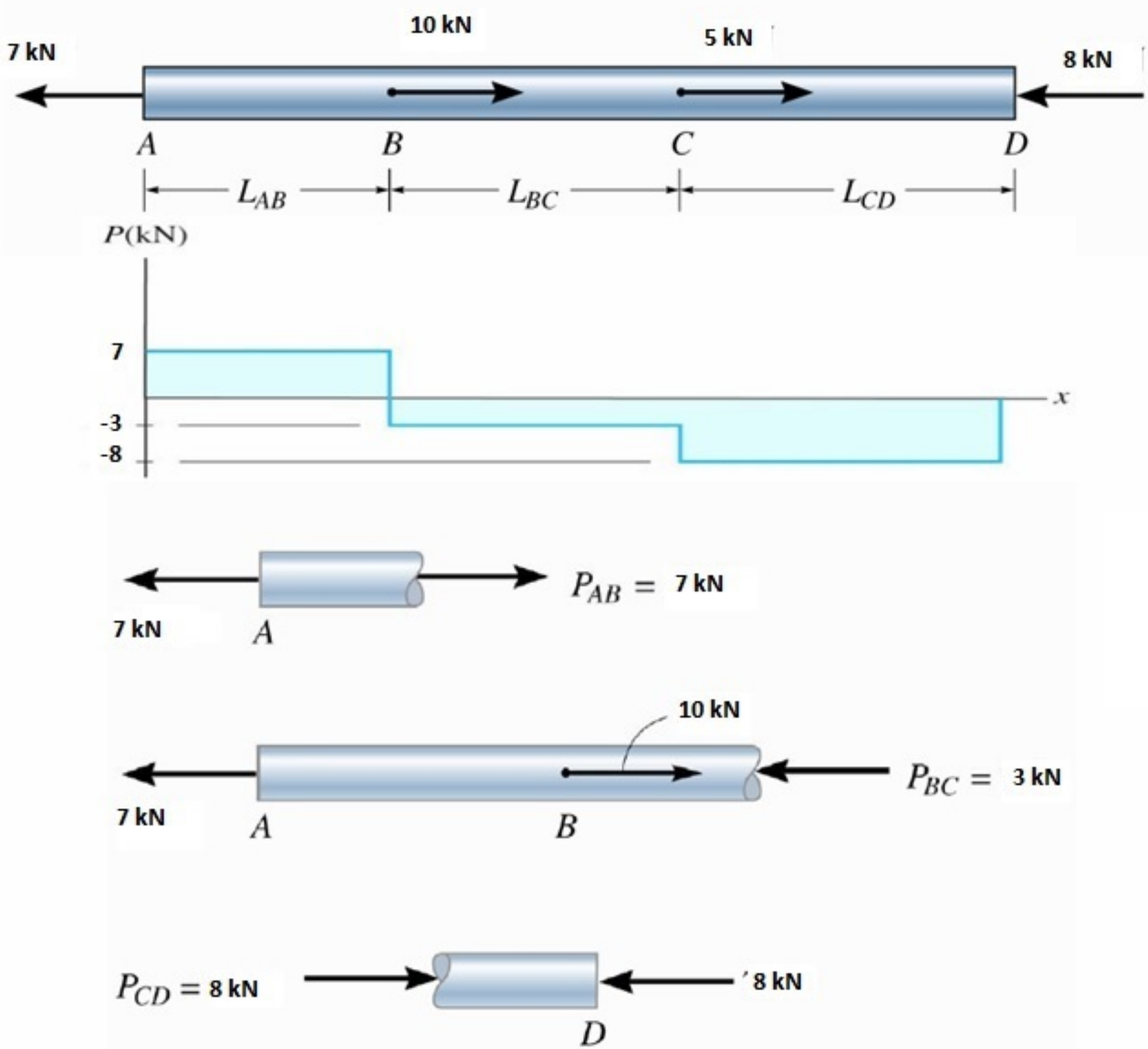
$$\delta = \sum \frac{PL}{AE}$$

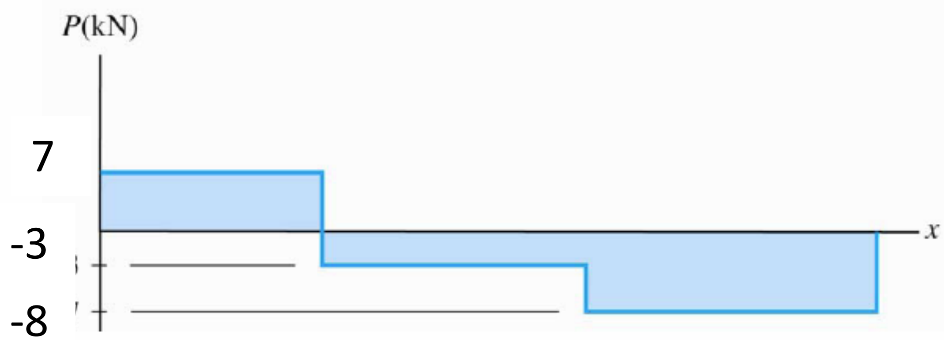
Axially Loaded Body - Elastic Deformation

- Tension and elongation are caused by positive force and displacement.
- Compression and contraction are caused by negative force and displacement.



FREE BODY DIAGRAM





Displacement from point A to point D

$$\delta_{AD} = \Sigma(PL)/(AE) = \frac{7\text{ kN } L_{AB}}{AE} + \frac{-3\text{ kN } L_{BC}}{AE} + \frac{-8\text{ kN } L_{CD}}{AE}$$

Positive answer implies that both ends A and D move away form each other

Negative answer implies that both ends A and D move closer to each other

Key Points!

- Stress and deformation within the region of applied load or supports tend to move at a distance removed from this region
- By considering applied internal load with stress and displacement with strain, displacement of one end of axially loaded member can be determined
- Obeying Hooke's Law, there will be no yielding causes by load because it behaves in linear elastic manner

Force & Displacement Analysis

- Axial force of a member must be determined separately if they varies
- Section must be created at random location *x* from one end of the member
- *P(x)* will represent the force act at each random points

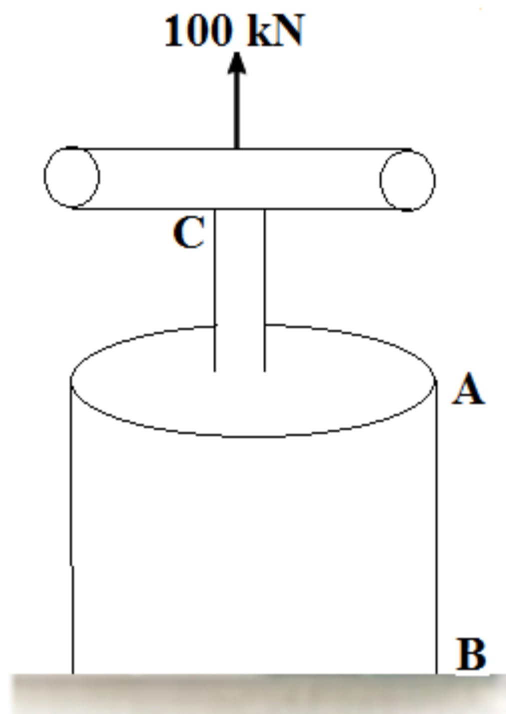
Force & Displacement Analysis

- A different cross-section area of a member must be represented by *A(x)*

- If cross sectional area or internal loading experienced changes, $\delta = \Sigma(PL)/(AE)$ are applied to each segment

Example 1

The air pump shown below consists of a tube *AB* with a cross-section area of 500 mm^2. It was made by stainless steel. At the rigid collar, a titanium rod having a diameter of 30 mm is attached and passes through the tube. The distance between B and C is 700mm, while the distance between A and B is 500 mm. A tensile load of 100 kN was given to the rod. Calculate the displacement of point *C* of the rod. Take E_{Ti} = 119 GPa, E_{st} = 200 GPa.



Example 1 (Solution)

- Find the displacement of end *B* with respect to end *C*.

$$\delta_{C/B} = \frac{PL}{AE} = \frac{\left[+100\left(10^3\right)\right](0.7)}{\pi(0.015)\left[119\left(10^9\right)\right]} = 0.000832183\text{m}$$

- Displacement of end *A* with respect to the end *B*,

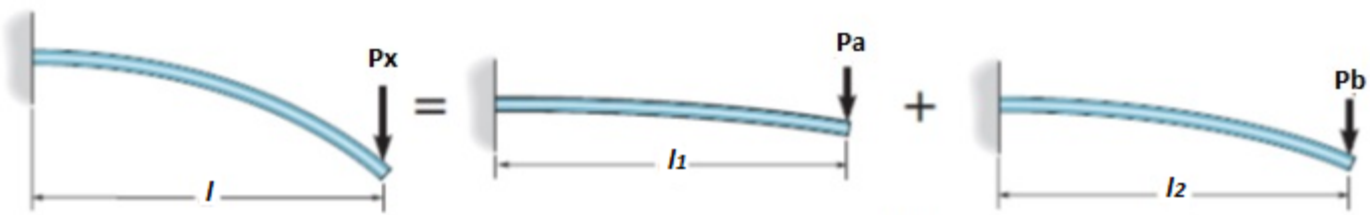
$$\delta_B = \frac{PL}{AE} = \frac{\left[100\left(10^3\right)\right](0.5)}{\left[500\left(10^{-6}\right)\right]\left[200\left(10^9\right)\right]} = 0.0005 = 0.0005\ \text{m}$$

- Both displacements are in positive direction, thus

$$\delta_C = \delta_B + \delta_{C/B} = 0.001332\ \text{m}$$

Principle of Superposition

- Apply on problems with complicated loadings.
- Loads are separated into components and the results of each component must be added.
- Applicable for **small deformation** and **elastic material**
- If $P_x = P_a + P_b$ and $l \neq l_1 \neq l_2$, then the deflection at x is sum of two cases, $\delta_x = \delta_{x1} + \delta_{x2}$



(a) (b)

Statically Indeterminate Axially Loaded Member

- A structural member is said to be **statically indeterminate,** if the force equilibrium condition alone cannot be used to solve the problem.

- To overcome the problem of statically indeterminate member, another condition at the constrain is needed. This condition is known as **compatibility conditions**.

Statically Indeterminate Axially Loaded Member

Steps for analysis:

Equilibrium

i. Identify all forces involved in a member by sketching free-body diagram

ii. If unknown is more than equation of equilibrium, the problem is considered as statically indeterminate

iii. List down all equation of equilibrium of the member

Statically Indeterminate Axially Loaded Member

Compatibility

i. Investigate the nature of movement of a member by drawing a displacement diagram

i. Find the compatibility conditions in terms of displacement

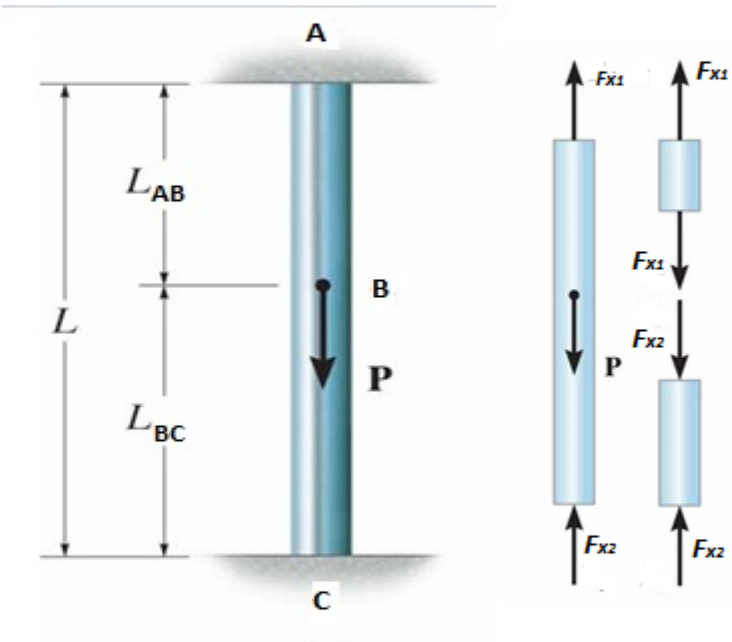
Statically Indeterminate Axially Loaded Member

Load-Displacement

i. Relate unknown displacement by using load-displacement equation

i. Substitutes equilibrium equation into load-displacement equation of vice-versa. Negative or positive value indicates the force direction

Statically Indeterminate Axially Loaded Member



$$+\uparrow \sum F = 0; \Rightarrow F_{X1} + F_{X2} - P = 0$$

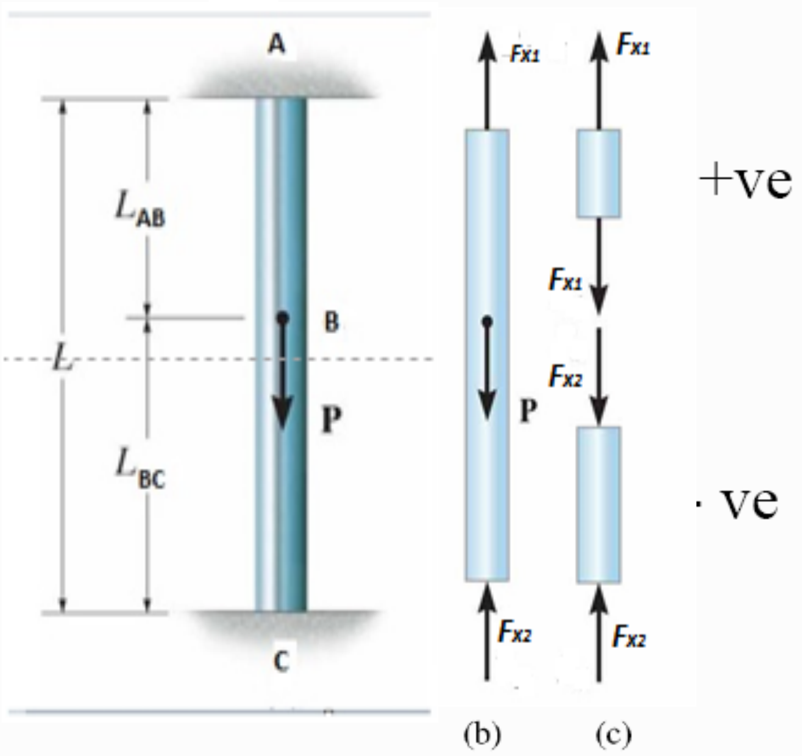
Since both end is fixed, thus:

$$\delta_{A/B} = 0$$

$$\frac{F_{x1}\left(L_{AC}\right)}{AE} - \frac{F_{x2}\left(L_{CB}\right)}{AE} = 0$$

The member is considered **statically indeterminate** because equilibrium condition is unable to determine the reaction forces

The problem is solved using compatibility equation



Given that both ends is fixed,

The relative displacement between both ends are zero

$$\delta_{A/B} = 0$$

$$\frac{F_{X1}\ L_{AC}}{A\ E} + - \frac{F_{X2}\ L_{CB}}{A\ E} = 0$$

$$F_{X1} = F_{X2}\, L_{CB} / L_{AC}$$

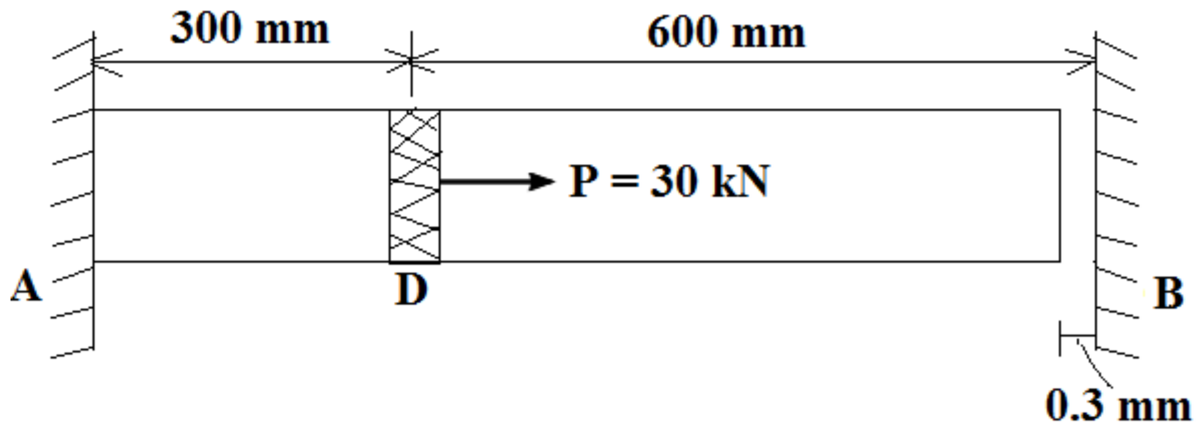
Key Points!

- In superposition, two conditions must be satisfied. They are: (i) the material must be elastic i.e., obey hooke's law, and (ii) deformation must be small

- For statically indeterminate, reaction member cannot be determined by only equation of equilibrium

Example 2

The steel rod below has a diameter of 12 mm. Point A is fixed end. Before load is aaplied, the gap of 0.3 mm. Find the reaction at point A and B if axial force of 30 kN is applied to the rod as shown. Given E_{st} = 200 GPa



Example 2 (Solution)

a) Equilibrium Condition :

It is assumed that force P is strong enough to cause end B to contact point C. However, the rod is considered as statically indeterminate because there is only one equation of equilibrium

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $\quad - F_A - F_B + 30(1000)\ N = 0 \quad (1)$

b) Compatibility :

Load P caused point B to reach point C and did not move further. Therefore,

$$\delta_{A/B} = 0.0003\ m$$

Example 2 (Solution)

c) Load-displacement relations:

$$\delta_{A/B} = 0.0003m = \frac{F_A L_{AD}}{AE} - \frac{F_B L_{DB}}{AE}$$

$$0.0003m = \frac{F_A(0.3m)}{\pi(0.006m)^2[200(10^9)N/m^2} - \frac{F_B(0.6m)}{\pi(0.006m)^2[200(10)^9 N/m^2}$$

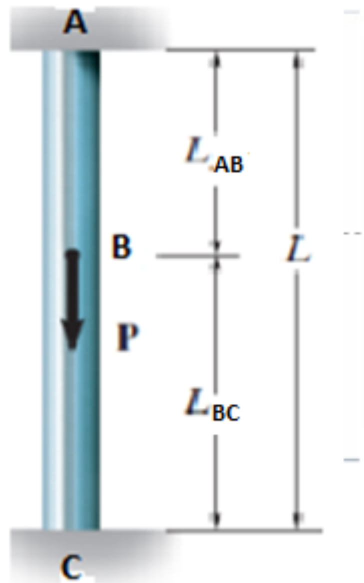
$$F_A(0.3m) - F_B(0.6m) = 6785.84Nm \qquad (2)$$

Substitutes Equation 1 into equation 2 or vice-versa,

F_A = 27.54 kN $\qquad$ F_B = 2.46 kN

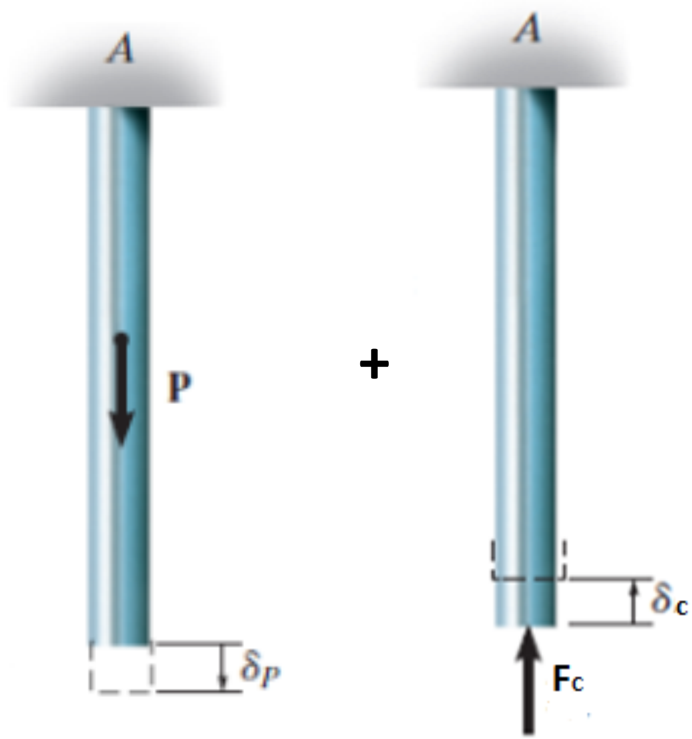
Force Method of Analysis

- Compatibility equation based on principle of superposition able to solve statically indeterminate problems



- If support at point C is removed temporarily, the bar is considered statically indeterminate

Force Method of Analysis



C is displaced downward by δ_P

End C is displaced upward by Fc with δ_B

Thus, δ_P - δ_B = 0

Compatibility & Equilibrium Analysis

Compatability

i. One of the support must be choose as redundant define the compatibility equation

ii. Using load-displacement relationship, find the equation of external load and redundant displacement in terms of loading

iii. Use compatibility equation to figure out the magnitude of redundant force

Compatibility & Equilibrium Analysis

Equilibrium

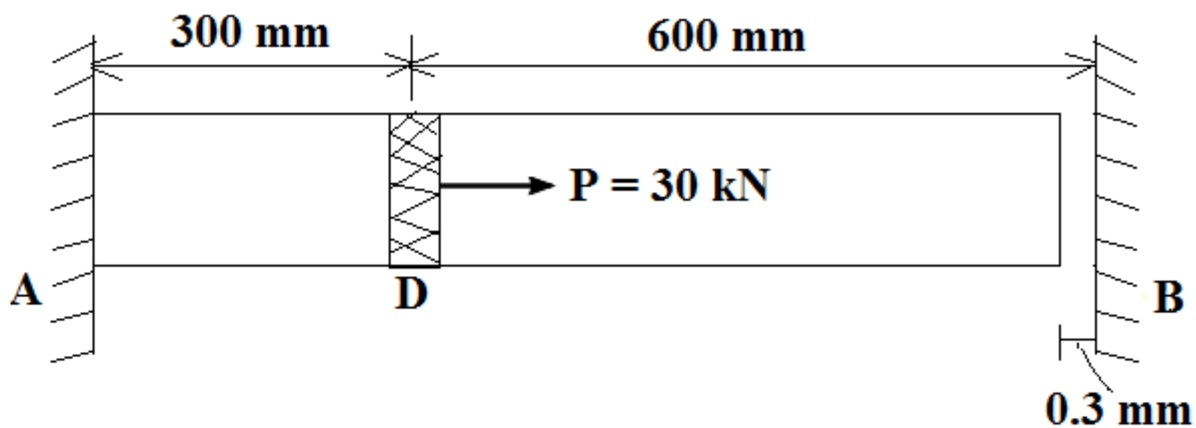
i. Establish Free Body Diagram

ii. Express equation of equilibrium for the member using result obtained from redundant

iii. Solve equations of all reactions

Example 3

The steel rod below has a diameter of 12 mm. Point A is fixed end. Before load is aaplied, the gap of 0.3 mm. Find the reaction at point A and C if axial force of 30 kN is applied to the rod as shown. Given E_{st} = 200 GPa



Example 3 (Solution)

Compatibility equation:

$$\delta_P - \delta_B = 0.0003 \qquad (1)$$

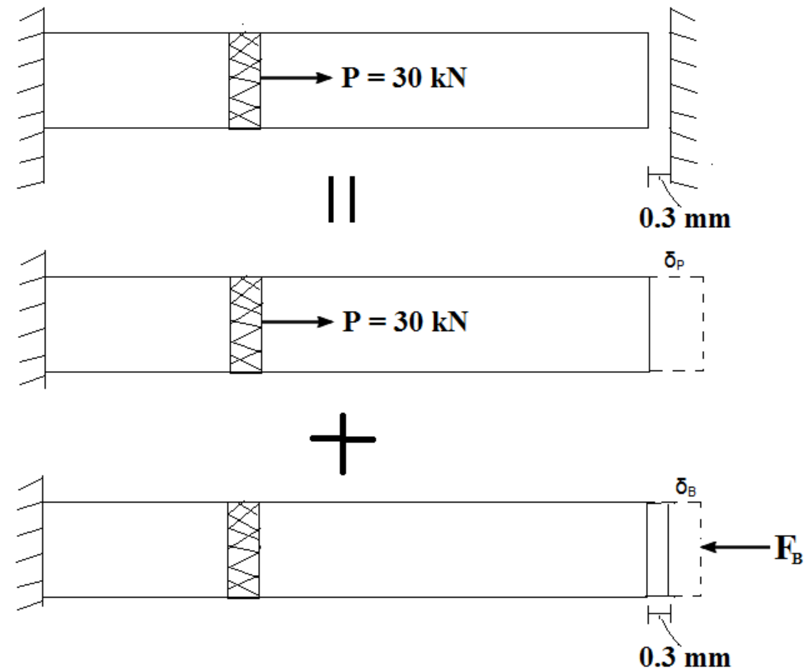
$$\delta_P = \frac{PL_{AD}}{AE} = \frac{30(10^3)\times 0.3}{\pi(0.006)^2\times 200(10^9)} = 0.0003979\ m$$

$$\delta_B = \frac{F_B L_{AB}}{AE} = \frac{F_B\times 0.9}{\pi(0.006)^2\times 200(10^9)} = 3.97887\times 10^{-8} m\ (F_B)$$

Substitute for δ_P and δ_B in equ (1)

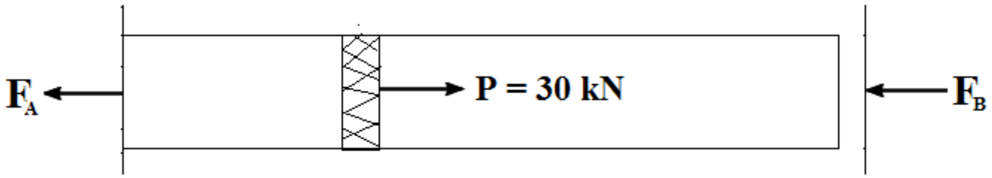
$$0.0003979 - 3.97887\text{x}10^{-8}\ (F_B) = 0.0003$$

$$F_B = 2.46\ \text{kN}$$



Example 3 (Solution)

FBD



Equation of Equilibrum:

$\Sigma F_x = 0$

$-F_A + 30\text{ kN} - 2.46\text{ kN} = 0$

$F_A = 27.52\text{ kN}$

$F_A = 27.54\text{ kN}$ $F_B = 2.46\text{ kN}$

Summary

- Saint Venant's Principle
- Elastic Deformation of an Axially Loaded member
- Principle of Superposition
- Statically Indeterminate Axially Loaded member
- Force Method of Analysis