

SOLID MECHANICS

BETM 2303

SHEAR STRESS AND STRAIN

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LESSON OUTCOME

1. To introduce and apply the concepts of shear stress.
2. To introduce and apply the concepts of normal and shear strain.

Average Shear Stress

The average shear stress is given by:

$$\tau_{avg} = \frac{V}{A}$$



Remember!

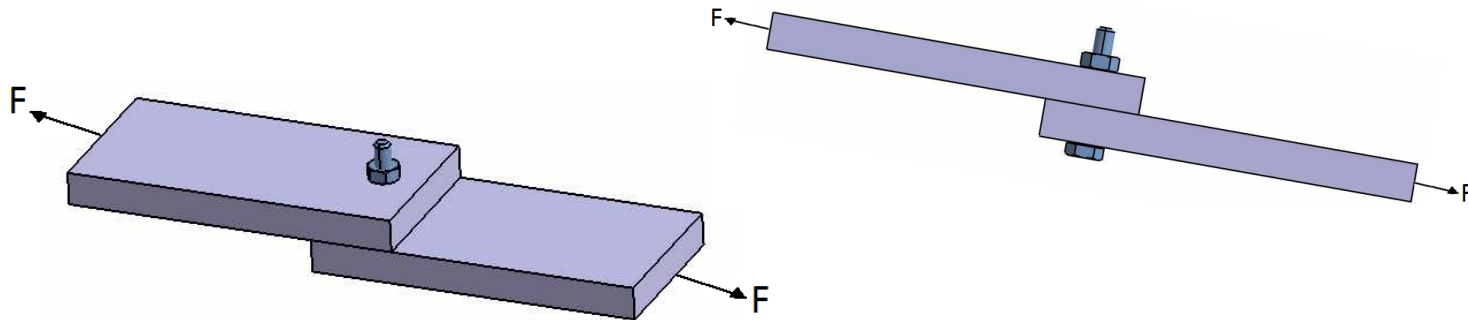
τ_{avg} is average value of shear stress

V is value of resultant internal
shear force using FBD and
equations of equilibrium

A is value of cross-sectional area

Single shear

- The cross-sectional area of bolt and bonding surface between the two members are subjected to one connection (single shear force).
i.e, for equilibrium, $V = F$
- The joint shown below illustrate typical examples of a single shear.



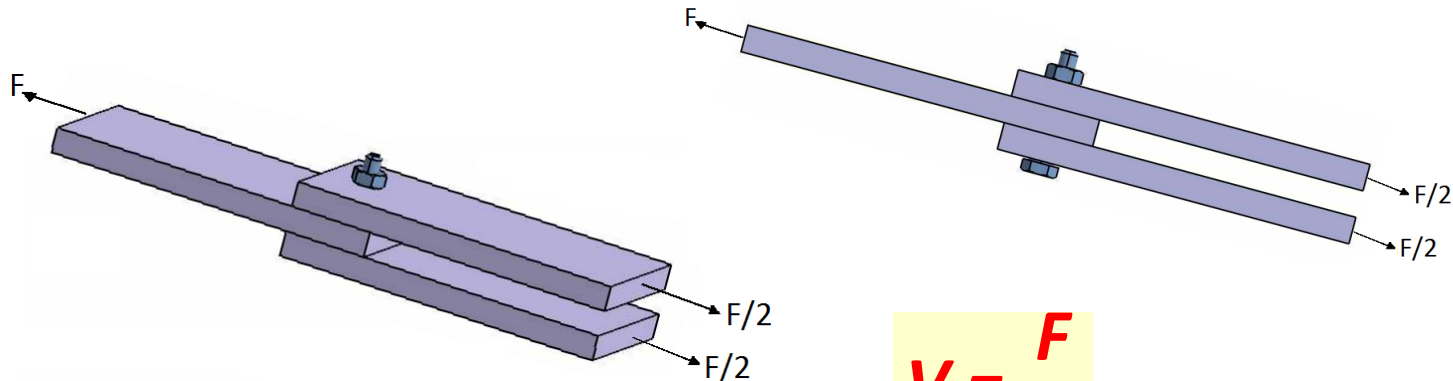
$$V = F$$

Double shear

- The cross-sectional area of bolt and bonding surface between two members subjected to two connections (double shear force)

i.e., for equilibrium, $V = F/2$

The joints shown below are examples of double-shear connections.



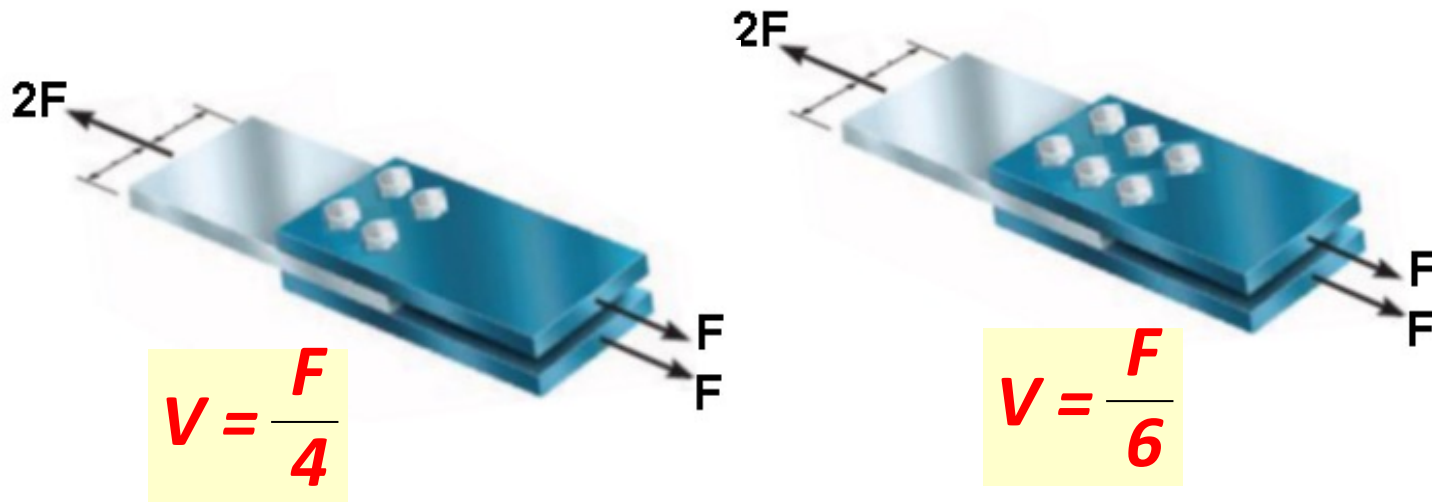
$$V = \frac{F}{2}$$

Multiple shear

- The cross-sectional area of bolt and bonding surface between two members subjected to multiple shear with n , connections

i.e., for equilibrium, $V = F/n$

The joints shown below are examples of multiple-shear connections.



Procedure for analysis

Resultant Internal shear force

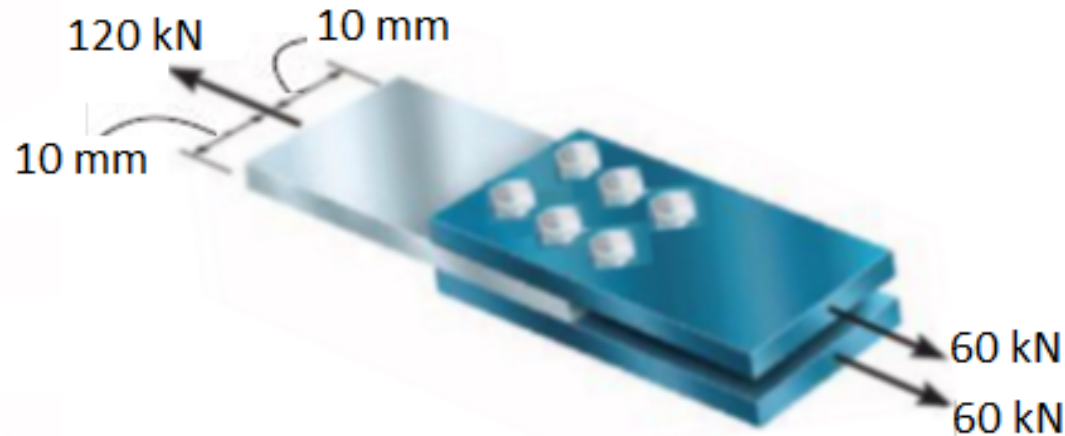
1. Cut the body where the average shear stress, τ_{avg} is to be calculated
2. Draw free-body diagram
3. Use equation of equilibrium (EoE) to compute the internal shear force V

Average shear stress

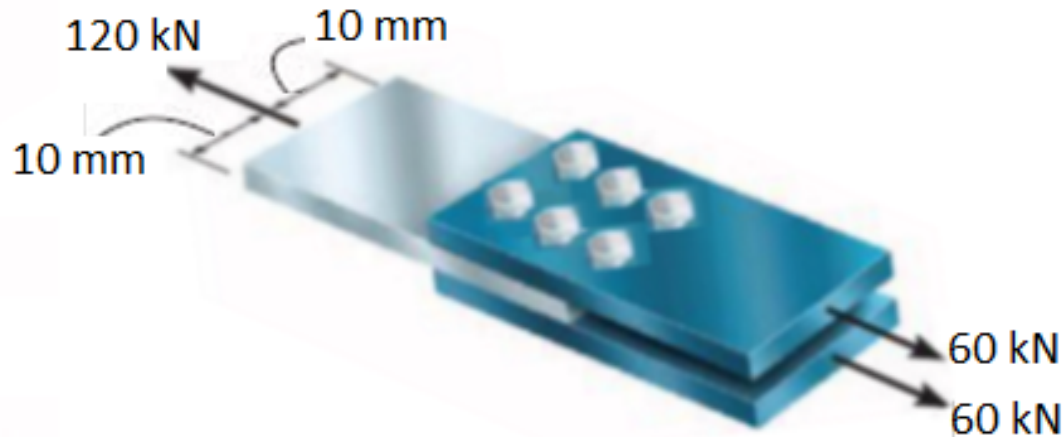
1. Calculate the cross-sectional area A
2. Compute average shear stress using $\tau_{avg} = V/A$

Example 1

A lap joint is fastened together using six bolts as depicted in the figure below. Each of the plate thickness, $t = 10$ mm. If the diameter of the bolts is 20 mm. Determine the average shear stress on the bolts.



Example 1 (SOLUTION)

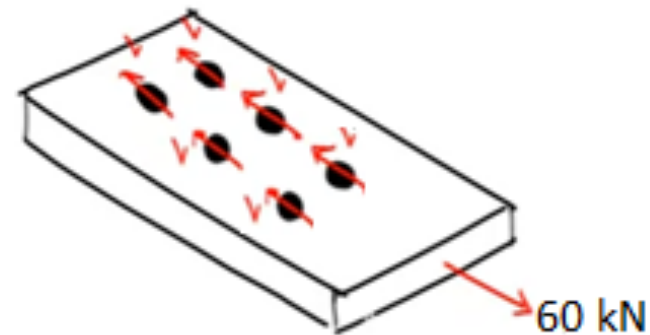


Using Equation of Equilibrium

$$V = \frac{F}{6} = \frac{60}{6} = 10 \text{ kN}$$



FBD



Example 1 (SOLUTION)

To find the average shear stress

$$\begin{aligned}
 \tau_{\text{avg}} &= \frac{V}{A} = \frac{10 \times 10^3 \text{ N}}{\frac{\pi}{4} (0.02 \text{ m})^2} \\
 &= 31.83 \text{ MPa}
 \end{aligned}$$

Allowable Stress

- As a designer/engineer/technology, your primary design consideration when designing a structural elements or mechanical components, is to design a member will not fail during useful operation
- In order to achieve this, the stress in the member must be set at a level where the member will be safe
- Select an allowable load that is less than the failure load of the member
- The common approach used is known as factor of safety (F.S.)

$$\text{F.S.} = \frac{F_{\text{fail}}}{F_{\text{allow}}}$$

Allowable Stress

- F.S. can also be expressed in terms of stress acting on the member as given below:

$$\text{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

Remember!

$$\text{F.S.} = \frac{\tau_{\text{fail}}}{\tau_{\text{allow}}}$$

Note: To avoid potential failure of the member, F.S. must be chosen to be greater than 1. However, the actual value will depend on the types of material used and the potential application of the member.

Design of Simple Connections

- To design a simple connection, one of the key parameter is the cross-sectional area over which critical stress is acting.
- To obtain the area of member subjected to a *normal force*,

$$A = \frac{P}{\sigma_{\text{allow}}}$$

- To obtain the area of member subjected to a *shear force*,

$$A = \frac{V}{\tau_{\text{allow}}}$$

Step for analysis

First consider the section where the critical stress is acting. When using average normal stress and shear stress equations

Internal Loading

1. Cut the member at the location where the critical stress is acting
2. Draw free-body diagram (FBD)
3. Use equations of equilibrium (EoE) to obtain internal resultant force

Design of Simple Connections

Step for Analysis

Required Area

- From the given allowable stress, compute the required area to withstand the load using

$$A = P/\sigma_{\text{allow}} \text{ (Normal stress)}$$

or

$$A = V/\tau_{\text{allow}} \text{ (Shear stress)}$$

Deformation

- Deformation is the change that occur in shape and size of a body when a force is applied to the body.
- Deformation may either be highly visible or practically invisible.
- Deformation of a body in general, occur in both the axial direction and lateral direction.

Strain

Normal Strain: is defined as ratio of change in length to the original length (i.e., change in length divided by original length).

Symbol: **ϵ (epsilon)**

Unit: no unit but sometimes report in mm/mm

The change in length, ΔL , If “pulls” on ΔA -> elongation

The change in length, ΔL , If “pushes” on ΔA -> contraction

$$\epsilon = \frac{\Delta L}{L_o} = \frac{L_f - L_o}{L_o}$$



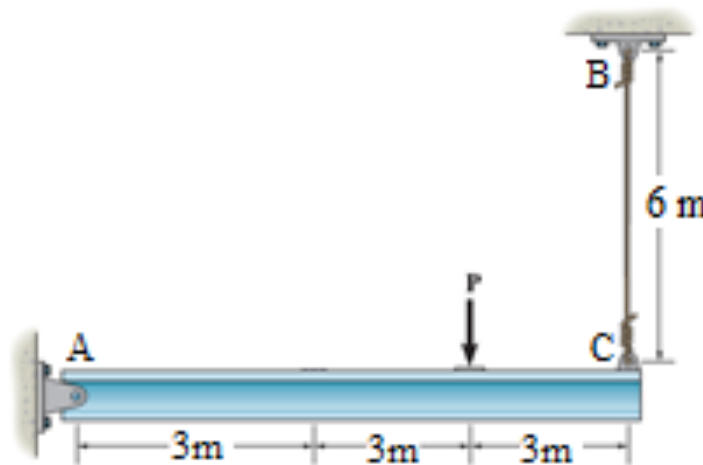
ΔL = change in length after a load is applied

L_o = original length before the load is applied

L_f = final length after the load is applied

Example 2

A rigid body has a pin support at point A and wire BC. Wire BC has a pin support at point B and free to move at point C. If the load P was applied on the body and causes the end C to move downward by 20 mm. Determine the normal strain in wire BC.



Example 2 (SOLUTION)

Normal Strain on wire BC, ϵ_{BC}

$$\epsilon_{BC} = \frac{\Delta L}{L_o}$$

$$\Delta L = 20 \text{ mm} = 0.02 \text{ m}$$

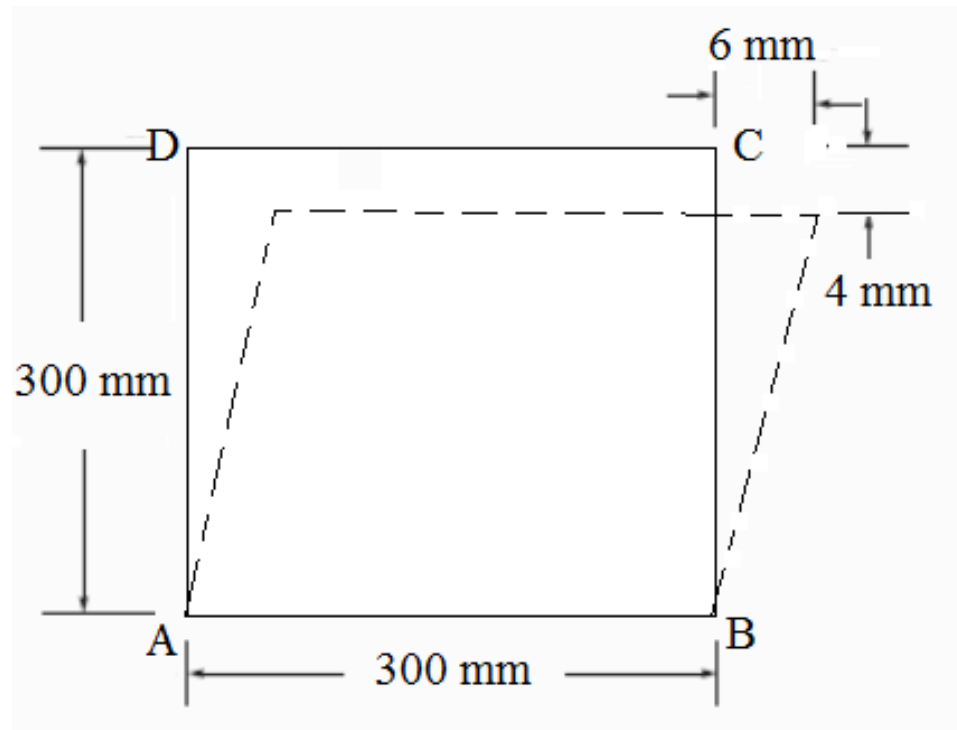
$$L_o = 6 \text{ m}$$

Then

$$\begin{aligned}\epsilon_{BC} &= \frac{0.02m}{6m} \\ &= 0.003333\end{aligned}$$

Example 3

From the figure below, as a result of loading, the plate is deformed into the dashed shape. Calculate the average normal strain along the side AD .



Example 3 (SOLUTION)

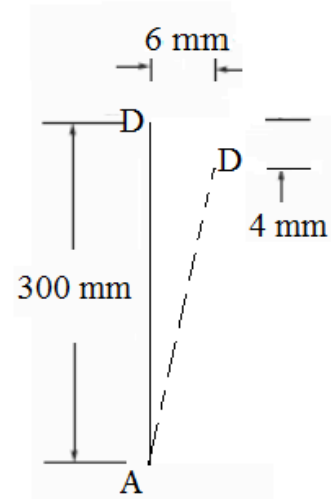
Normal Strain

- Line AD , coincident with the y axis, becomes line AD' after deformation, thus the length of this line is

$$AD' = \sqrt{(300 - 4)^2 + 6^2} = 296.06 \text{ mm}$$

- The average normal strain for AB is therefore

$$(\epsilon_{AD})_{avg} = \frac{AD' - AD}{AD} = \frac{296.06 - 300}{300} = -1.313(10^{-2})$$



- The negative sign indicates the strain causes a contraction of AB .

Shear Strain: is defined as the measure of change in angle between two line segments.

Symbol: γ (gamma)

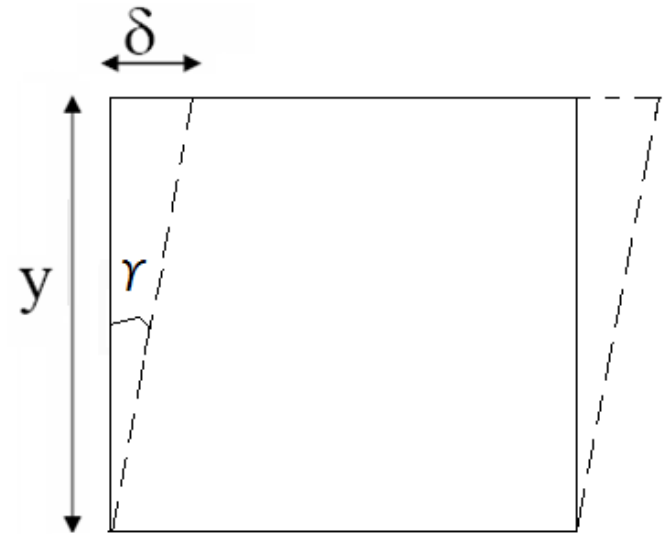
Unit: radians

$$\gamma = \theta_o - \theta_f$$

Remember!

θ_o = original angle

θ_f = final angle



Dashed line = deformed shape

2.2 Strain

Since $\theta_o = 90^\circ$, then the equation can be re-written as

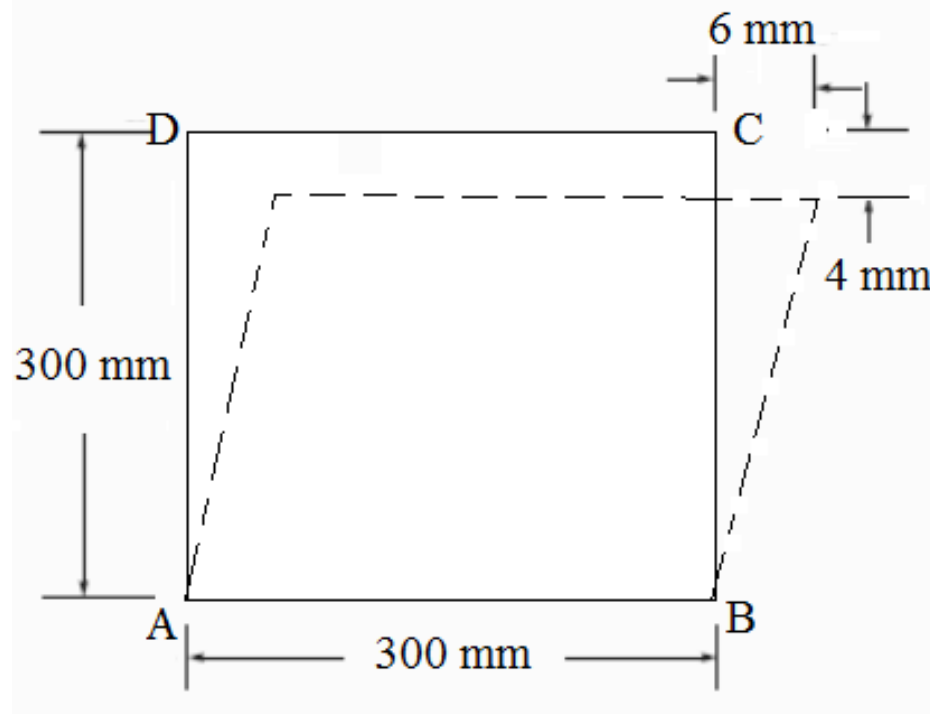
$$\gamma = \frac{\pi}{2} - \theta_f (\text{radians})$$



- If θ_f smaller than $\pi/2 \rightarrow \gamma$ is positive.
- If θ_f bigger than $\pi/2 \rightarrow \gamma$ is negative.

Example 4

From the figure below, as a result of loading, the plate is deformed into the dashed shape. Calculate the average shear strain in the plate at point A.



Example 4 (SOLUTION)

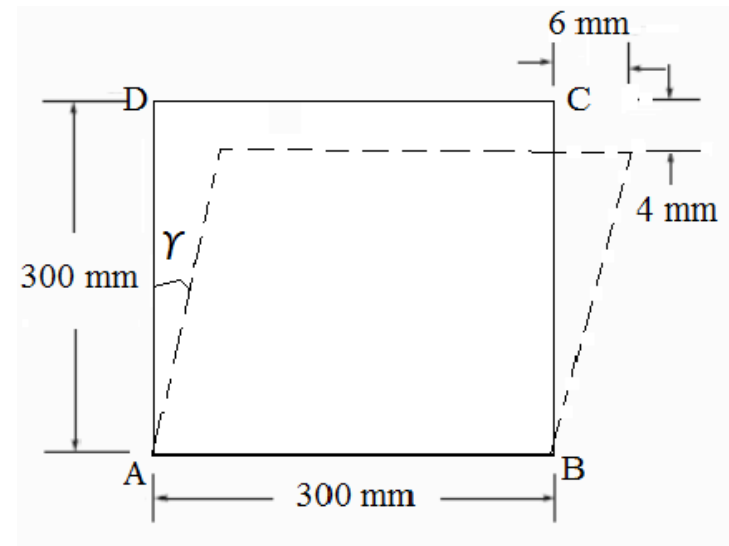
Shear Strain

- As noted, the once 90° angle BAC between the sides of the plate, referenced from the x, y axes, changes to θ' due to the displacement of B to B' .
- Since then γ is the angle shown in the figure.

$$\gamma = \frac{\pi}{2} - \theta_f (\text{radians})$$

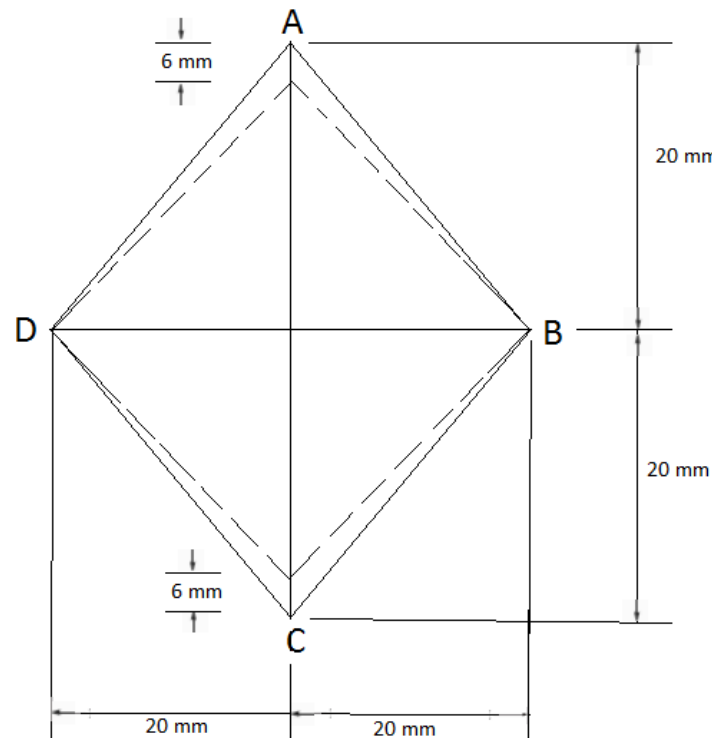
- Thus,

$$\gamma = \tan^{-1} \left(\frac{6}{300 - 4} \right) = 0.0203 \text{ rad (Ans)}$$



Try this?

The figure below shows a square plate, as a result of loading, the corners A and C are deformed into the dashed shape. Calculate the average shear strain in the plate at point A and B.



Self-review Questions

1. When designing a structural member, the stress in it must be restricted to a safe level, one method used is the factor of safety (F.S).

What is factor of safety (F.S)

- (a) stress failed plus stress allowed
- (b) stress allowed divided by stress failed
- (c) stress failed multiply by stress allowed
- (d) stress allowed multiply by stress failed
- (e) stress failed divided by stress allowed

Self-review Questions

2. When a force is applied to a body, deformation occurs as a results of which of the following:

(i) change in shape

(ii) change in size

(iii) no changes

(a) i

(b) ii

(c) i, ii

(d) i, ii, iii

(e) iii

Self-review Questions

3. In solid mechanics, normal strain can be defined as
- (a) when someone is under mental, spiritual and financial emotion
 - (b) change in length divided by original length
 - (c) force divided by the cross sectional area
 - (d) change in angle divided by area
 - (e) change in length divided by area

Self-review Questions

4. In solid mechanics, shear strain is defined as
- (a) change in length divided by original length
 - (b) change in angle between two small line segments
 - (c) change in area divided by original area
 - (d) change in length in the material
 - (e) change in length divided by original angle

Self-review Questions

5. A rigid body has an original length of 50 mm, when subjected to external load, it elongate by 0.1 mm. Determine the normal strain in the body

- (a) 0.002
- (b) 500
- (c) 5
- (d) 0.998
- (e) 0.5

Self-review Answers

1. e

2. c

3. b

4. b

5. a

Summary

- Average Shear Stress
 - Single Shear
 - Double Shear
 - Multiple Shear
- Allowable Stress
- Design of Simple Connections
- Deformation
- Strain
 - Normal Strain
 - Shear Strain