Solid Mechanics
BETM 2303
Week 6 – Axial Load (Thermal Stress & Stress Concentration)

Mohamed Saiful Firdaus Hussin, Olawale Ifayefunmi

mohamed.saiful@utem.edu.my, olawale@utem.edu.my

Lesson Outcome

- ✓ To explain the effect of temperature change towards a body dimension.
- ✓ To solve the problem related to stress distribution within a localized region.

Thermal Stress

✓ Generally,

- ↑ temperature → body expansion
- ↓ temperature → body contraction

✓ Homogenous and isotropic material resulted in linear relation between temperature and expansion/contraction

Thermal Stress

Thus,

$$\delta_T = \alpha \Delta T L$$

α linear coefficient of thermal expansion

ΔT temperature change

L iniatial length

δ_T change in length of the member

Thermal Stress

- Thermal displacement of a statically indeterminate body will be influenced by the supports, resulting in thermal stresses

Example :

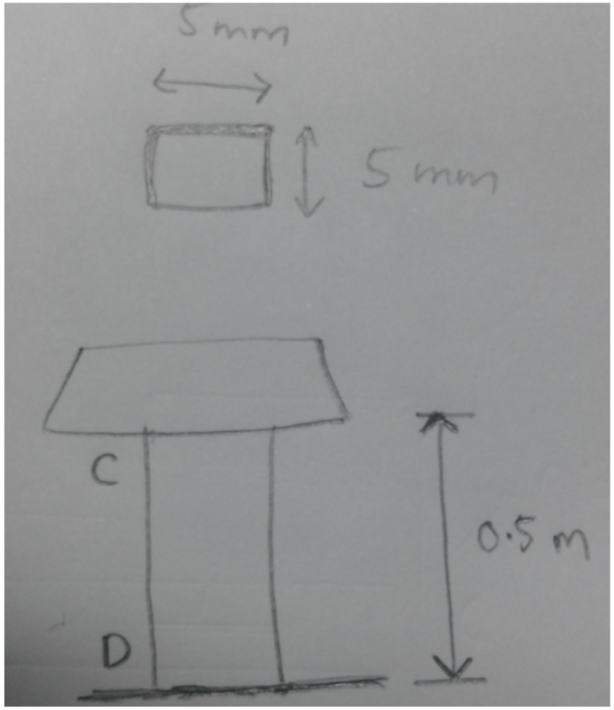


Figure 6.1

Thermal Stress

Stainless steel sheet shown by Figure 6.1 is constraint between two fixed supports, given initial temperature T_0 = 25°C. Determine the normal thermal stress of the bar when the temperature is increased to T_1 = 55°C.

Solution :

Draw free body diagram of the steel bar

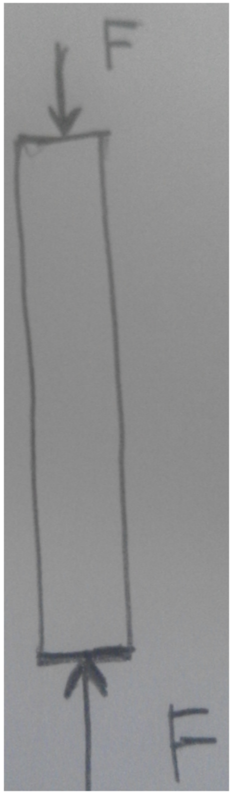


Figure 6.2

Thermal Stress

Force at point C and D are same since there is no external load acting on the body

$$F_C = F_D = F$$

However, force cannot be determined by equilibrium because the body is statically indeterminate.

Thermal Stress

Thermal displacement δ_T at C act against force F. Thus,

$$\delta_{C/D} = 0 = \delta_T - \delta_F$$

Using thermal and load-displacement relationship,

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

Thermal Stress

Thus,

$$F = \alpha \Delta T A E$$

$$F = [12(10^{-6})/°C](55°C - 25°C)(0.005m)^2(200(10^6)kN/m^2]$$

$$F = 1.5kN$$

Thermal Stress

The average normal compressive stress is,

$$\sigma = \frac{F}{A}$$

$$\frac{1.5kN}{(0.005m)^2} = 60000kPa = 72MPa$$

Stress Concentration

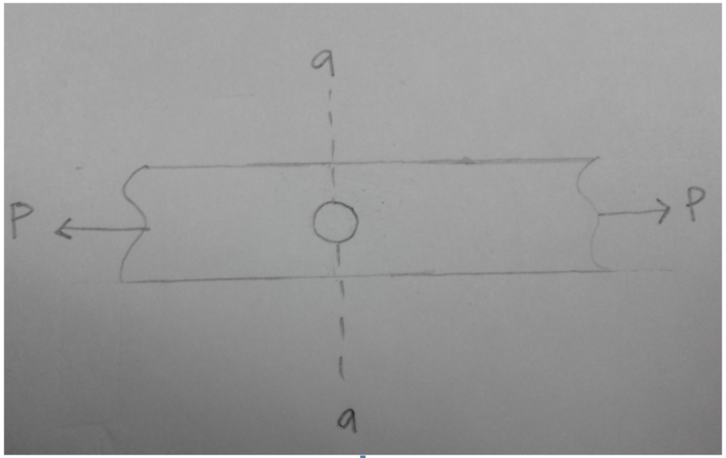
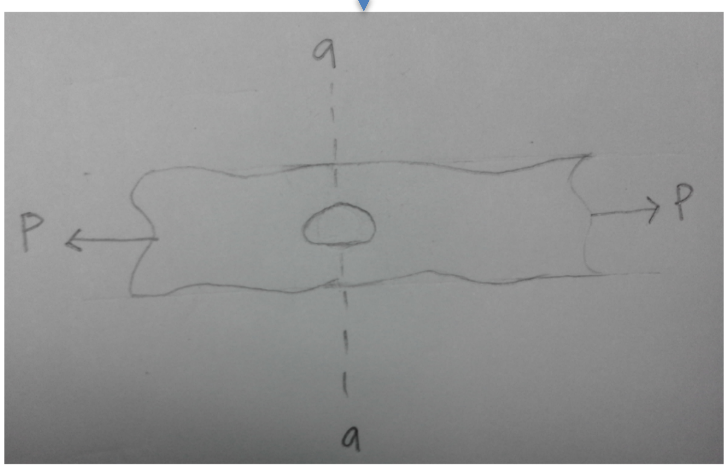


Figure 6.3

Undergoes Distortion



- Axial force P is subjected to a rod
- The rod deflected to irregular pattern after undergoes distortion
- At section a where cross sectional is minimum, the normal stress is maximum

Stress Concentration

- Stress distribution upon this section can be calculated using theory of elasticity or Hooke's Law, given that the material behaves elastically

- Magnitude of resultant force from stress distribution is required in order to be equal to P

$$P = \int_A \sigma \, dA$$

Stress Concentration

- In stress analysis, maximum stress of a section must be known. The body will be designed to resist the stress when outside load is applied

- Stress concentration factor K is the ratio of maximum stress to the average normal stress acting at the cross section

$$K = \frac{\sigma_{\max}}{\sigma_{ave}}$$

Stress Concentration

- Material property does not influence the value ok *K*, but geometry and discontinuity

- As discontinuity decreased, the stress concentration will increase

- A bar needs a change in the cross section when the edge produced the *K* value greater than 3

Stress Concentration

- $K > 3$ indicates that max. normal stress, $\sigma_{\max}$ is three times greater than average normal stress, σ_{ave}
- The value of *K* can be reduced by using fillet at the edge of a structure
- Grooves and holes can also be used to reduce the value of *K*, helping stress and strain to be spread fairly throughout a body

Stress Concentration

- When static load is applied to a ductile material, it is not suitable to use *K* because when stress greater than proportional limit, material will experience crack

- When a body is subjected to fatigue loadings, stress concentration will cause failure to its structure and elements

Key Points!

- Change in the area of a cross section will produce stress concentration. Bigger change yield larger stress concentration

- In design and analysis, *stress concentration factor K* is important in determining maximum stress acting on smallest area of cross section on a body

Example

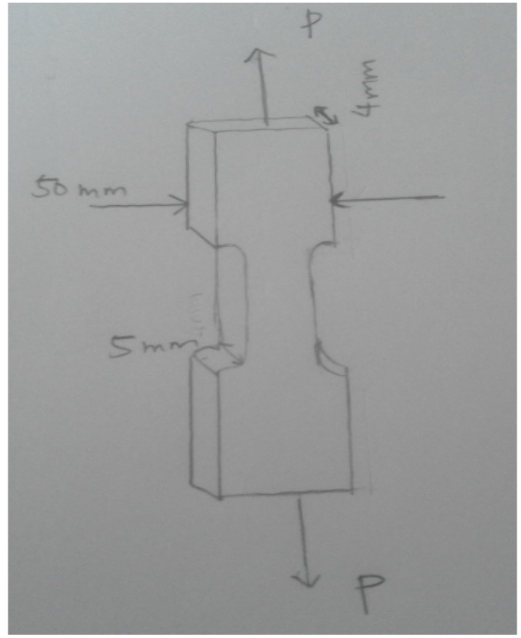


Figure 6.4

A steel bar above has the σ_Y = 250 MPa. Calculate maximum applied load P that can be put without causing yield to the steel and maximum value of P that the bar can support

Example

$$\frac{r}{h} = \frac{5mm}{50mm - 10mm} = 0.125$$

$$\frac{w}{h} = \frac{50mm}{50mm - 10mm} = 1.25$$

Thus, $K = 1.75$

Maximum load before yielding occurs at $\sigma_{max} = \sigma_Y$

Example

- Average normal stress, $\sigma_{avg} = P/A$

$$\sigma_{\max} = K\sigma_{avg}$$

$$\sigma_{\Upsilon} = K\left(\frac{P_{\Upsilon}}{A}\right)$$

$$250(10^6)Pa = 1.75\left[\frac{P_{\Upsilon}}{(0.004m)(0.040m)}\right]$$

Example

$$P_{\Upsilon} = 22.86kN$$

Maximum applied load P can be given without causing yield is **22.86 kN**

Next,

$$\sigma_{\Upsilon} = \frac{P_P}{A}$$

Example

$$250(10^6)Pa = \frac{P_P}{(0.004m)(0.04m)}$$

$$P_p = 40kN$$

Maximum value of P that the steel bar can support is **40 kN**

Self-review Questions

1. An axially loaded member is said to be statically indeterminate, when the problem

(A) cannot be solved by equation of equilibrium only

(B) can be solved by equation of equilibrium only

(C) can be solved by simultaneous equation only

(D) can be solved by compatibility equation only

(E) none of the above

Self-review Questions

2. For elastic axially loaded member, deformation can be expressed as

(A) $\sigma = \frac{P(x)}{A(x)}$

(B) $\varepsilon = \frac{d\delta}{dx}$

(C) $\delta = \frac{PL}{AE}$

(D) $\sigma = E\varepsilon$

(E) $\tau = \frac{Tr}{J}$

Self-review Questions

3. What is Saint Venant's Principle?

(A) Stress is proportional to strain

(B) Deformation at the support is the same

(C) forces in x and y direction and moments about the fixed point

(D) If the location of cross-sectional area is away from given load and support, stress distribution will tend to disappear

(E) none of the above

Self-review Questions

4. A member having a cross-sectional area of $10mm^2$ is subjected to an axial load of 20kN. Determine the deformation on the member, if the original length is 50mm and Young's modulus is 200 GPa.

(A) 2000 m

(B) 2×10^{-11} m

(C) 3.142 m

(D) 0.0005 m

(E) none of the above

Self-review Questions

5. Which equation is capable of solving statically indeterminate member?

(A) Equation of equilibrium

(B) Simultaneous equation

(C) Compatibility Equation

(D) Equation of equilibrium & compatibility equation

(E) Equation of equilibrium & simultaneous equation

Self-review Questions

6. Aluminium steel having initial temperature of 50$\circ$C was heated up to a temperature of 80$\circ$C. Determine the change in length of the member, if the original length is 10mm. The linear coefficient of thermal expansion for aluminium is $12x10^{-6}\circ C^{-1}$

(A) 3.6×10^{-6} m

(B) 6.0×10^{-6} m

(C) 9.6×10^{-6} m

(D) 15.6×10^{-6} m

(E) None of the above

Self-review Answers

1. A

2. C

3. D

4. D

5. D

6. A

Summary

- Temperature change will lead a member to change its length, given that it is homogenous and isotropic, expressed by

$$\delta = \alpha \Delta T L$$

- Stress concentration is caused by holes and sharp transitions at cross section. Maximum stress at cross section can be calculated using

$$\sigma_{\max} = K \sigma_{avg}$$