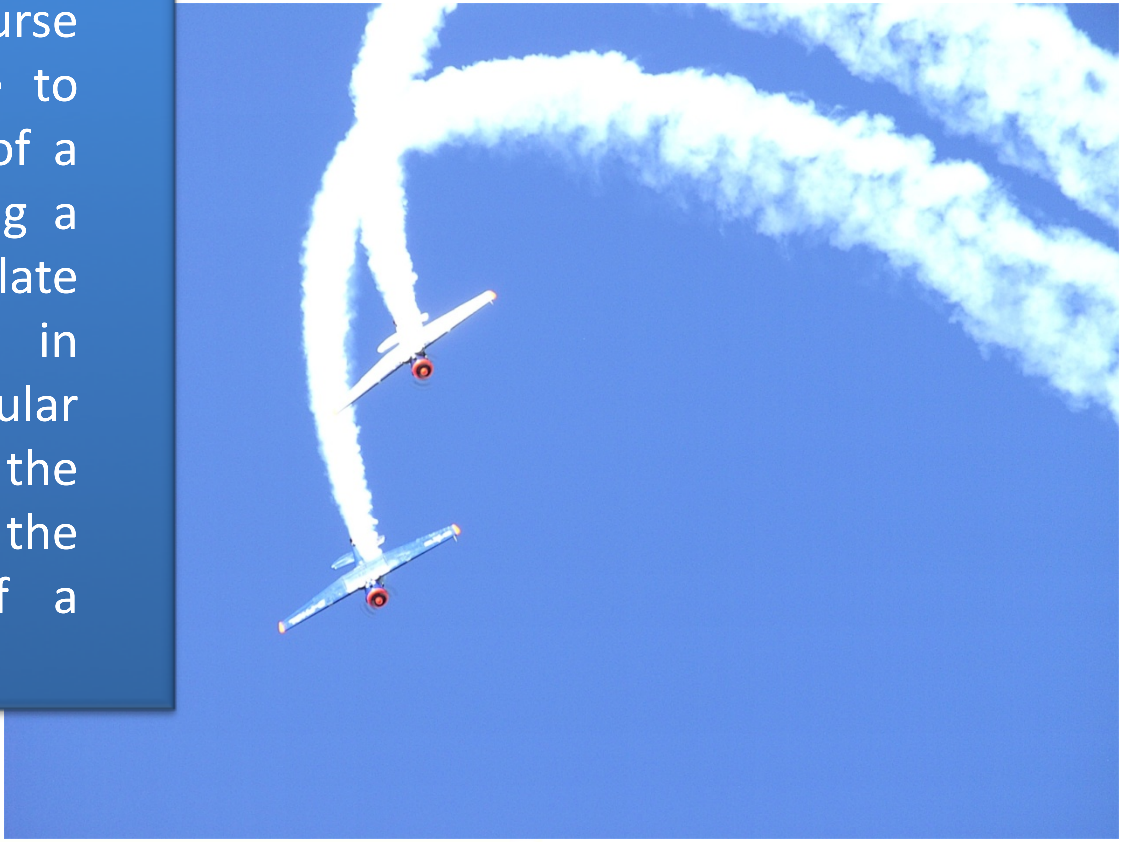
ENGINEERING DYNAMICS

CURVILINEAR MOTION

CURVILINEAR MOTION

The Objectives:

At the end of the course students will be able to describe the motion of a particle traveling along a curved path, relate kinematic quantities in terms of the rectangular components of the vectors and analyze the free-flight motion of a projectile.



CURVILINEAR MOTION

General Curvilinear Motion	
A particle moving along a curved path.	3d motion that can be described as a vector.

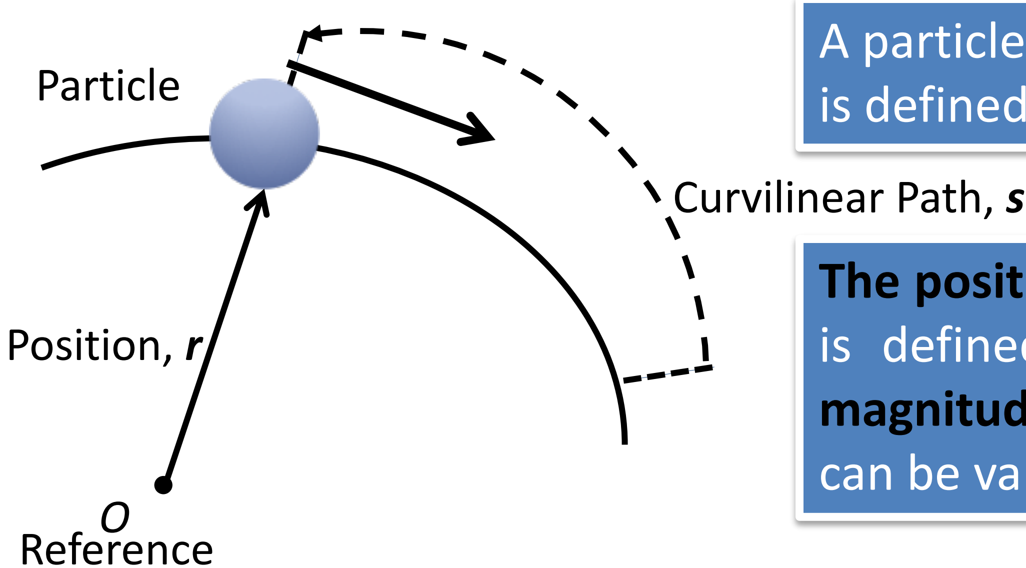


Figure 5 : The position of particle at curvilinear path

A particle moves along a curvilinear path is defined by the function, *s*. (Figure 5)

The position of the particle at any instant is defined by the vector ***r* = *r(t)***. The **magnitude** and **direction** of position, ***r*** can be vary with time.

CURVILINEAR MOTION

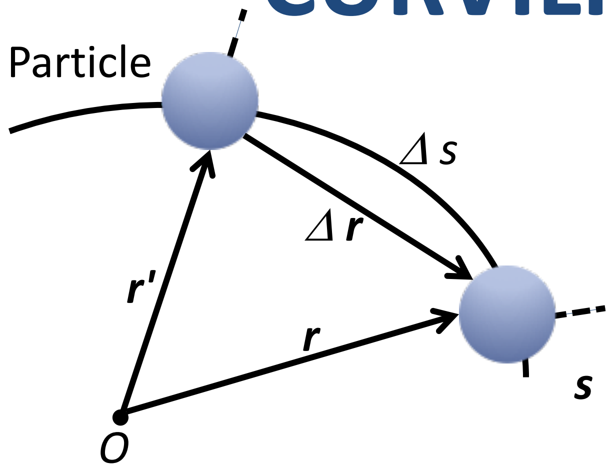


Figure 6 : The displacement of particle at curvilinear path

As shown in Figure 6, the particle moves a along the curve path with distance Δs during time interval Δt.

The **displacement** is determined by the difference between current and past vector.

$$\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$$

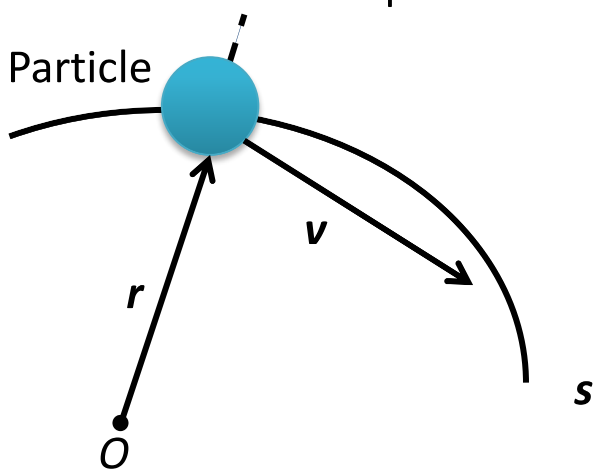


Figure 7 : The velocity of particle at curvilinear path

- ❑The **velocity** of particle at curvilinear path shown in Figure 7 represents the rate of change in the position of a particle.
- ❑The magnitude of v is called the speed.
- ❑The velocity vector, $\boldsymbol{v}$, is always tangent to the path of motion.

CURVILINEAR MOTION

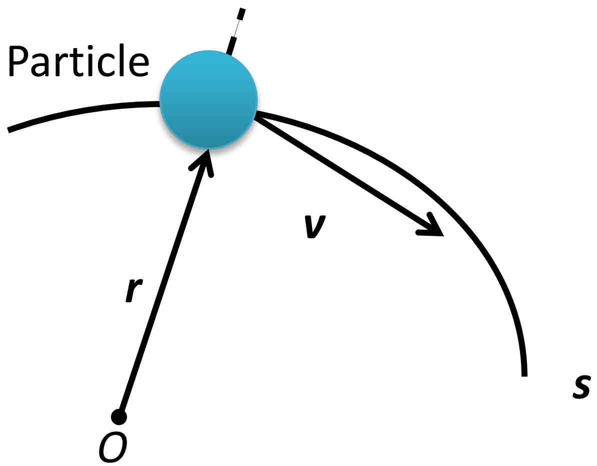


Figure 7 : The velocity of particle at curvilinear path

The **average velocity** of the particle during the time interval Δt is along path s is defined as

$$\mathbf{v}_{avg} = \Delta \mathbf{r}/\Delta t \,.$$

The instantaneous velocity $\mathbf{v}$ is the time-derivative of position $\mathbf{r}$ when the time interval Δt is taken as close as possible to zero second.

$$\mathbf{v} = d\mathbf{r}/dt \,.$$

The magnitude may vary with time.

CURVILINEAR MOTION

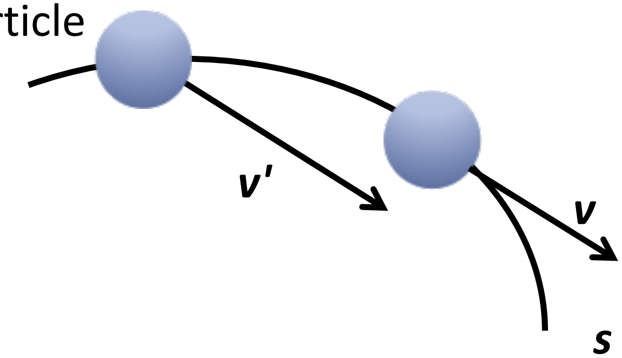


Figure 8 : The acceleration of particle at curvilinear path

- As shown in Figure 8, the particle's velocity changes from ***v*** to ***v'*** over a time interval ***Δt.***
- **An Acceleration** represents the rate of change in the velocity of a particle.

- The average acceleration can be defined as the rate of change in the velocity of particle in time interval ***Δt.***

$$a_{avg} = \Delta v/\Delta t = (v - v')/\Delta t$$

- The instantaneous acceleration of the particle moving at curvilinear path is the time derivative of the particle velocity when ***Δt*** is taken as close as possible to zero second.

$$a = dv/dt = d^2r/dt^2$$

CURVILINEAR MOTION

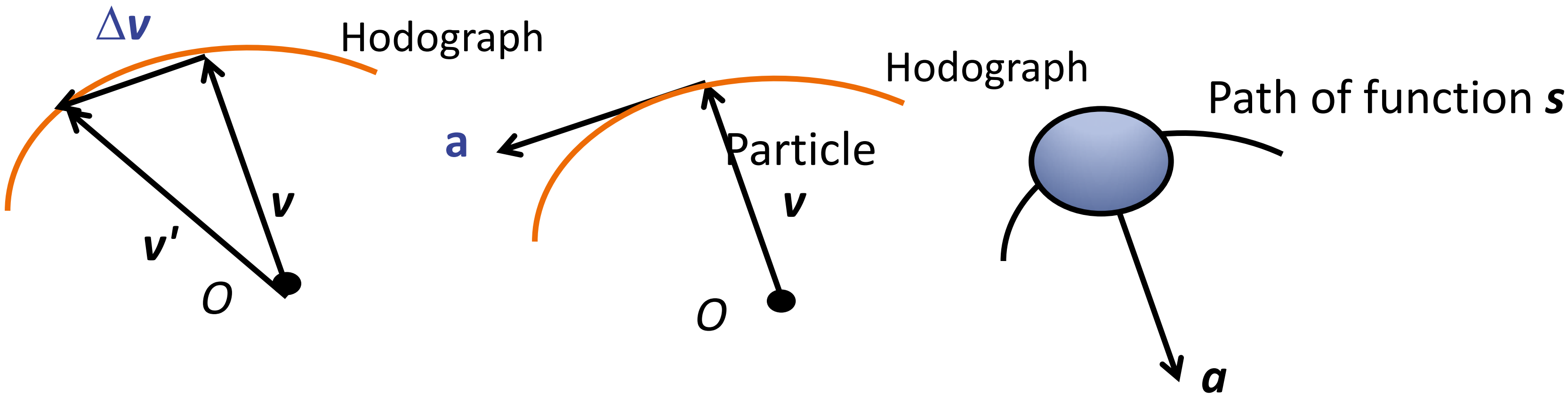


Figure 9 : Hodograph

Hodograph

- As shown in Figure 9, a plot of the locus of points defined by the arrowhead of the velocity vector.
- The acceleration vector is always tangent to the hodograph, but generally **not tangent** to the path of function ***s***.

CURVILINEAR MOTION

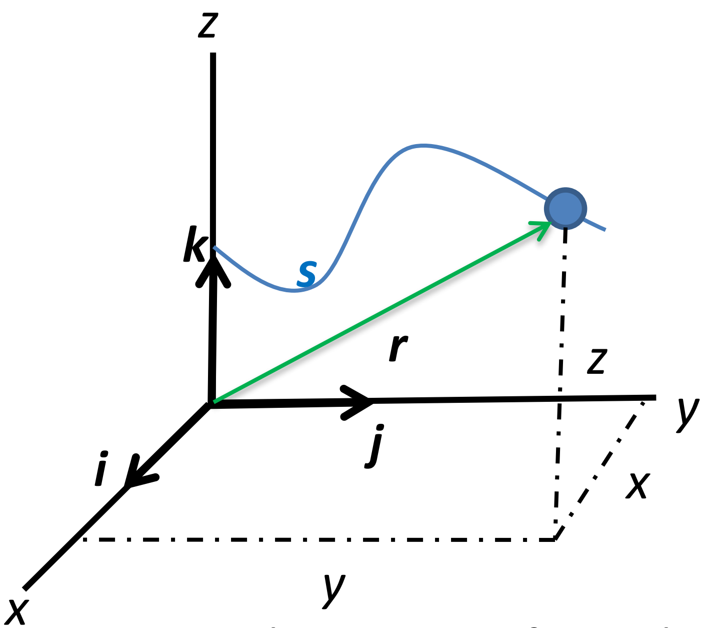


Figure 10 : The position of particle in rectangular components

Usually, it is common to describe the motion of a particle by using x, y, z or **rectangular components** relative to a **fixed frame of reference**.

The **position** (Figure 10) of the particle can be defined at any instant by using the **position vector**.

It is also can be represent by **functions of time**.

Position vector.

- $\boldsymbol{r} = \boldsymbol{x}\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$

Functions of time.

- x = x(t),
- y = y(t) and
- z = z(t) .

CURVILINEAR MOTION

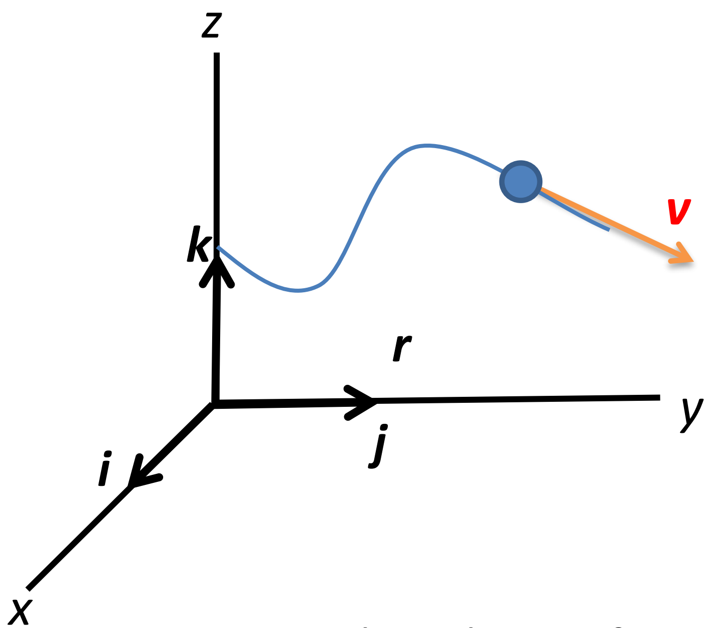


Figure 11 : The velocity of particle in rectangular components

Velocity

- The velocity is the time derivative of the position vector.(Figure 11)

  $\boldsymbol{v} = d\boldsymbol{r}/dt = d(x\boldsymbol{i})/dt + d(y\boldsymbol{j})/dt + d(z\boldsymbol{k})/dt$

  Or

  $v = v_x \boldsymbol{i} + v_y \boldsymbol{j} + v_z \boldsymbol{k}$
- Where,

  $v_x = \dot{x} = dx/dt$; $v_y = \dot{y} = dy/dt$; $v_z = \dot{z} = dz/dt$
- with the **magnitude**

  $$v = [(v_x)^2 + (v_y)^2 + (v_z)^2]^{0.5}$$
- and the **direction** of $\boldsymbol{v}$ is **tangent** to the path of motion.

CURVILINEAR MOTION

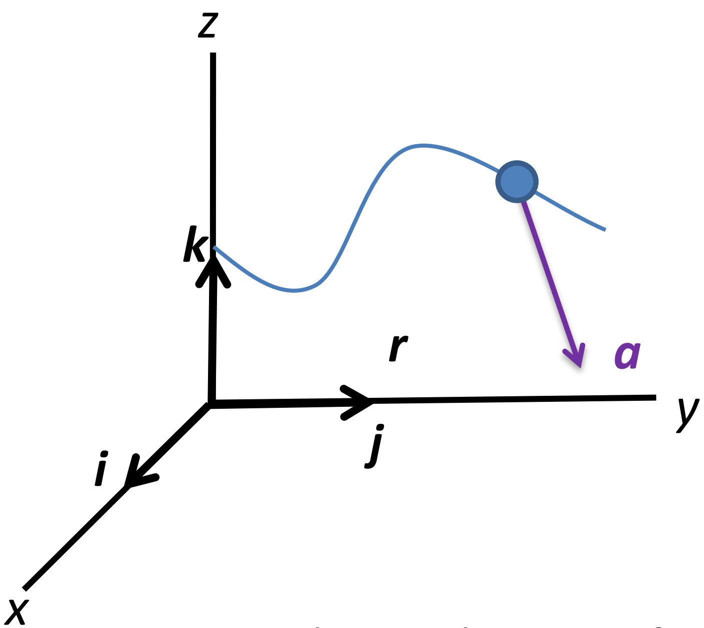


Figure 12 : The acceleration of particle in rectangular components

Acceleration

- The acceleration is the time derivative of the velocity vector.

  $\mathbf{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2 = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$

- Where,

  $a_x = \dot{v}_x = d\ddot{x}/dt$; $a_y = \dot{v}_y = d\ddot{y}/dt$;

  $a_z = \dot{v}_z = d\ddot{z}/dt$

- with the **magnitude**

  $a = [(a_x)^2 + (a_y)^2 + (a_z)^2\,]^{0.5}$

- and the **direction** of ***a*** is **not tangent** to the path of motion.

EXAMPLE 2

Problem

- The box slides down the slope described by the equation $y = (0.02x^2)$ m, where x is in meters. Given that the $v_x = -3$ m/s, $a_x = -1.5$ m/s^2 at $x = 5$ m. Determine the y components of the velocity and the acceleration of the box at $x = 5$ m.

Strategy

- Take the 1st time derivative of the path`s equation to get the particle's velocity.
- And take the second time derivative of the path's equation to determine the acceleration.

EXAMPLE 2

Determine the y-component of velocity by taking a time derivative of the position $y = (0.02x^2)$

- derivative of the velocity to determine the acceleration.

$$\dot{y} = 2\,(0.02)\,x\,\dot{x} = 0.04\,x\,\dot{x}$$

Determine the acceleration component by taking a time derivative of the velocity $\dot{y}$.

$$\Rightarrow \quad \ddot{y} = 0.04\,\dot{x}\,\dot{x} + 0.04\,\ddot{x}\,x$$

Substituting the *x*-component of the acceleration, velocity at x=5 into $\dot{y}$ and $\ddot{y}$.

EXAMPLE 2

Since $\dot{x} = v_x = -3$ m/s, $\ddot{x} = a_x = -1.5$ m/s^2 at $x = 5$ m

$\Rightarrow \quad \dot{y} = 0.04\, x\, \dot{x} = 0.04\,(5)\,(-3) = -0.6$ m/s

$$\begin{aligned} \Rightarrow \quad \ddot{y} &= 0.04\, \dot{x}\, \dot{x} + 0.04\, x\, \ddot{x} \\ &= 0.04\,(-3)^2 + 0.04\,(5)\,(-1.5) \\ &= 0.36 - 0.3 \\ &= 0.06 \text{ m/s}^2 \end{aligned}$$

At $x = 5$ m

$v_y = -\,0.6$ m/s $= 0.6$ m/s $\downarrow$

$a_y = 0.06$ m/s^2 $\uparrow$

MOTION OF A PROJECTILE

A good free kicker instinctively knows at what angle, θ, and initial velocity, he must kick the ball to make a goal. The motion itself call as projectile motion. We going to analyze the motion in order to understand it.

MOTION OF A PROJECTILE

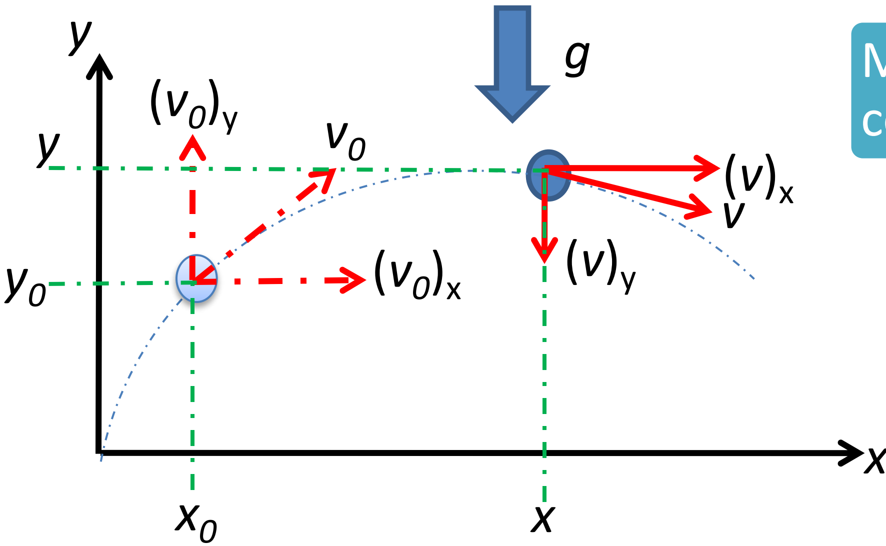


Figure 13 : The projectile motion

Motion of projectile is shown in Figure 13. It`s consist of two rectilinear motions.

- Horizontal direction (zero acceleration)
- Vertical direction

  (constant acceleration i.e gravity)

Analysis of the motion (Figure 14)

- Let the particle move with initial velocity v_0 at position (x_0, y_0).
- After t second, the particle arrived at new position (x, y) with velocity v.

MOTION OF A PROJECTILE

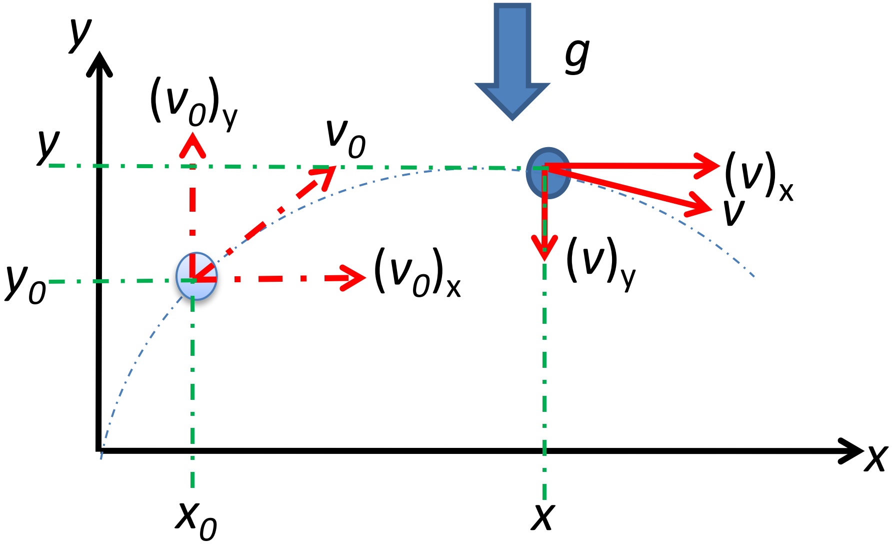


Figure 13 : The projectile motion

Assumption

- No air resistance.
- Positive direction for *y*-axis(↑ +ve).
- Positive direction for *x*-axis(⟶ +ve).

Horizontal direction

- Since $a_x = 0$,

$$v_x = v_{ox}$$

- And the position in the *x* direction is :

$$x = x_o + (v_{ox})\, t$$

Vertical direction

- Since $a_y = -g$,
- Constant acceleration equation can be applied.

$$v_y = v_{oy} - g\, t$$

$$y = y_o + (v_{oy})\, t \; - \tfrac{1}{2}\, g\, t^2$$

$$v_y^{\,2} = v_{oy}^{\,2} - 2\, g\, (y - y_o)$$

EXAMPLE 3

Problem

- The kid kick the ball with initial velocity v_A = 10 m/s at angle θ = 30 ° as shown in the figure below. Determine the velocity v_C and horizontal distance x_c when the ball arrived at point *C*.

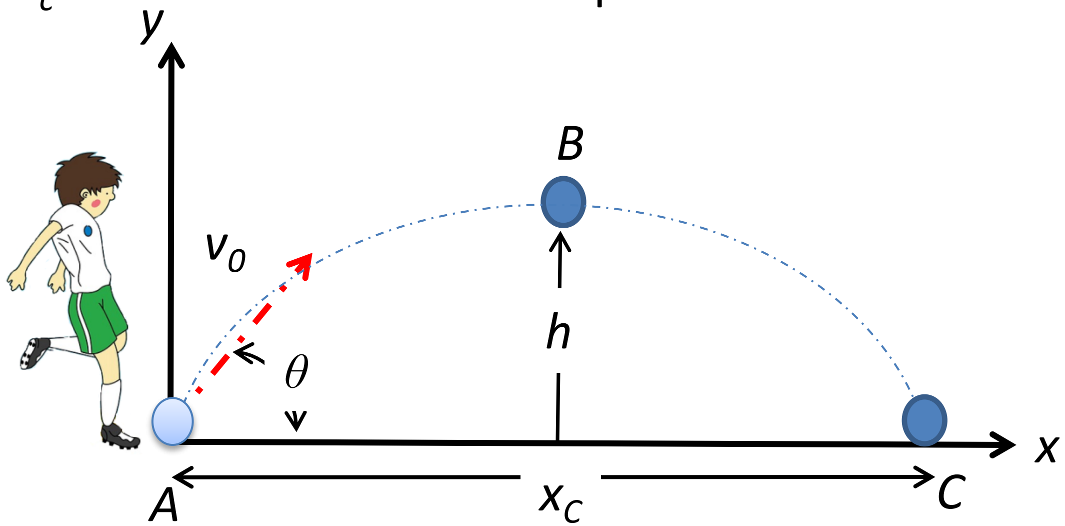


Figure 14 : The free kick

Strategy

- Solved using the kinematic equation for projectile motion.

EXAMPLE 3: SOLUTION

Using $v_{Ax} = 10 \cos 30$ and $v_{Ay} = 10 \sin 30$

- We can write

$$v_x = 10 \cos 30$$
$$v_y = 10 \sin 30 - (9.81)\, t$$
$$x = (10 \cos 30)\, t$$
$$y = (10 \sin 30)\, t - \tfrac{1}{2} (9.81)\, t^2$$

Since y = 0 at C

$$0 = (10 \sin 30)\, t - \tfrac{1}{2} (9.81)\, t^2 \Rightarrow t = 0,\ 1.019 \text{ s}$$

EXAMPLE 3: SOLUTION

Velocity components at C are;

$v_{Cx} = 10 \cos 30$
$= 8.66 \text{ m/s} \rightarrow$

$v_{Cy} = 10 \sin 30 - (9.81)(1.019)$
$= -5 \text{ m/s} = 5 \text{ m/s} \downarrow$

$$\therefore v_C = \sqrt{8.66^2 + (-5)^2} = 10 \text{ m/s}$$

Horizontal distance the ball travels is;

$x_c = (10 \cos 30)\, t$
$x_c = (10 \cos 30)\, 1.019 = 8.83 \text{ m}$