CALCULUS FOR TECHNOLOGY (BETU 1023)

WEEK 4

DIFFERENTIATION

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LEARNING OUTCOMES

By the end of this topic, students are able to:

- Define implicit and parametric functions.
- Able to solve implicit and parametric functions.

Diff. of Implicit Functions

Until now, what we have learned was to differentiate with respect to a variable.

For example, to differentiate:

$$y = 5x^3 - 4 \quad \rightarrow \quad \frac{dy}{dx} = 15x^2$$

x and *y* can be separated

We can directly differentiate the equation with respect to *x* since *y* and *x* appears alone on one side of the equation respectively. This equation is called an **explicit equation**.

Diff. of Implicit Functions

However, consider a case where we have x and y on each side which cannot be separated such as

$$x^2 + 2xy = y^2$$

x and *y* cannot be separated

This is called an **implicit equation**. Thus, to differentiate this equation we use the implicit technique of differentiation.

How to differentiate implicit function?

Step 1
- Differentiate both sides of the equation.

Step 2
- Solve the differential equation

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EXAMPLE

Find $\dfrac{dy}{dx}$ of the following implicit functions.

(a) $x^2 + y^2 - 2x = \sin y$

(b) $x^2 y^3 - e^y = e^{2x}$

(c) $x^2 - xy + y^2 = 7 \quad \text{at } x = 3, y = 2$

Solution: (a)

$$x^2 + y^2 - 2x = \sin y$$

Consider this is an implicit equation.

$$2x + 2y\frac{dy}{dx} - 2 = \cos y\frac{dy}{dx}$$

Differentiate with respect to *x*

$$\frac{dy}{dx}[2y - \cos y] = 2 - 2x$$

Collecting $\frac{dy}{dx}$ on a side

$$\frac{dy}{dx} = \frac{2-2x}{2y - \cos y}$$

Simplify.

Solution: (b)

Use product rule

$$x^2y^3 - e^y = e^{2x}$$

$$\left[x^2\left(3y^2\frac{dy}{dx}\right) + 2xy^3\right] - e^y\frac{dy}{dx} = 2e^{2x}$$

$$\frac{dy}{dx}\left[3x^2y^2 - e^y\right] = 2e^{2x} - 2xy^3$$

$$\frac{dy}{dx} = \frac{2e^{2x} - 2xy^3}{3x^2y^2 - e^y}$$

Solution: (c)

$$x^2 - xy + y^2 = 7$$

$$2x - \left[x\frac{dy}{dx} + y \right] + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}[2y - x] = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Try It Yourself 1

Find $\frac{dy}{dx}$ for the following implicit functions.

(a) $x^3 + y = 1 + y^3$

(b) $\ln y = y - x^2$

(c) $x^3 + y^3 - 3xy^2 = 8$

(d) $\sin(x + y) = 1 + y$

Solution

(a) $\text{Ans}: \dfrac{3x^2}{3y^2-1}$

(b) $\text{Ans}: \dfrac{2xy}{y-1}$

(c) $\text{Ans}: \dfrac{3x^2}{3y^2-1}$

(d) $\text{Ans}: \dfrac{\cos(x+y)}{1-\cos(x+y)}$

Diff. of Parametric Functions

We have seen that we can obtain the differential coefficient of implicit functions using the implicit differentiation. In some cases, implicit functions can be expressed in terms of parameters. In this sections we introduce differentiation of **parametric functions**. The implicit relationship of *x* and *y* can be expressed in a simpler form by using a third variable known as parameter. For example,

$$x = t^2 \quad \text{and} \quad y = t^3 + t$$

To solve for $\frac{dy}{dx}$, we need to use the chain rule that is

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

where we differentiate both *x* and *y* with respect to *t*.

EXAMPLE

Find $\dfrac{dy}{dx}$ of the following implicit functions.

(a) $x = 2t \quad , \quad y = 4 - 4t - 4t^2$

(b) $x = \dfrac{t-3}{t} \quad , \quad y = \dfrac{t^2+4}{t}$

(c) $x = e^t \quad , \quad y = \sin t$

Solution: (a)

$$x = 2t \quad , \quad y = 4 - 4t - 4t^2$$

$$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = -4 - 8t$$

Differentiate each term with respect to *t*

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= (-4 - 8t) \times \frac{1}{2}$$

$$= -2 - 4t$$

Use Chain Rule then Simplify.

Solution: (b)

$$x = \frac{t-3}{t} \quad , \quad y = \frac{t^2+4}{t}$$

$$= 1 - 3t^{-1} \qquad = t + 4t^{-1}$$

$$\frac{dx}{dt} = 3t^{-2} \qquad \frac{dy}{dt} = 1 - 4t^{-2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \left(1 - 4t^{-2}\right) \times \left(\frac{1}{3t^{-2}}\right)$$

$$= \frac{1}{3}\left(t^2 - 4\right)$$

Solution: (b)

$$x = e^t \quad , \quad y = \sin t$$

$$\frac{dx}{dt} = e^t \qquad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= (\cos t) \times \left(\frac{1}{e^t} \right)$$

$$= e^{-t} \cos t$$

Try It Yourself 2

Find $\frac{dy}{dx}$ for the following implicit functions.

(a) $x = 2t^2 - 4$, $y = 3t^4$

(b) $x = 2t^2 + 4t$, $y = 3t^2 + 2$

(c) $x = u^2 - u$, $y = u^3 - u^2$

(d) $x = \frac{1}{t-1}$, $y = t^3$

(e) $x = t^2 + 3$, $y = t(t^2 + 3)$

Solution

(a) $3t^2$

(b) $\dfrac{3t}{2(t+1)}$

(c) $\dfrac{u(3u-2)}{2u-1}$

(d) $-3t^2(t-1)^2$

(e) $\dfrac{3(t^2+1)}{2t}$

REFERENCES

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- Bivens, I.C., Stephen, D., & Howard, A. (2012). Calculus Early Transcedentals (10th ed.). John Willey & Sons Inc.