BETM 3583

Vibration Analysis and Monitoring

Ahmad Yusuf Ismail[1]
Mohd Afdhal bin Shamsudin[2]
Nur Rashid bin Mat Nuri @ Md Din[3]
Muhamad Azwar bin Azhari[4]

[1]ahmadyusuf.ismail@utem.edu.my
[2]afdhal@utem.edu.my
[3]nrashid@utem.edu.my
[4]azwar@utem.edu.my

Contents

Learning Outcome

1. Understand the concept and application of Probability distribution and density
2. Understand the concept and application of Fourier Analysis
3. Understand the concept and application of Cepstrum analysis

Probability Distribution & Density

In Topic 2 → Random Vibration

To characterize random signals in the way of their instantaneous value is distributed can be expressed in terms of "Probability Distribution"

Probability Distribution & Density

What is Probability ?

Concept of probability →

In a short run, chance behavior can be unpredictable,
But in long run, it can have a regular and predictable pattern.

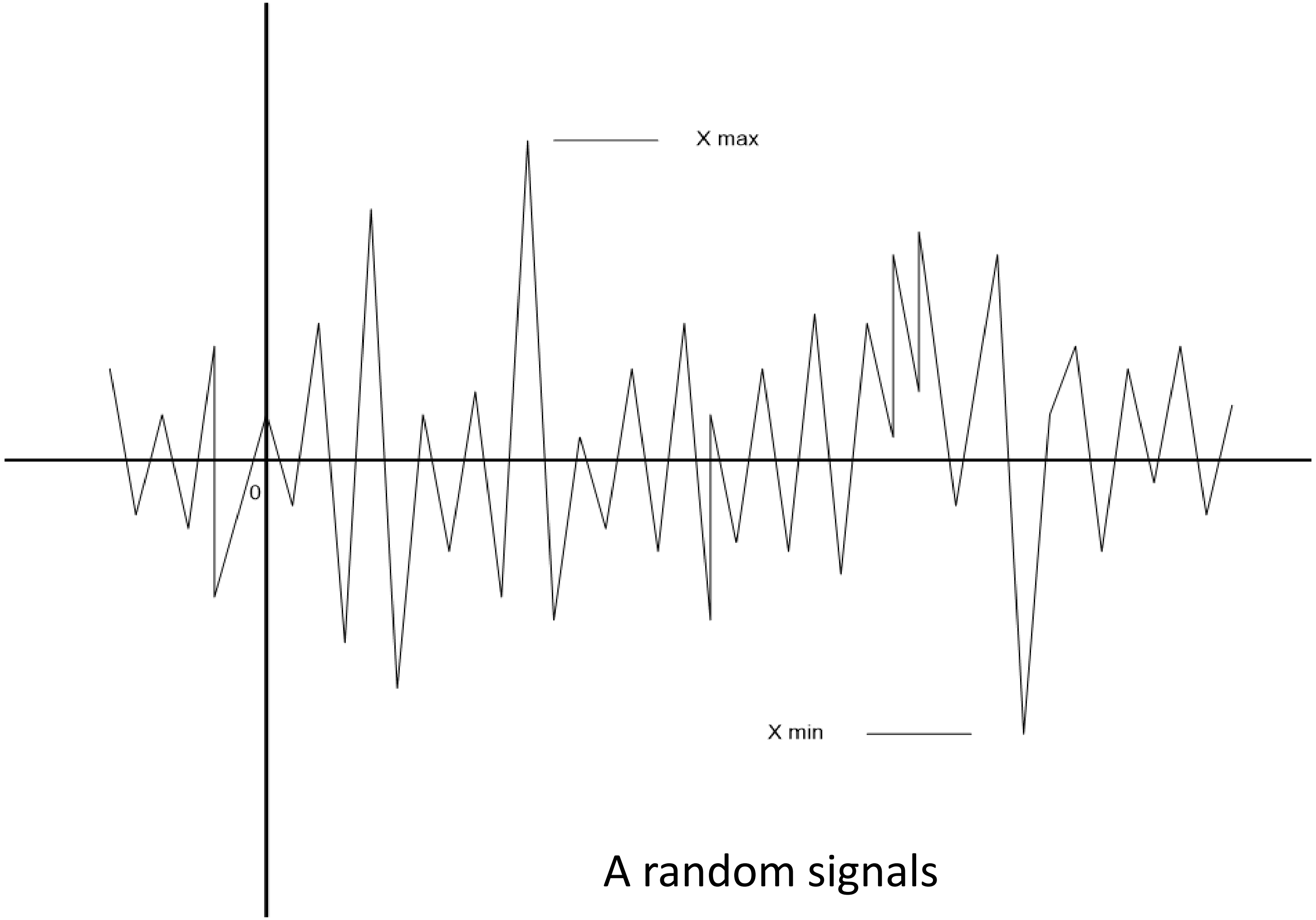
So, Probability → the proportion of times an interest outcome will occur in long repetition series.

Probability Distribution & Density

Probability Distribution

A probability of a measureable subset to occur in a random experiments.

Probability Distribution & Density



A random signals

Probability Distribution & Density

Figure above shows a typical random signals

Minimum value : x_{min}

Maximum value : x_{max}

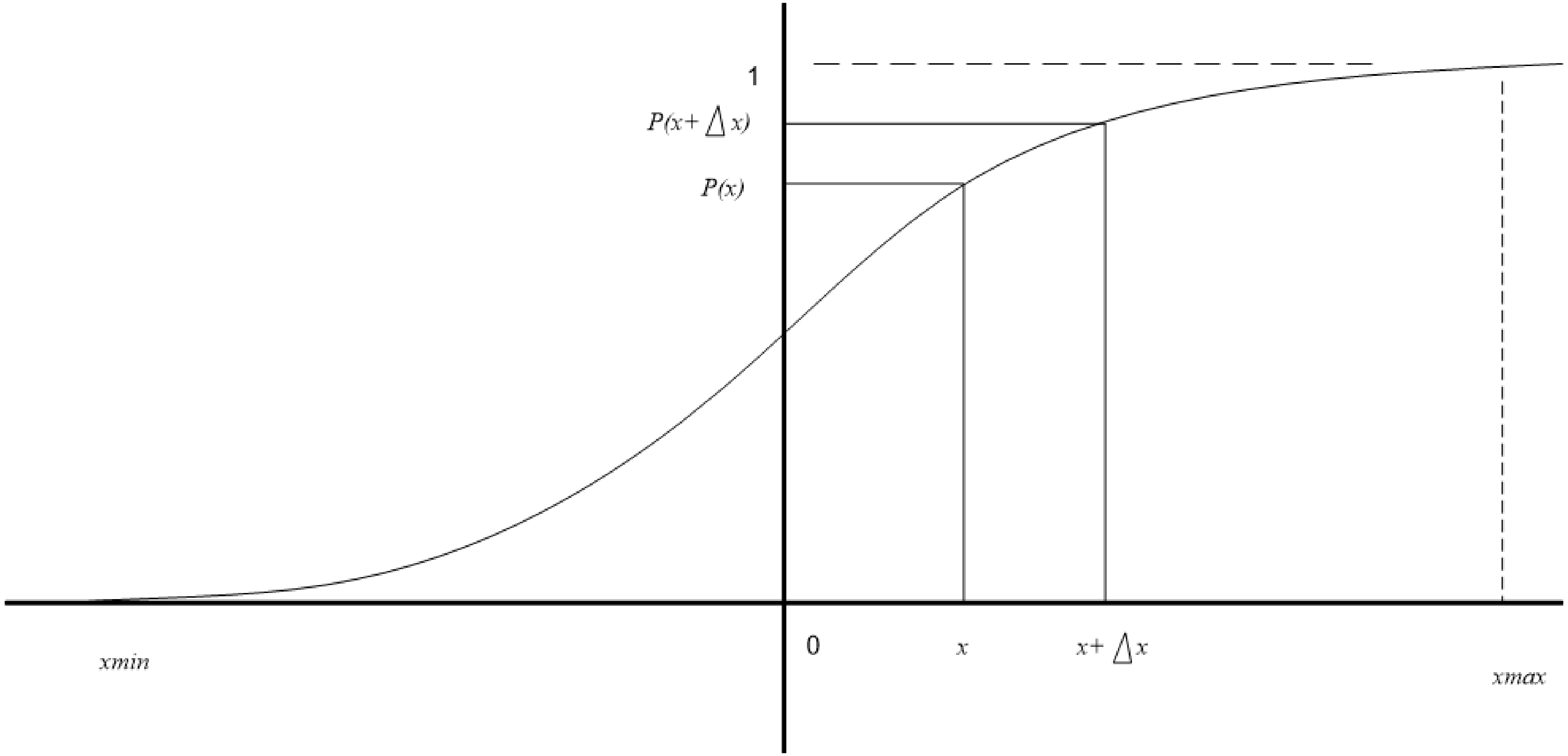
Probability :

$$P(x) = \Pr[x(t) \leq x]$$

Probability of a particular sample is less than or equal to x.

Probability Distribution & Density

P(x) must have Probability distribution form :



Probability Distribution & Density

Probability density *p(x)* is given by:

$$p(x) = \lim_{\Delta x \to 0} \frac{P(x + \Delta x) - P(x)}{\Delta x} = \frac{dP(x)}{dx}$$

Probability Distribution & Density

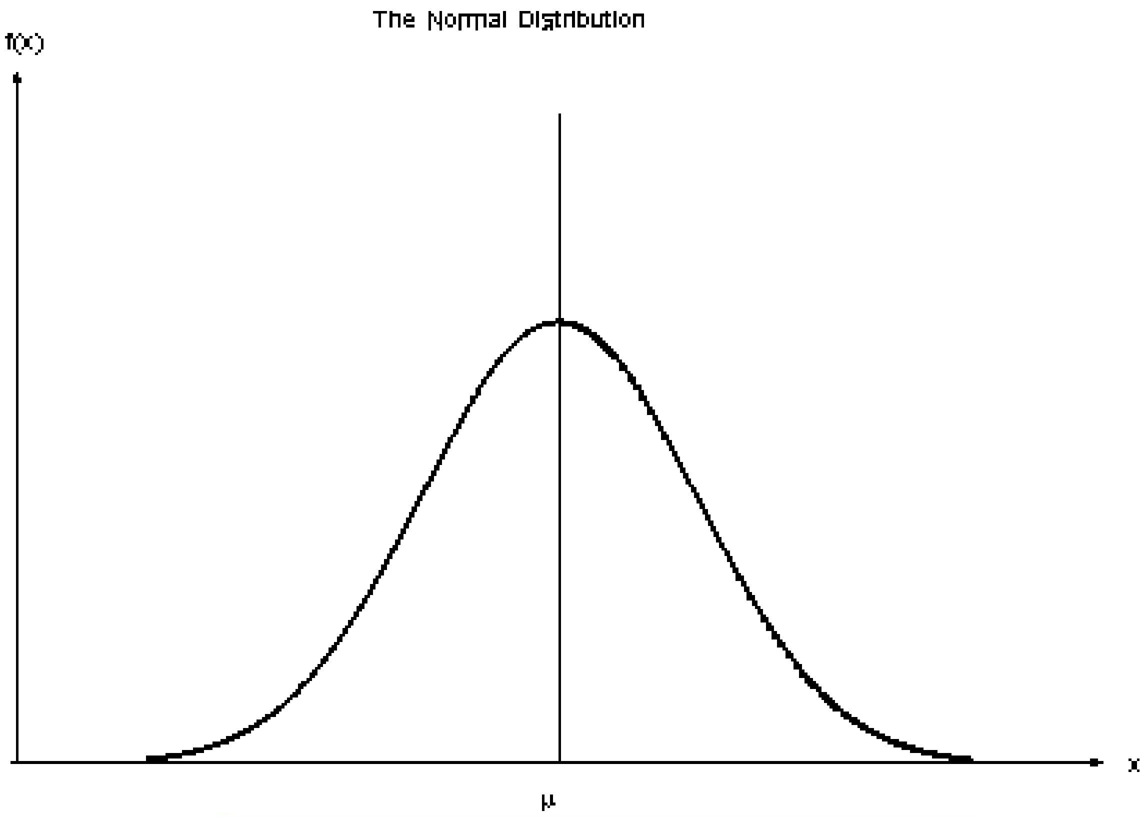
Since *p(x)* is derivation of *P(x)* over *dx,* :

$$\int_{-\infty}^{\infty} p(x)dx = \int_{-\infty}^{\infty} dP(x) = [P(\infty) - P(-\infty)] = 1$$

Evident that the total area under the curve must always be one.

Probability Distribution & Density

For the so-called Gaussian Random Signals, the probability density function is given by :



Probability Distribution & Density

Gaussian Random Signals → Statistical Parameter

Mean Value

$$\mu = \int_{-\infty}^{\infty} xp(x)dx$$

At the line of simmetry

Probability Distribution & Density

Gaussian Random Signals → Statistical Parameter

Variance

$$\sigma^2 = \int_{-\infty}^{\infty} [x-\mu]^2 \, p(x) dx$$

This is ‘moment inertia’ about the mean value

Probability Distribution & Density

Gaussian Random Signals → Statistical Parameter

Skewness

$$S = \frac{\int_{-\infty}^{\infty} [x-\mu]^3 p(x)dx}{\sigma^3}$$

Zero (0) for symmetrical function and

large for assymmetrical function

Probability Distribution & Density

Gaussian Random Signals → Statistical Parameter

Kurtosis

$$K = \frac{\int_{-\infty}^{\infty} [x-\mu]^4 \, p(x)dx}{\sigma^4}$$

Large for ‘Spiky’ signals

Fourier Analysis

Fourier Analysis

Fourier Analysis

Fourier Analysis is used to express signals as a summation of sinusoidal components.

In machine vibration analysis → it is used for periodic signals (machine rotating with constant speed)

Fourier Analysis

For a periodic signal *g(t)* with period *T*

$$g(t) = g(t + nT)$$

It is given that

$$g(t) = \frac{a_o}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega_o t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_o t)$$

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Fourier Analysis

where

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\cos(k\omega_o t)dt$$

$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\sin(k\omega_o t)dt$$

Fourier Analysis

The total component at frequency is ω_k

$$\omega_k = k\omega_o = a_k \cos(k\omega_k t)dt + b_k \sin(k\omega_k t)dt$$

Or can be written as

$$C_k \cos(\omega_k t + \phi_k)$$

Fourier Analysis

Where

$$C_k = \sqrt{a_k^{\ 2} + b_k^{\ 2}}$$

And

$$\phi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

Cepstrum Analysis

Cepstrum Analysis

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Cepstrum Analysis

→Results of Inverse Fourier Transform

Useful to differentiate multiple faults which is impossible/difficult to be seen in common Spectrum/FFT/others

Cepstrum Analysis

Cepstrum Analysis

Can be used for :

1. Gearbox monitoring and rolling element bearing
2. Gearbox testing
3. Bearing fault
4. Echo detection
5. Speech analysis

Thank you

Q n A