# MECHANISM DESIGN

# CHAPTER 5:
# ACCELERATION ANALYSIS

Shamsul Anuar Shamsudin
Mohd Nizam Sudin

# INTRODUCTION

- When speed changes, the object experiences acceleration if the speed is increasing or deceleration if the speed is decreasing.

- From Newton Law, the acceleration and force are related. One can produce the other.

$$F = ma$$

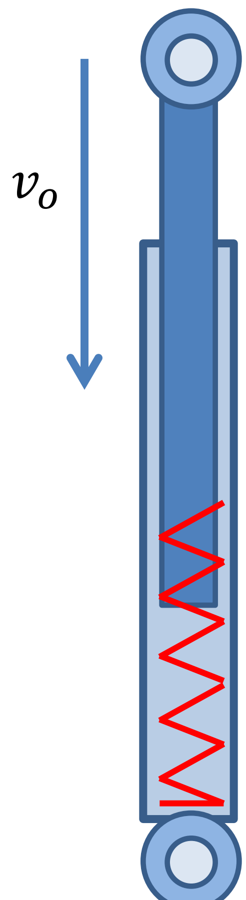
- There are two types of acceleration considered here
  - ❑ Linear
  - ❑ Angular

- Main equations:

$$a^t = r\alpha = r\frac{d^2\theta}{dt^2}$$

$$a = \frac{d^2 s}{dt^2}$$

# EXAMPLE 1




An absorber is pushed at a rate of 8 cm/s. When the source of shock ceases, it takes an additional 0.5 sec to stop.

Find the acceleration and the displacement of the rod.

[You may have to review some basic rigid-body or particle acceleration in dynamics]

$$s = v_i t + \frac{at^2}{2}$$

$$v_f = v_i + at$$

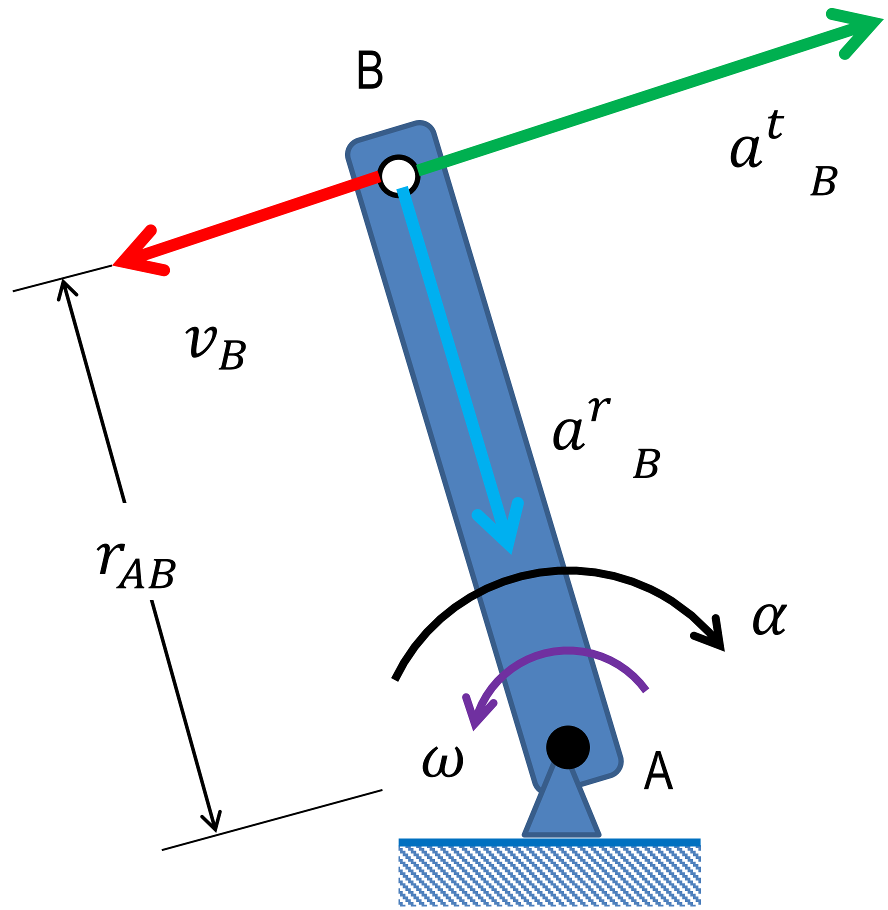
$${v_f}^2 = {v_i}^2 + 2as$$

$$s = \frac{(v_i + v_f)t}{2}$$

# LINEAR ACCELERATION DUE TO ROTATION

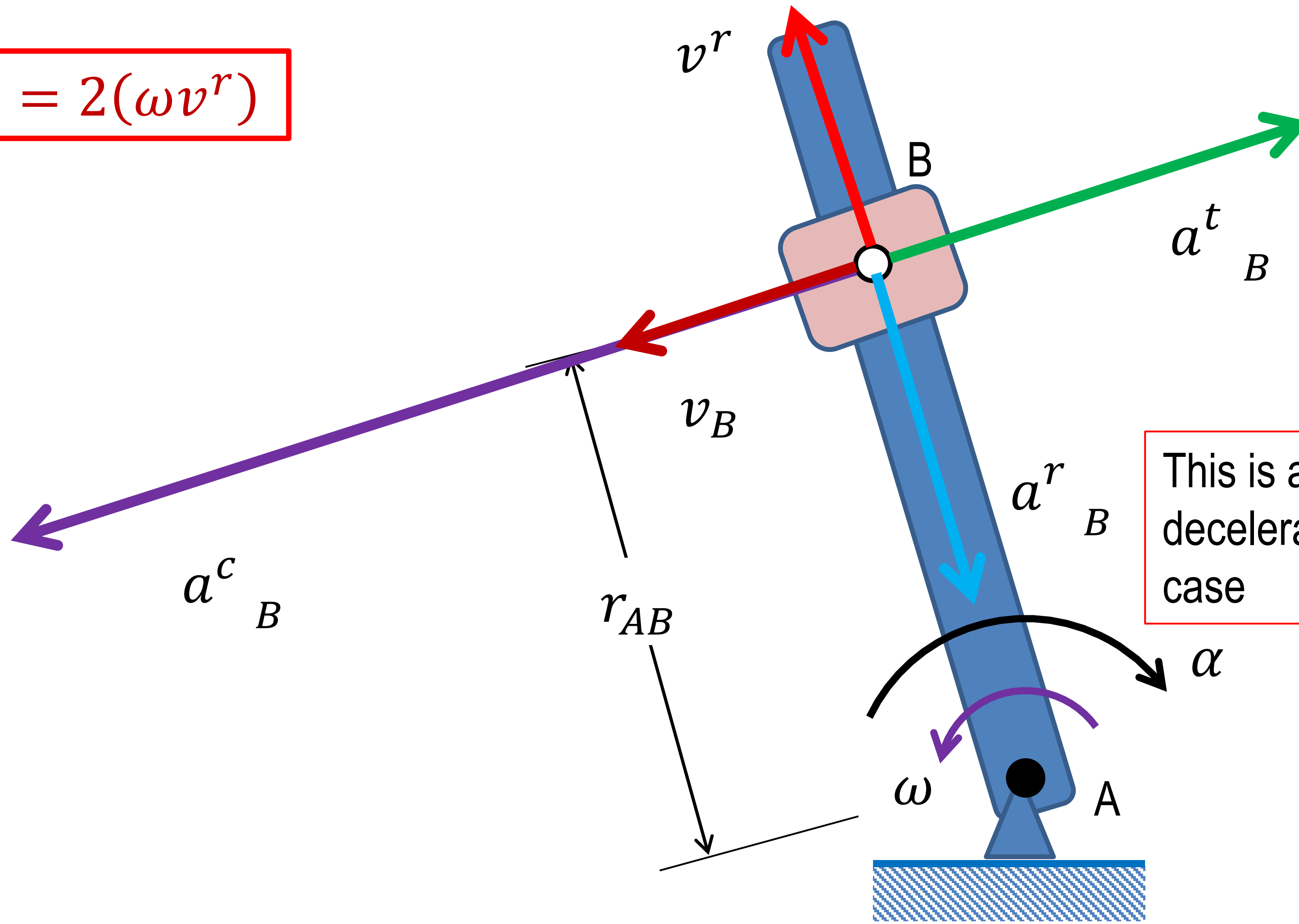
$$a^t = r\alpha$$

$$a^r = r\omega^2 = \frac{v^2}{r}$$



# CORIOLIS ACCELERATION OF A COLLAR

$$\boxed{a^c = 2(\omega v^r)}$$




This is a deceleration case

# EXAMPLE 2

Point P is moving at 50 cm/s. At the position shown, the link start to slow down, then stop in 5 seconds.

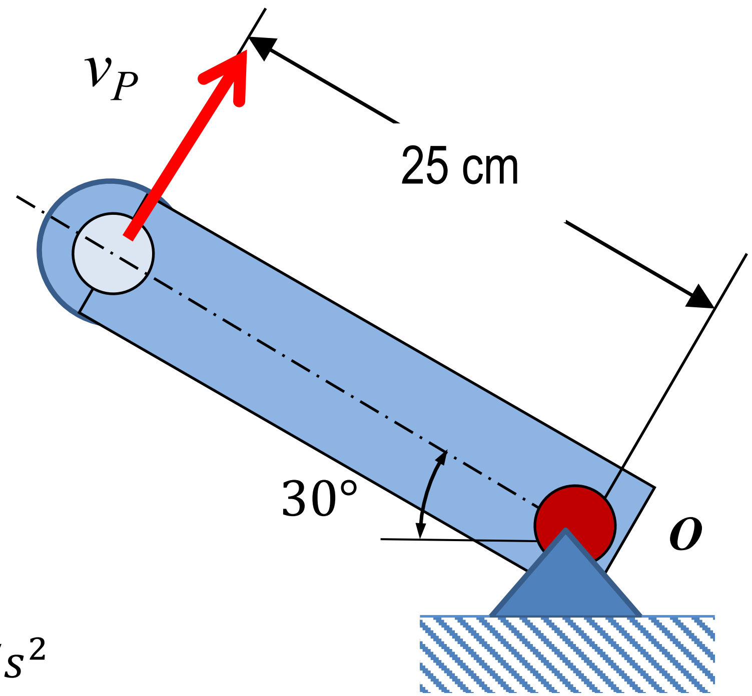
Calculate the acceleration of point P.

$$\omega = \frac{0.5\ m/s}{0.25\ m} = 2\ rad/s$$

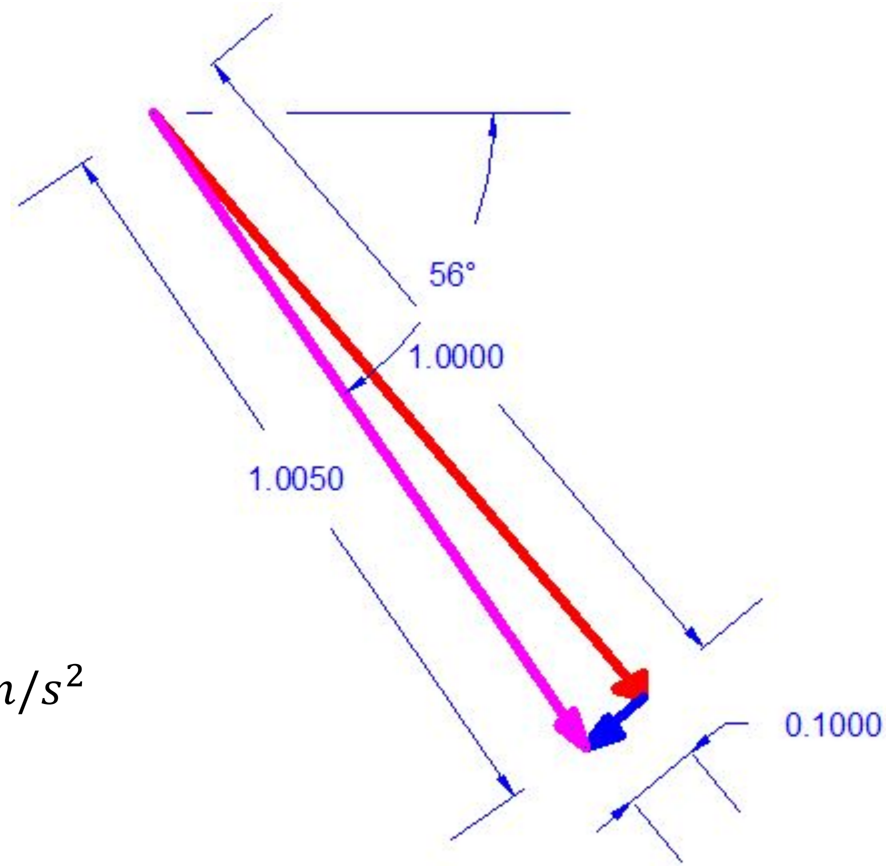
$$\alpha = \frac{(0 - 2\ rad/s)}{5\ s} = -0.4\ rad/s^2$$

$$a_P{}^t = (0.25\ m)(-0.4\ rad/s^2) = -0.1\ m/s^2$$

$$a_P{}^r = (0.25\ m)(2\ rad/s)^2 = 1\ m/s^2$$



# EXAMPLE 2 - Solution

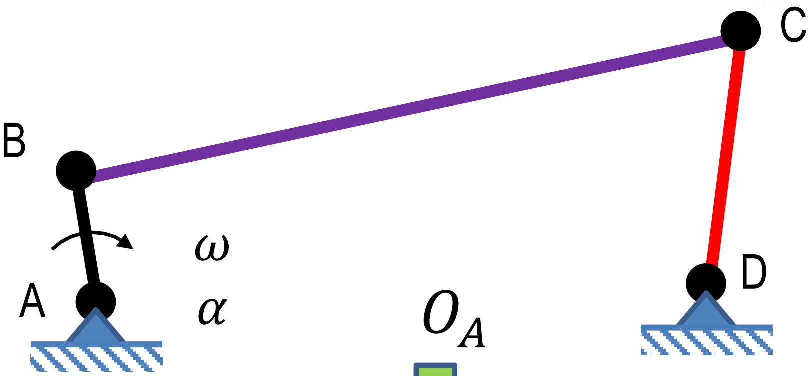


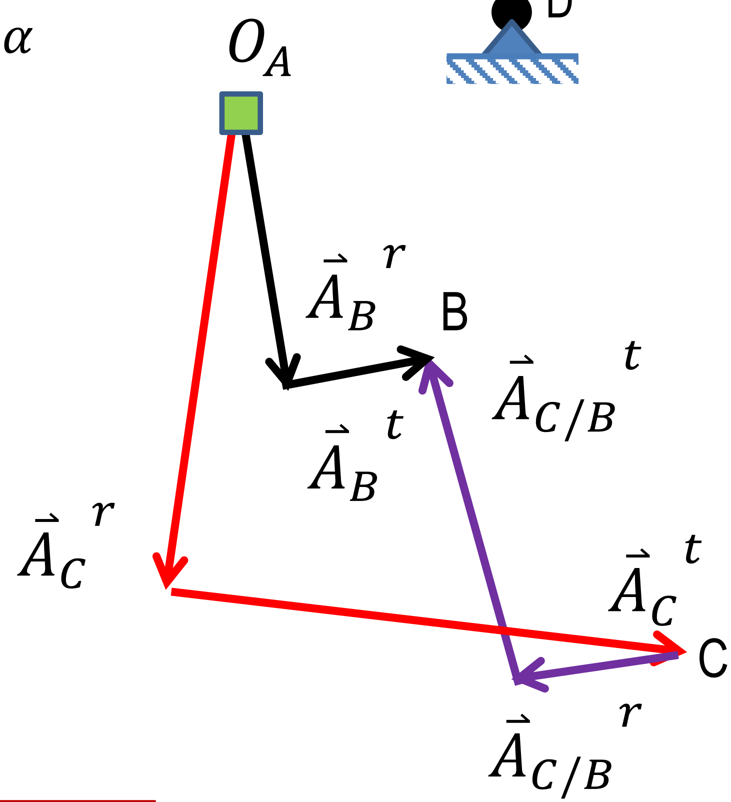

$$a_P = \sqrt{1^2 + (-0.1)^2} = 1.005\, m/s^2$$

$$\theta_{a_P} = -56°$$

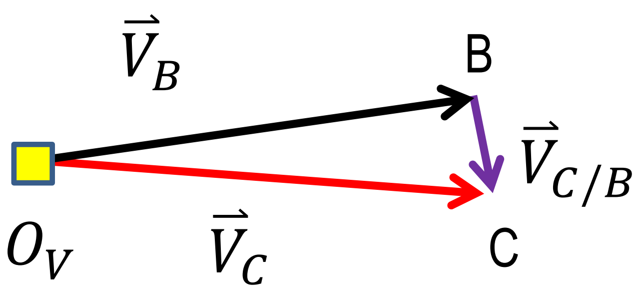
# VECTOR POLYGONS

Use graphical (CAD) approach or analytical. Do position → velocity → acceleration.



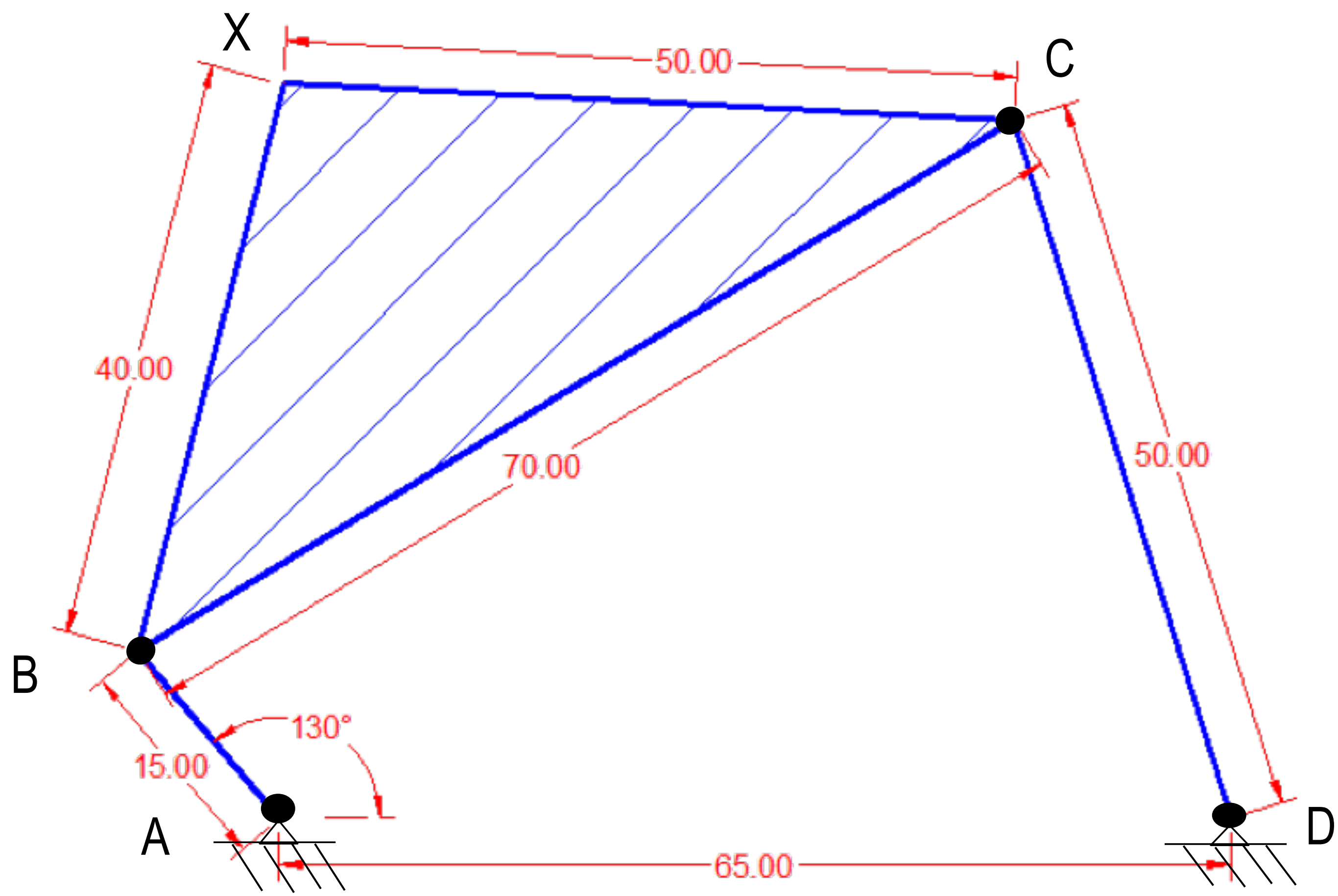

$$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B$$




$$\vec{A}_B{}^r + \vec{A}_B{}^t = \vec{A}_C{}^r + \vec{A}_C{}^t + \vec{A}_{C/B}{}^r + \vec{A}_{C/B}{}^t$$

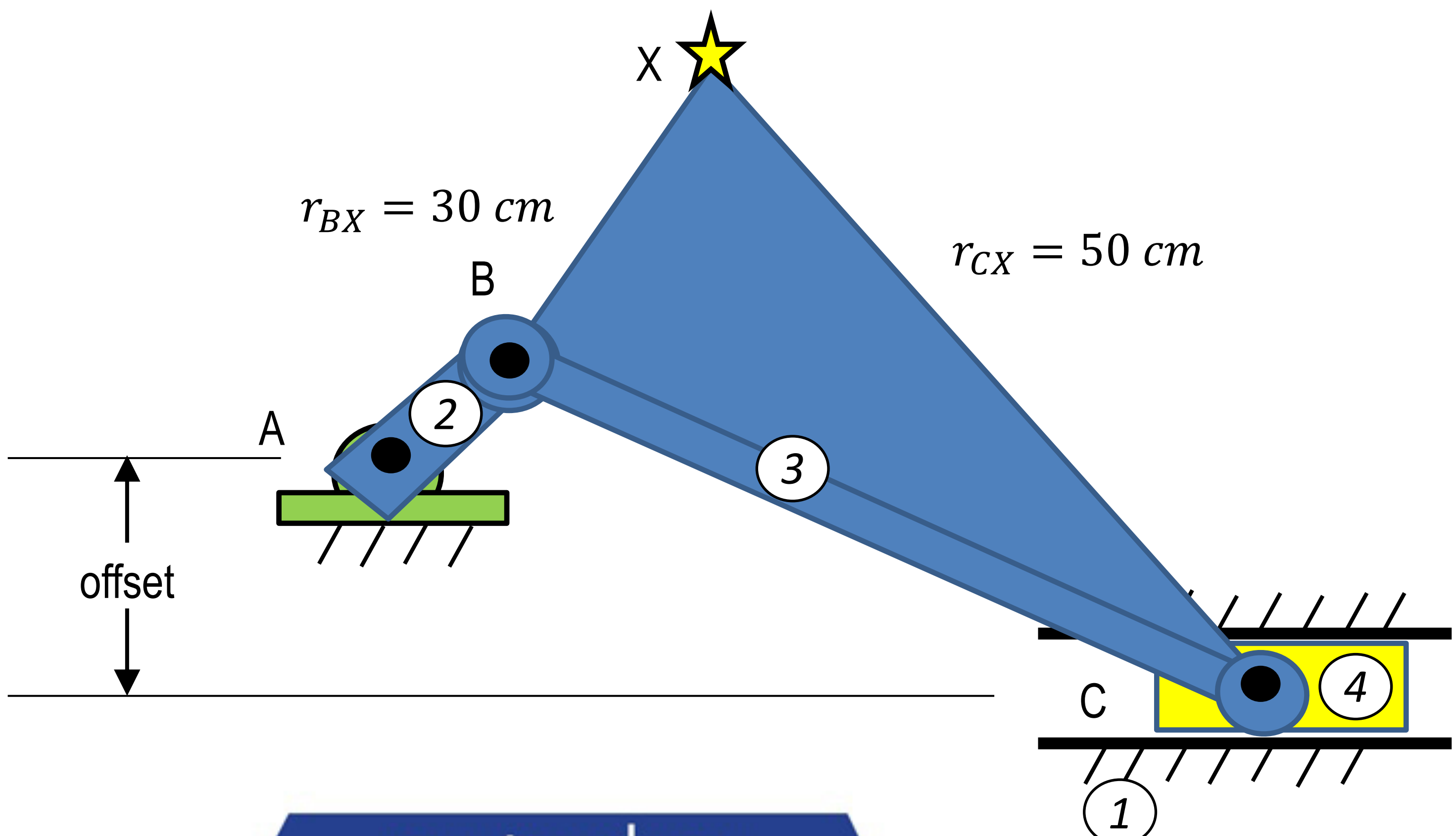
# EXERCISE 1

The 15 mm crank rotates at 1000 rpm, CCW. Find the linear velocity of the X. Get the acceleration of point X if the crank is slowing down at a rate of 10 rad/$s^2$.



# EXERCISE 2

The 10 cm crank revolves at 100 rpm, counterclockwise. The connecting rod is 40 cm. The offset is 7.5 cm. Crank angle is 30°. The crank is speeding up at 5 rad/s$^2$.
Find the velocity and acceleration of point X.



# ANALYTICAL SOLUTIONS: SLIDER-CRANK

Position

$$\theta_3 = \sin^{-1}\left[\frac{r_1 + r_2 \sin\theta_2}{r_3}\right]$$

$$r_4 = r_2 \cos\theta_2 + r_3 \cos\theta_3$$

Velocity

$$\omega_3 = -\omega_2\left[\frac{r_2 \cos\theta_2}{r_3 \cos\theta_3}\right]$$

$$v_4 = \omega_2 r_2 \sin\theta_2 - \omega_3 r_3 \sin\theta_3$$

Acceleration

$$\alpha_3 = \frac{{\omega_2}^2 r_2 \sin\theta_2 + {\omega_3}^2 r_3 \sin\theta_3 - \alpha_2\, r_2 \cos\theta_2}{r_3 \cos\theta_3}$$

$$\alpha_4 = -\alpha_2 r_2 \sin\theta_2 - \alpha_3 r_3 \sin\theta_3 - {\omega_2}^2 r_2 \cos\theta_2 - {\omega_3}^2 r_3 \cos\theta_3$$

# ANALYTICAL SOLUTIONS: FOUR-BAR

Position

$$BD = \sqrt{{r_1}^2 + {r_2}^2 - 2r_1 r_2 \cos\theta_2}$$

$$\gamma = \cos^{-1}\left[\frac{{r_3}^2 + {r_4}^2 - BD^2}{2r_3 r_4}\right]$$

*Circuit 1:*

$$\theta_3 = 2\tan^{-1}\left[\frac{-r_2\sin\theta_2 + r_4\sin\gamma}{r_1 + r_3 - r_2\cos\theta_2 - r_4\cos\gamma}\right]$$

$$\theta_4 = 2\tan^{-1}\left[\frac{r_2\sin\theta_2 - r_3\sin\gamma}{r_4 - r_1 + r_2\cos\theta_2 - r_3\cos\gamma}\right]$$

*Circuit 2:*

$$\theta_3 = 2\tan^{-1}\left[\frac{-r_2\sin\theta_2 - r_4\sin\gamma}{r_1 + r_3 - r_2\cos\theta_2 - r_4\cos\gamma}\right]$$

$$\theta_4 = 2\tan^{-1}\left[\frac{r_2\sin\theta_2 + r_3\sin\gamma}{r_4 - r_1 + r_2\cos\theta_2 - r_3\cos\gamma}\right]$$

# ANALYTICAL SOLUTIONS: FOUR-BAR

Velocity

$$\omega_3 = -\omega_2 \left[\frac{r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin \gamma}\right]$$

$$\omega_4 = -\omega_2 \left[\frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin \gamma}\right]$$

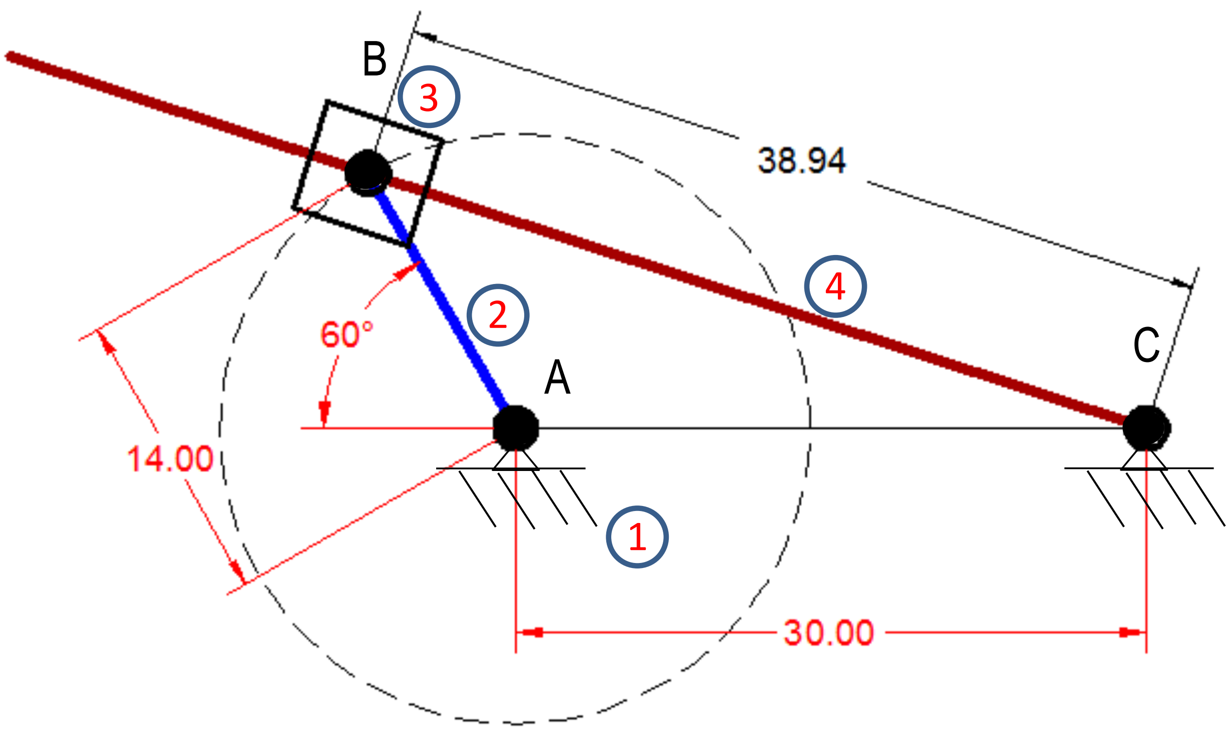
Acceleration

$$\alpha_3 = \frac{\alpha_2 r_2 \sin(\theta_2 - \theta_4) + {\omega_2}^2 r_2 \cos(\theta_2 - \theta_4) + {\omega_3}^2 r_3 \cos(\theta_4 - \theta_3) - {\omega_4}^2 r_4}{r_3 \sin(\theta_4 - \theta_3)}$$

$$\alpha_4 = \frac{\alpha_2 r_2 \sin(\theta_2 - \theta_3) + {\omega_2}^2 r_2 \cos(\theta_2 - \theta_3) + {\omega_3}^2 r_3 - {\omega_4}^2 r_4 \cos(\theta_4 - \theta_3)}{r_4 \sin(\theta_4 - \theta_3)}$$

# EXAMPLE 3

The crank turns 400 rpm CCW. Analyze the acceleration.



# EXAMPLE 3 – Graphical Solution

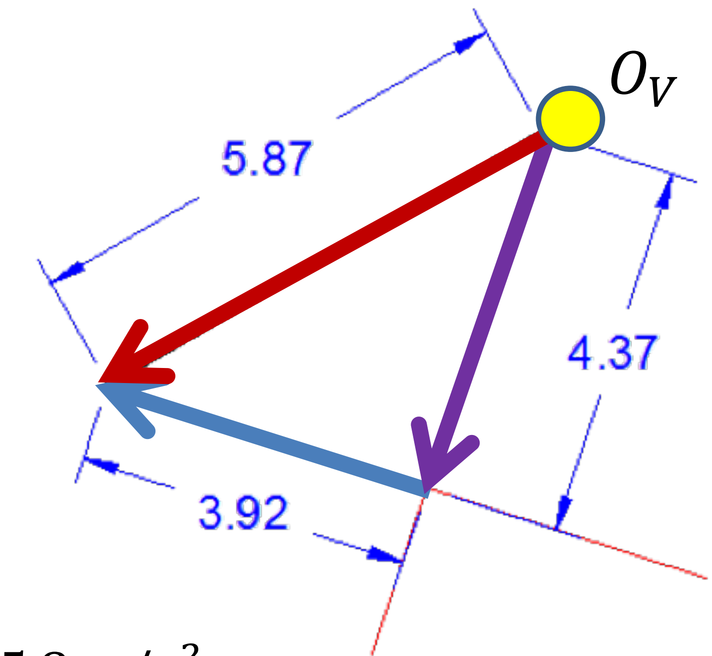
$$\omega_2 = (400)\frac{\pi}{30} = 41.9\ rad/s$$

$$v_B = r_2\omega_2 = (0.14)(41.9) = 5.87\ m/s$$

$$v_{B4} = 4.37\ m/s$$

$$v_{B2/B4} = 3.92\ m/s$$

$$\omega_4 = \frac{4.37\ m/s}{0.3894\ m} = 11.22\ rad/s$$




$$\begin{aligned} a_{B2}{}^{t} &= (0.14\ m)(0\ rad/s^2) \\ &= 0 m/s^2 \end{aligned}$$

$$a_{B2}{}^{r} = (0.14\ m)(41.9\ rad/s)^2 = 245.8\ m/s^2$$

# EXAMPLE 3 – Graphical Solution

$$a_{B4}{}^{r} = \frac{v_{B4}{}^{2}}{r_{B4}} = \frac{4.37^2}{0.3894} = 49.04\ m/s^2$$

$$a_{B2/B4}{}^{c} = 2v_{B2/B4}\omega_4 = 2(3.92)(11.22) = 87.96\ m/s^2$$

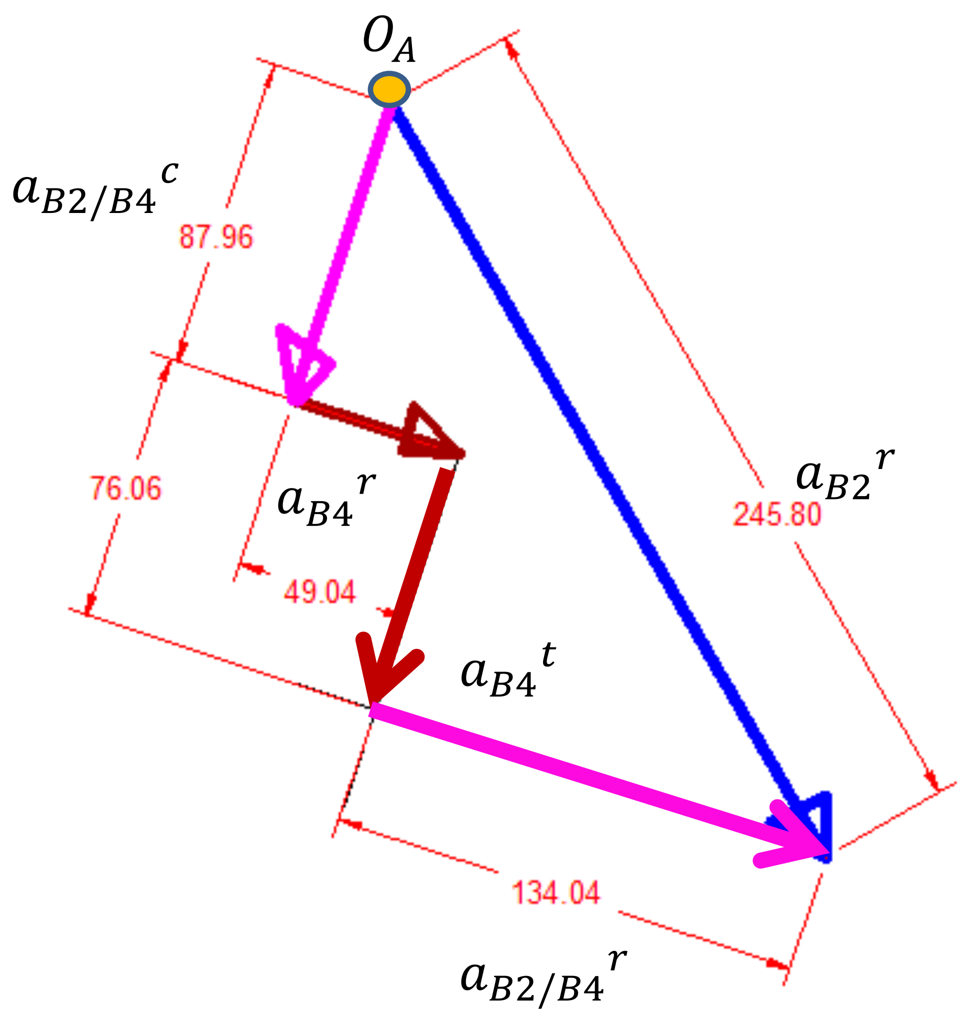
$$a_{B2/B4}{}^{r} = ?$$

$$a_{B2/B4}{}^{t} = 0$$

$$a_{B4}{}^{t} = 0$$

$$a_{B2/B4}{}^{r} = ?$$

# EXAMPLE 3 – Graphical Solution



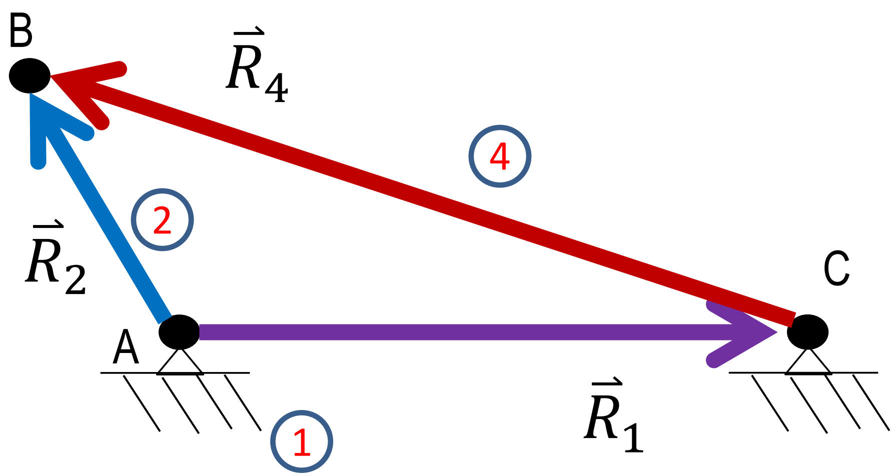
# EXAMPLE 3 – Vector Solution

## Position

$$\vec{R}_2 = \vec{R}_1 + \vec{R}_4$$

$$r_2 \begin{Bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{Bmatrix} = \begin{Bmatrix} r_{1x} \\ r_{1y} \end{Bmatrix} + r_4 \begin{Bmatrix} \cos\theta_4 \\ \sin\theta_4 \end{Bmatrix}$$

2 unknowns: $\theta_4$ and $r_4$



# EXAMPLE 3 – Vector Solution

$$14\begin{Bmatrix}\cos 120\\ \sin 120\end{Bmatrix} = \begin{Bmatrix}30\\ 0\end{Bmatrix} + r_4\begin{Bmatrix}\cos\theta_4\\ \sin\theta_4\end{Bmatrix}$$

$$14\begin{Bmatrix}\cos 120\\ \sin 120\end{Bmatrix} = \begin{Bmatrix}30\\ 0\end{Bmatrix} + \vec{R}_4$$

$$\vec{R}_4 = \begin{Bmatrix}-37\\ 12.12\end{Bmatrix}$$

$$r_4 = \sqrt{(-37)^2 + 12.12^2} = 38.93\ cm$$

$$\theta_4 = \tan^{-1}\left(\frac{12.12}{-37}\right) + 180 = 162°$$

# EXAMPLE 3 – Vector Solution

Velocity

$$r_2\omega_2\begin{Bmatrix}-\sin\theta_2\\ \cos\theta_2\end{Bmatrix} = 0 + r_4\omega_4\begin{Bmatrix}-\sin\theta_4\\ \cos\theta_4\end{Bmatrix} + v_4\begin{Bmatrix}\cos\theta_4\\ \sin\theta_4\end{Bmatrix}$$

2 unknowns: $\omega_4$ and $v_4$

$$(0.14)(41.9)\begin{Bmatrix}-\sin 120\\ \cos 120\end{Bmatrix} = 0 + (0.3893)\omega_4\begin{Bmatrix}-\sin 162\\ \cos 162\end{Bmatrix} + v_4\begin{Bmatrix}\cos 162\\ \sin 162\end{Bmatrix}$$

$$\begin{bmatrix}-0.1203 & -0.9511\\ -0.3702 & 0.3090\end{bmatrix}\begin{Bmatrix}\omega_4\\ v_4\end{Bmatrix} = \begin{Bmatrix}-5.08\\ -2.93\end{Bmatrix}$$

$$\begin{Bmatrix}\omega_4\\ v_4\end{Bmatrix} = \begin{Bmatrix}11.19\ rad/s\\ 3.93\ m/s\end{Bmatrix}$$

# EXAMPLE 3 – Vector Solution

## Acceleration

$$r_2 {\omega_2}^2 \begin{Bmatrix} -\cos\theta_2 \\ -\sin\theta_2 \end{Bmatrix} + r_2 \alpha_2 \begin{Bmatrix} -\sin\theta_2 \\ \cos\theta_2 \end{Bmatrix}$$

$$= r_4 {\omega_4}^2 \begin{Bmatrix} -\cos\theta_4 \\ -\sin\theta_4 \end{Bmatrix} + r_4 \alpha_4 \begin{Bmatrix} -\sin\theta_4 \\ \cos\theta_4 \end{Bmatrix} + v_4 \omega_4 \begin{Bmatrix} -\sin\theta_4 \\ \cos\theta_4 \end{Bmatrix} + a_4 \begin{Bmatrix} \cos\theta_4 \\ \sin\theta_4 \end{Bmatrix}$$

$$+ v_4 \omega_4 \begin{Bmatrix} -\sin\theta_4 \\ \cos\theta_4 \end{Bmatrix}$$

2 unknowns: $\alpha_4$ and $a_4$

# EXAMPLE 3 – Vector Solution

Acceleration

$$\alpha_2 = 0$$

$$(0.14)(41.9)^2 \begin{Bmatrix} -\cos 120 \\ -\sin 120 \end{Bmatrix} + 0$$
$$= (0.3893)(11.19)^2 \begin{Bmatrix} -\cos 162 \\ -\sin 162 \end{Bmatrix} + (0.3893)\alpha_4 \begin{Bmatrix} -\sin 162 \\ \cos 162 \end{Bmatrix}$$
$$+ 2(3.93)(11.19) \begin{Bmatrix} -\sin 162 \\ \cos 162 \end{Bmatrix} + a_4 \begin{Bmatrix} \cos 162 \\ \sin 162 \end{Bmatrix}$$

$$\begin{bmatrix} -0.1203 & -0.9511 \\ -0.3702 & 0.3090 \end{bmatrix} \begin{Bmatrix} \alpha_4 \\ a_4 \end{Bmatrix} = \begin{Bmatrix} 103.71 \\ -114.14 \end{Bmatrix}$$

$$a_{B4}{}^{t}(r_4) = \alpha_4$$
$$a_{B2/B4}{}^{r} = a_4$$

$$\boxed{\begin{Bmatrix} \alpha_4 \\ a_4 \end{Bmatrix} = \begin{Bmatrix} 196.55\ rad/s^2 \\ -133.90\ m/s^2 \end{Bmatrix}}$$

# MANY THANKS! ☺

Main References:

[1] Myszka, David H., 2012. Machines and mechanism: applied kinematic analysis, 4th ed., Prentice Hall, New York.

[2] http://www.physicsclassroom.com/class/1DKin/Lesson-6/Kinematic-Equations

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