MECHANISM DESIGN
CHAPTER 6:
GEARS AND OTHER MECHANISMS

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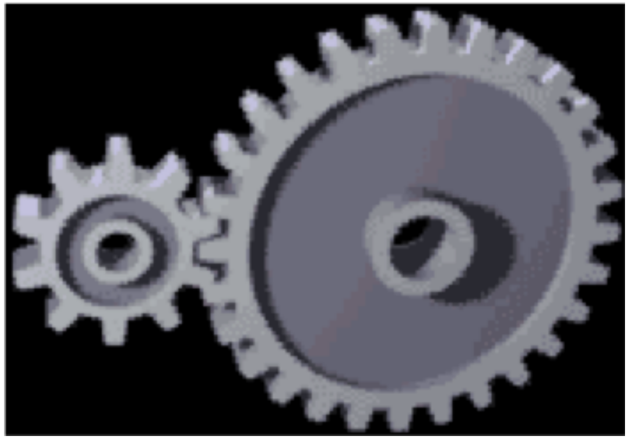
GEARS

The main objectives to use gears are to transmit angular motion and torques from an input source to an output.

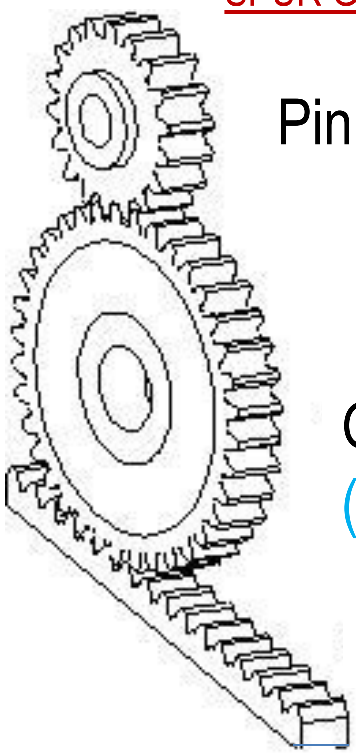
- Friction rollers can do the same too, but friction may reduce and affect efficiency adversely.
- Over the centuries, many shapes of gear tooth were invented, but modern design uses involute curves as part of the profile.
- The most common gear is the **spur gear**.



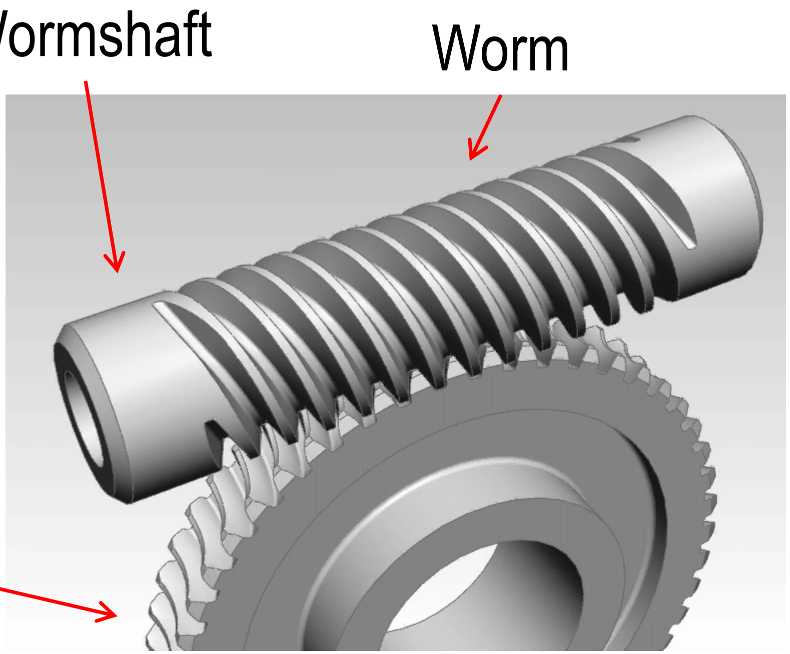
Source:
https://en.wikipedia.org/wiki/Gear#/media/File:Gears_animation.gif

GEAR TYPES

SPUR GEARS



RACK AND PINION

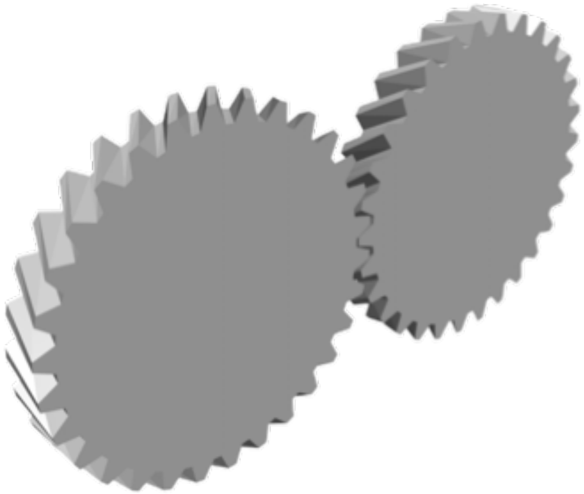


WORM GEARS

Source:
https://commons.wikimedia.org/wiki/File:Spur_gear.JPG

Source:
https://en.wikipedia.org/wiki/Worm_drive#/media/File:Worm_Gear.gif

GEAR TYPES



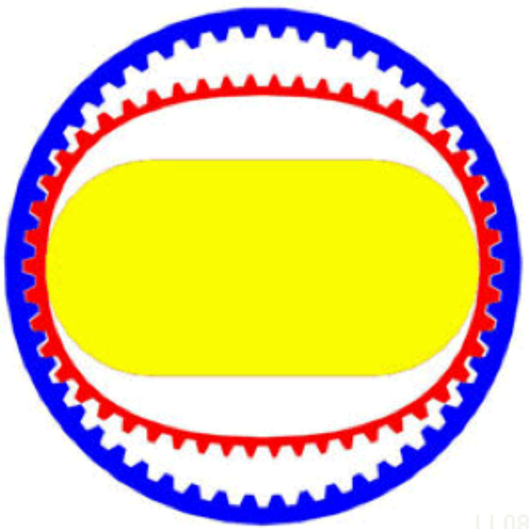
PARALLEL HELICAL GEARS – RH & LH

SPIRAL BEVEL GEARS

Source:
https://da.wikipedia.org/wiki/Tandhjul#/media/File:Anim_engrenages_helicoidaux.gif

Source:
https://en.wikipedia.org/wiki/Bevel_gear#/media/File:Gear-kegelzahnrad.svg

GEAR TYPES



HARMONIC DRIVE

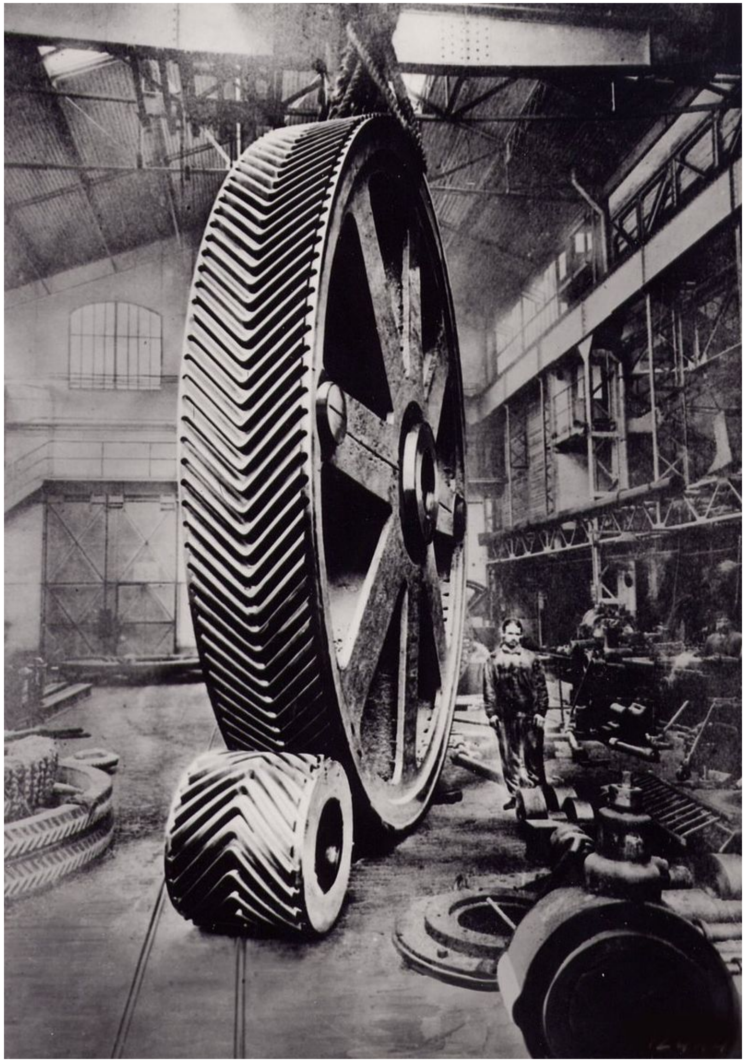
PLANETARY OR EPICYLCIC GEARS

Source:
https://en.wikipedia.org/wiki/Harmonic_drive#/media/File:Harmonic_drive_animation.gif

Source:
https://de.wikipedia.org/wiki/Umlaufr%C3%A4dergetriebe#/media/File:Planeetwielmechanisme.gif

GEAR TYPES

HERRINGBONE GEARS



Source:
https://en.wikipedia.org/wiki/Herringbone_gear#/media/File:Engrenages_-_85.488_-.jpg

GEAR SIZES

Gears can differ in size due to their **teeth number (*N*)**. Since there cannot be partial teeth involved, N must be a positive integer. Sometimes, next to the value is 'T' that denotes teeth.

Another measure of size is **Diametral Pitch (P_d) or DP**. This can be in numbers like 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 32, and 48. The unit for P_d is teeth per inch or T/in. P_d gets larger, the size gets smaller.

$$P_d = \frac{N}{d}$$

Martin Sprocket or Boston Gears may have available stocks information as such:

http://www.martinsprocket.com/docs/default-source/catalog-gears/spur-gears.pdf?sfvrsn=14

http://www.bostongear.com/smartcat/pdf/116-006-24.pdf

GEAR SIZES

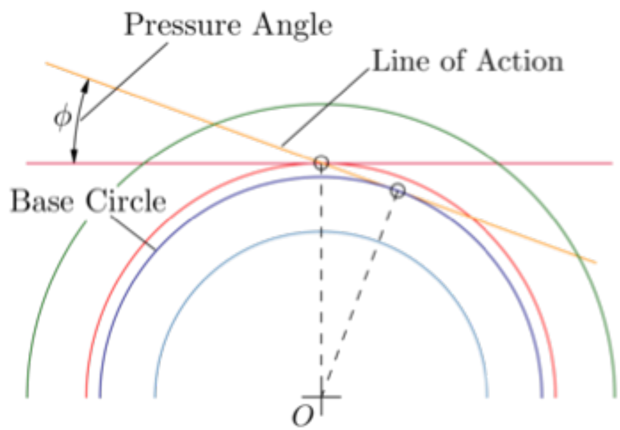
$$m = \frac{d}{N}$$

The reciprocal is **module m** in metric. The unit is mm/T or just mm. Common values are 1, 1.25, 3, 3.5, 4, 5, 5.5, 6. 7 and 8, just to name a few. The bigger the number, the bigger the size.

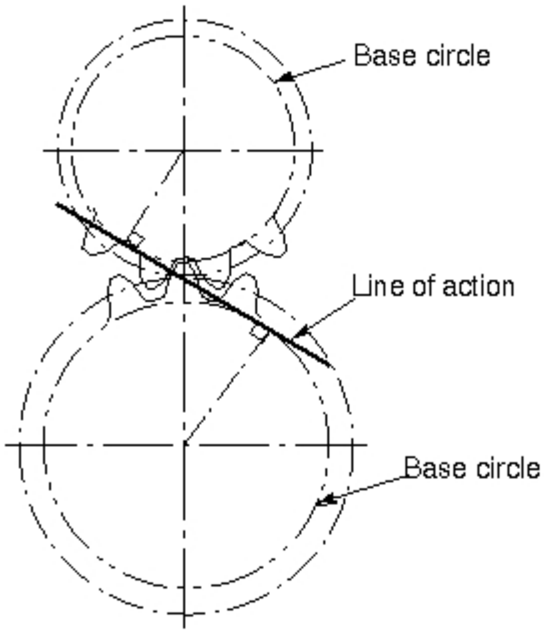
KHK Gears may have available stocks information as such:

http://www.khkgears.co.jp/en/khk_products/stock_gears_introduction.html#hira

PRESSURE ANGLE (ϕ)



Source:
https://commons.wikimedia.org/wiki/Unwin%27s_Construction#/media/File:Unwin%27s_Construction_2.svg



Source:
https://en.wikipedia.org/wiki/Gear#/media/File:Action_line.jpg

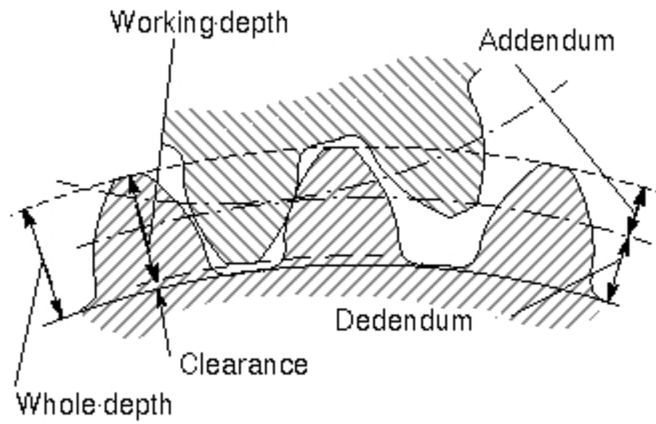
Standard pressure angles: $\phi = 14\frac{1}{2}^{o}, \; 20^{o}, \; 25^{o}$

Basically, for mating gears, they must have the **same DP and ϕ**. More theory at:

https://www.bostongear.com/pdf/gear_theory.pdf

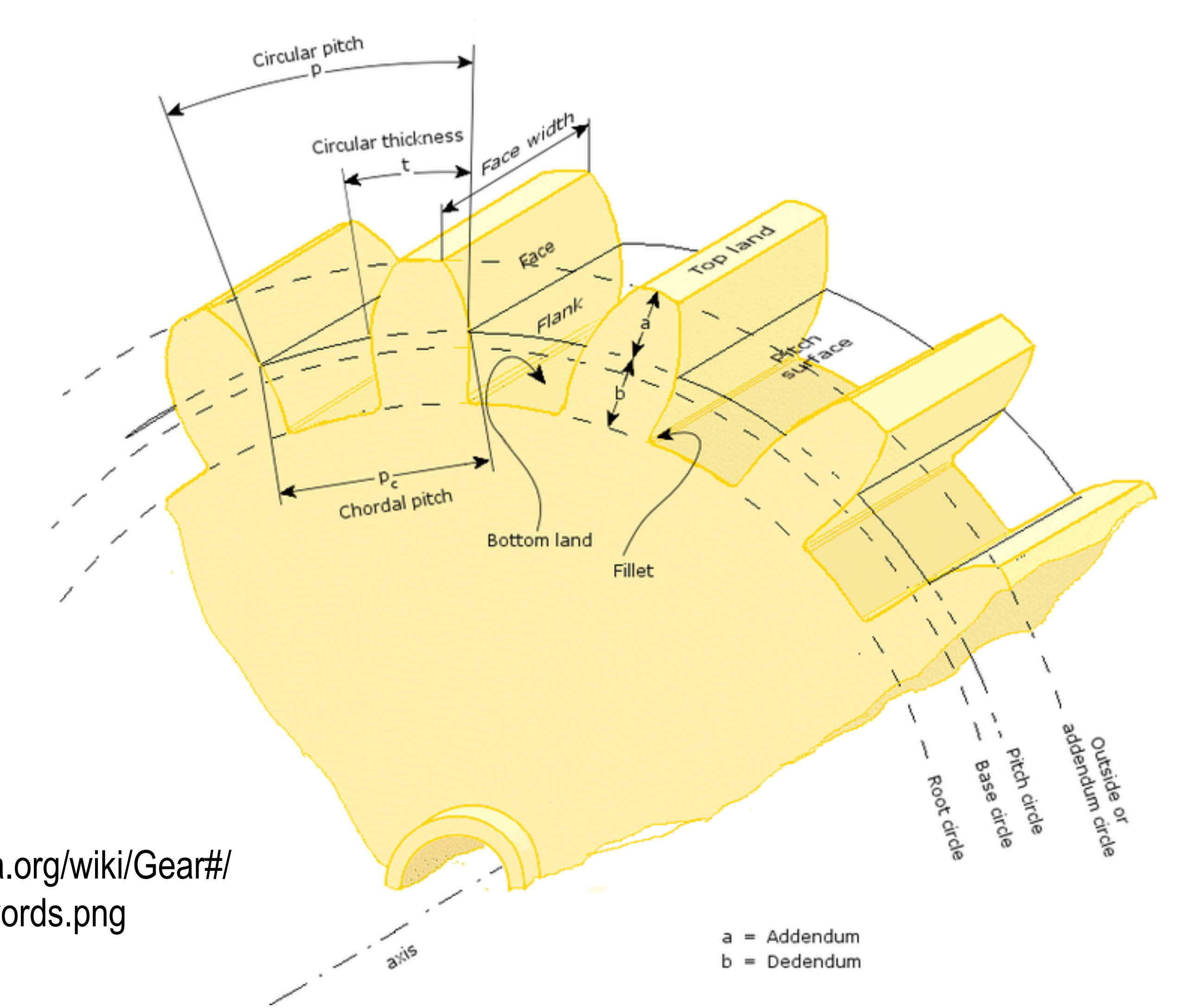
NOMENCLATURE

Source:
https://commons.wikimedia.org/wiki/File:Principal_dimensions.jpg



- Base circle $$d_b = d \cos \phi$$
- Circular Pitch $$p = \frac{\pi d}{N}$$
- Face Width $$F = \frac{12}{P_d}$$
- Addendum $$a = \frac{1}{P_d}$$
- Dedendum $$b = \frac{1.25}{P_d}$$

NOMENCLATURE



Source:
https://en.wikipedia.org/wiki/Gear#/media/File:Gear_words.png

EXAMPLE 1

Find the diametral pitch (DP or P_d) and the circular pitch of a 40-tooth gear, which has a pitch diameter (D or d) of 36.00 in.

Solution

Given: $N = 40\ T,\ D = 36.00\ in$

$T = teeth$

Find: $P_d,\ p_c$

Solutions: $P_d = \frac{N}{D} = \frac{40}{(36\ in)} = 1.1111\ T/in$ is the diametral pitch.

Circular pitch, $p_c = \frac{\pi}{P_d} = \frac{\pi}{1.1111\ T/in} = 2.8274\ in/T$

CONTACT RATIO (m_p)

Average number of teeth in contact at any instance.

$$m_p = \frac{Z}{p_b}$$

Where (i) Base pitch (p_b):

$$p_b = \frac{\pi d_1 \cos\phi}{N_1} = \frac{\pi d_2 \cos\phi}{N_2}$$

And (ii) Length of contact path (Z):

$$Z = \sqrt{(r_2 + a)^2 - (r_2 \cos\phi)^2} - r_2 \sin\phi + \sqrt{(r_1 + a)^2 - (r_1 \cos\phi)^2} - r_1 \sin\phi$$

- ❑ Gears with larger m_p have smoother load transfer, hence quieter operations.

$$a = \frac{1}{P_d}$$

- ❑ Common values are between 1.4 and 1.5, but must be > 1.20

EXAMPLE 2

A 22-tooth pinion mates with a 42-tooth gear. The gears are full depth, have diametral pitch of 16 teeth/in, and are cut with a 20° pressure angle. Find the contact ratio.

Solution

Given: $N_1 = 22\ T,\ N_2 = 42\ T,\ P_d = 16\ T/in,\ \emptyset = 20^\circ$

Find: m_p

Solutions:

$$m_p = \frac{P_d\left[\sqrt{(r_{p1}+a_1)^2-(r_{p1}\cos\emptyset)^2}+\sqrt{(r_{p2}+a_2)^2-(r_{p2}\cos\emptyset)^2}-CD\sin\emptyset\right]}{\pi\cos\emptyset}$$

$$r_{p1} = \frac{N_1}{2P_d} = \frac{22}{2(16)} = 0.6875\ in$$

$$r_{p2} = \frac{N_2}{2P_d} = \frac{42}{2(16)} = 1.3125\ in$$

$$CD = r_{p1} + r_{p2} = 0.6875 + 1.3125 = 2.0\ in$$

$$a_1 = a_2 = a = {}^{1}/_{P_d} = {}^{1}/_{16} = 0.0625\ in$$

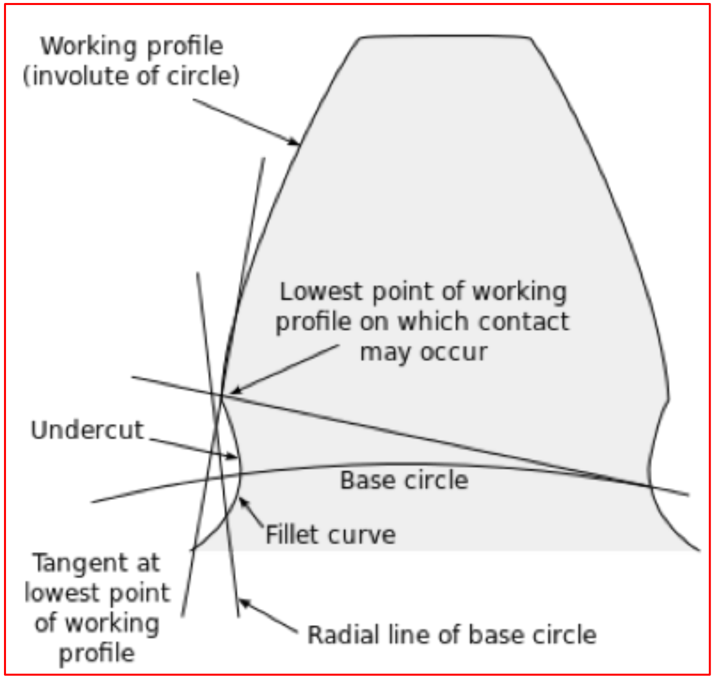
Therefore

$$m_p = \frac{16\left[\sqrt{(0.6875+0.0625)^2-(0.6875\cos 20)^2}+\sqrt{(1.3125+0.0625)^2-(1.3125\cos 20)^2}-2\sin 20\right]}{\pi\cos 20}$$

$$= 1.6518\ T$$

INTERFERENCE

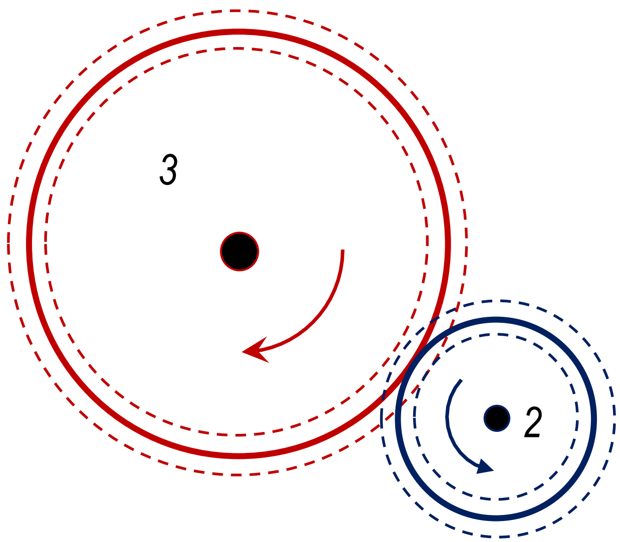
- ✓ This happens when tip of pinion clashes with the base of its mating gear.
- ✓ According to Boston Gear, this is prevalent when the number of teeth is small.
- ✓ For 14 ½ ° PA, the minimum is 32 T; for 20° 18 T; while for 25° 12 T.
- ✓ Below these numbers the gears may need undercutting but there are limits with this approach too and it weakens the gear teeth.



Source:
https://commons.wikimedia.org/wiki/File:Undercuts.svg

GEAR KINEMATICS

An important concept is the gear ratio that relates the gear speed from input to output. Some use the term Velocity Ratio (VR) that evolves into Train Value (TV) or *e*.



Directions (in planar cases)

+ω : CCW

-ω: CW

+VR: similar direction with that of pinion; e.g. An external gear drives an internal gear.

-VR: opposite direction to that of pinion; e.g. An external gear drives another external gear with parallel axes.

Common forms:

$$VR = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_3}{d_2}$$

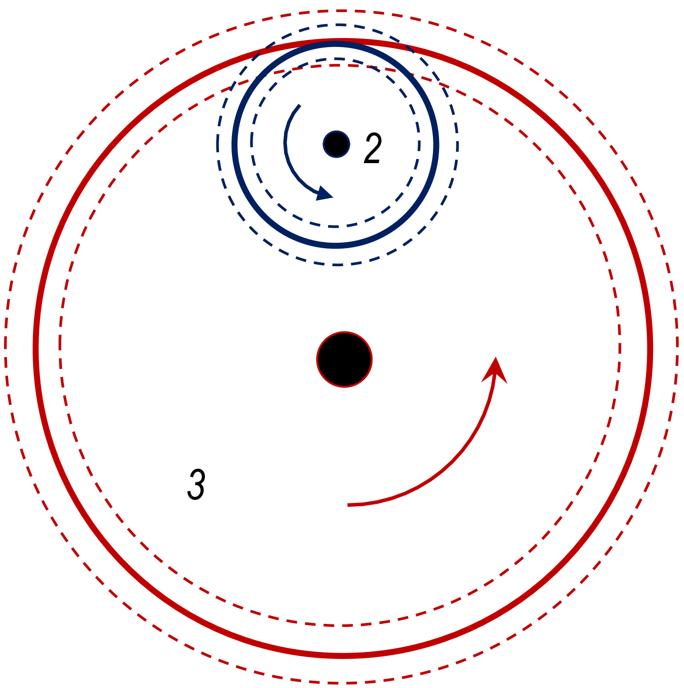
GEAR RATIO - CONTINUED

Many define the gear ratio as the ratio of faster input gear over the slower gear.

$$VR = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}$$

However, some may define the gear ratio as the ratio of slower output gear over the faster pinion.

$$n_{2/3} = \frac{\omega_3}{\omega_2} = \frac{N_2}{N_3}$$

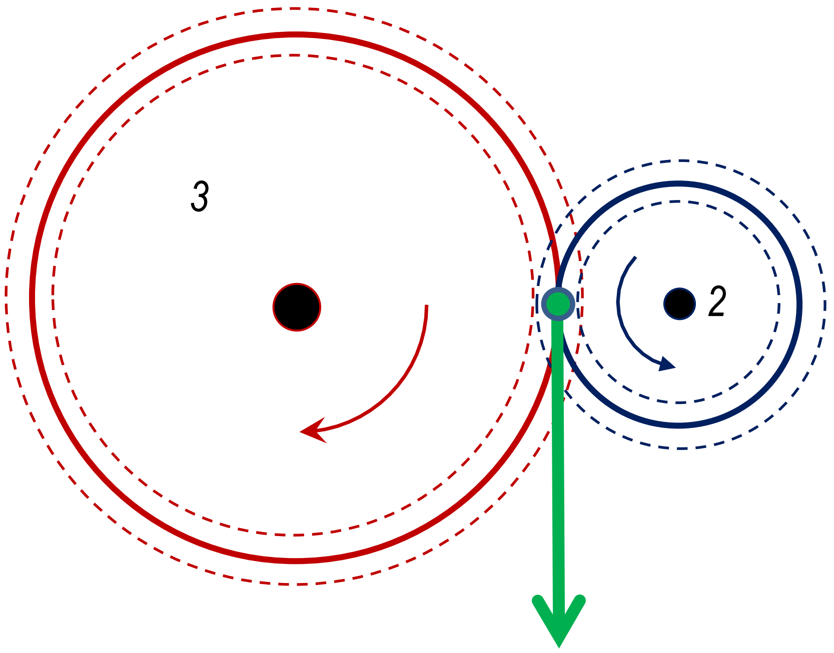


Nevertheless, it depends on what needs to be solved.

LINEAR VELOCITY

This is also called pitch line speed v_t. Sometime the unit is feet per minute (fpm). Angular speed should be in rad/s or rps.

The pitch line speed is used to find the suitable lubrication.



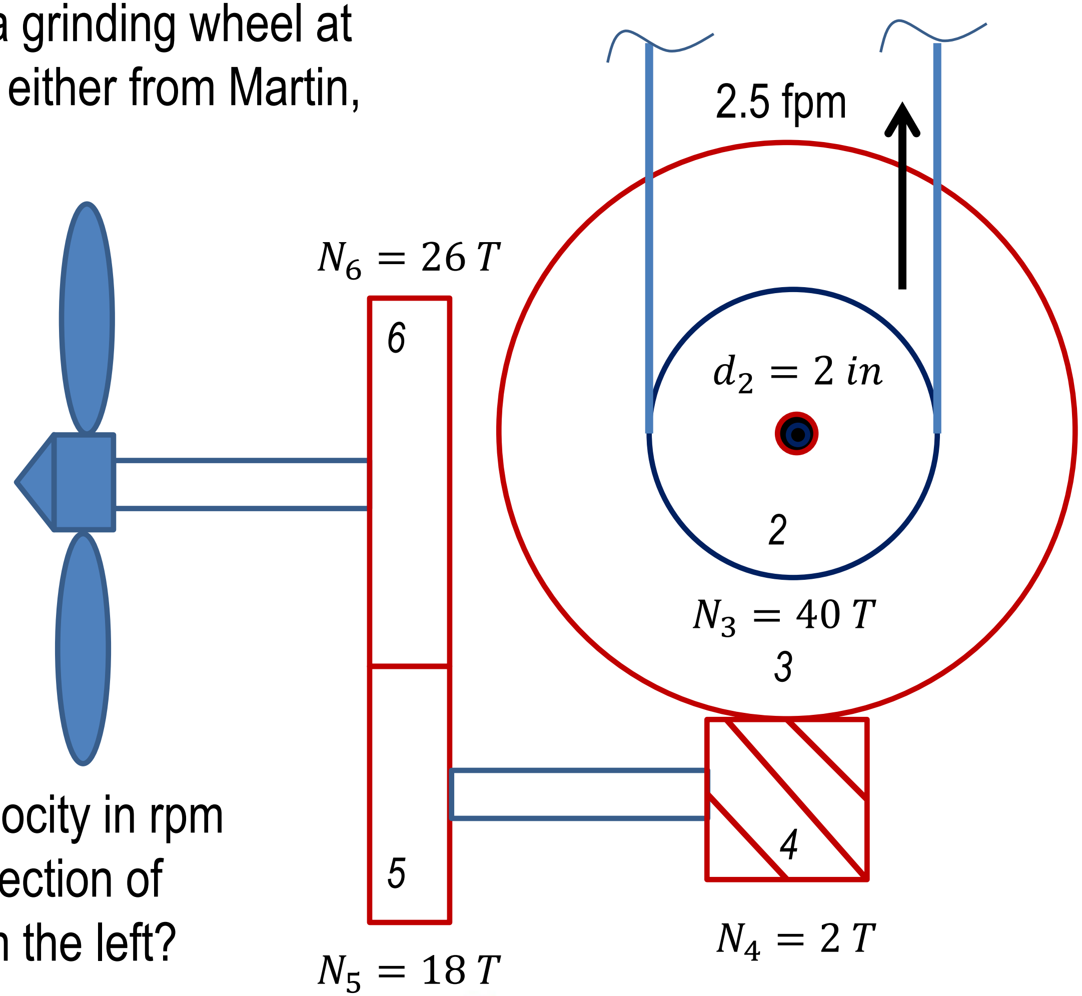
$$v = r_1\omega_1 = r_2\omega_2$$

EXERCISE 1

Design a gear set that transmits power from a 4 hp motor at 1000 rpm to a grinding wheel at 250 rpm. Use the catalog either from Martin, Boston Gears or KHK.

EXERCISE 2

Determine the angular velocity in rpm of the fan. What is the direction of rotation when viewed from the left?

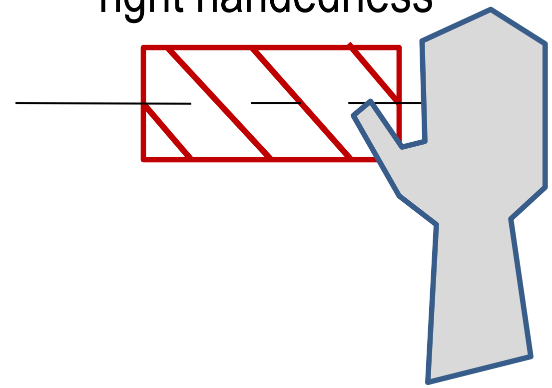


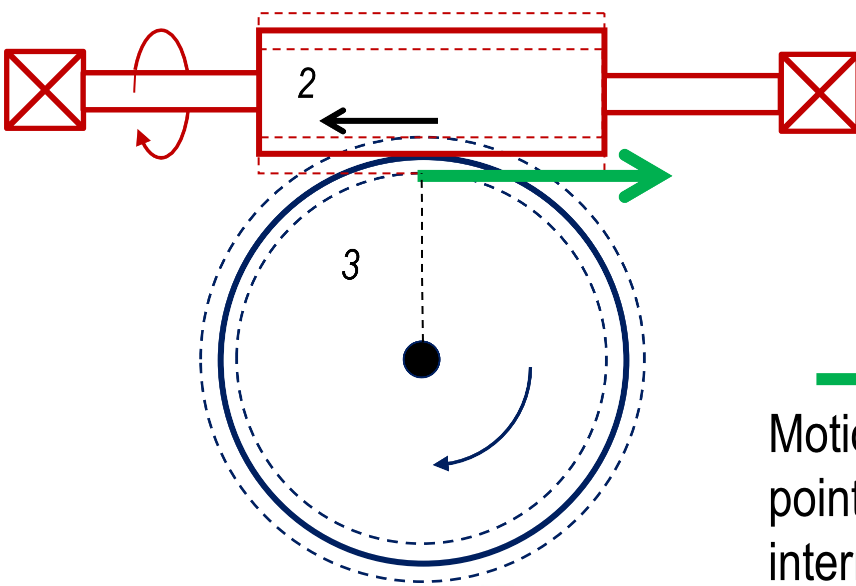
WORM GEARS

- Can achieve high ratios in small spaces.
- Shafts are perpendicular and non-intersecting
- **Self-locking: a must when involved heavy load.**
- Unfortunately, only 40-50% efficient.

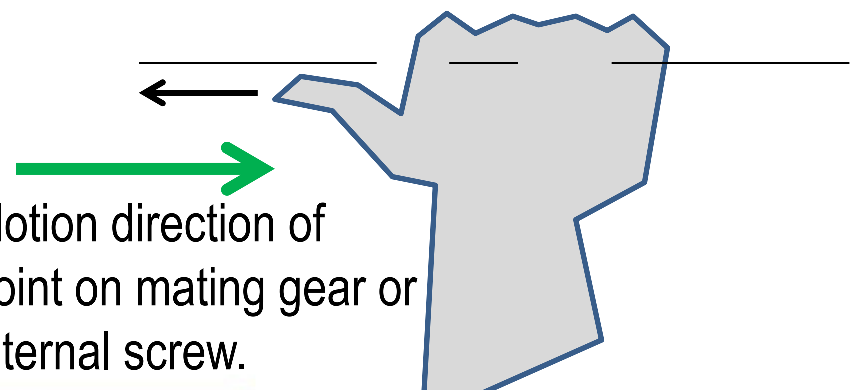
RH CASES

Palm down; thumb shows right handedness





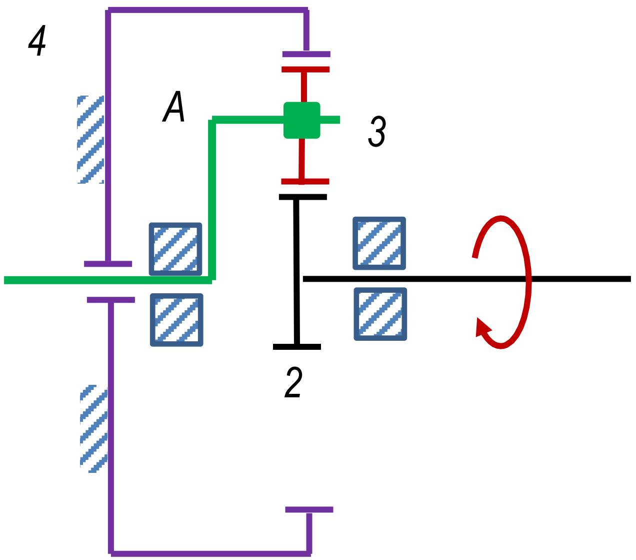
Curl fingers around axis. Thumb shows screw motion



Motion direction of point on mating gear or internal screw.

EPICYCLIC OR PLANETARY GEARS

An epicyclic gear train with arm *A* as an output and sun *2* as an input as shown. The tooth numbers are N_2 = 15; N_4 = 60. If annulus or ring gear *4* is locked i.e. $\omega_4 = 0 \text{ rpm}$, find the gear ratio of sun to arm.



Step	Number of rotation		
	A	*4*	*2*
(a) Give all gears +1 rotation	+1	+1	+1
(b) Hold arm *A*; rotate all gears	0	−1	*X*
(c) Resulting motion [=step (a) + step (b)]	+1	0	*Y*

GEAR SPEED ANALYSIS USING THE TABLE METHOD

Step	Number of rotation		
	A	*4*	*2*
(a) Give all gears +1 rotation	+1	+1	+1
(b) Hold arm *A*; rotate all gears	0	−1 $= \omega_4$	*X* $= \omega_2$
(c) Resulting motion [=step (a) + step (b)]	+1 $= \frac{\omega_A}{\omega_A}$	0 $= \frac{\omega_4}{\omega_A}$ 0, since this is fixed	*Y* $= \frac{\omega_2}{\omega_A}$

Use as input in this step

In step (b):
Since in the end $N_4 = 0 \text{ rpm}$, '-1' is given here so that the summation in row (c) will give '0'.

$$X = (-1)\left(+\frac{60}{N_3}\right)\left(-\frac{N_3}{15}\right) = +4$$

The relationship between an external gear and internal (ring) gear is that they rotate in the same direction. Hence, we give a '+' sign for their ratio. However, for external-external gears, their rotation directions are opposite, hence the '–' sign.

In step (c):

$$Y = +1 + X = +1 + 4 = +5$$

GEAR SPEED ANALYSIS USING THE FORMULA METHOD

$$\frac{\omega_L - \omega_A}{\omega_F - \omega_A} = \pm \frac{\prod N_p}{\prod N_g}$$

where ω_F is the angular speed of the first gear, ω_L is angular speed of the last gear, is $\prod N_p$ is the product of all teeth of pinion gears or gears that act as driver gears, and $\prod N_g$ is the product of driven gears.

$$\frac{0 - \omega_A}{\omega_2 - \omega_A} = \left(-\frac{N_2}{N_3}\right)\left(+\frac{N_3}{N_4}\right) = \left(-\frac{15}{N_3}\right)\left(+\frac{N_3}{60}\right)$$

$$-\omega_A = \left(-\frac{1}{4}\right)(\omega_2 - \omega_A)$$

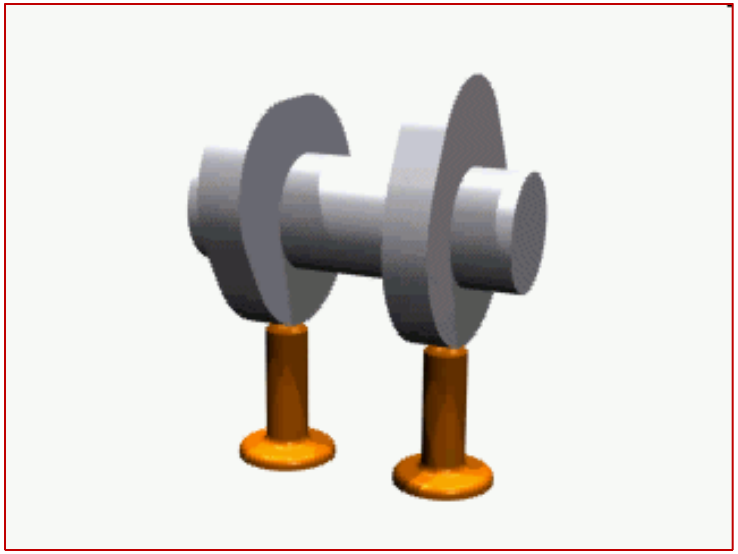
$$\frac{\omega_2}{\omega_A} = +5$$

CAMS

A cam can produce a mechanical motion where the input is rotational and output linear. Some output can be rotational too like a rocker arm follower.

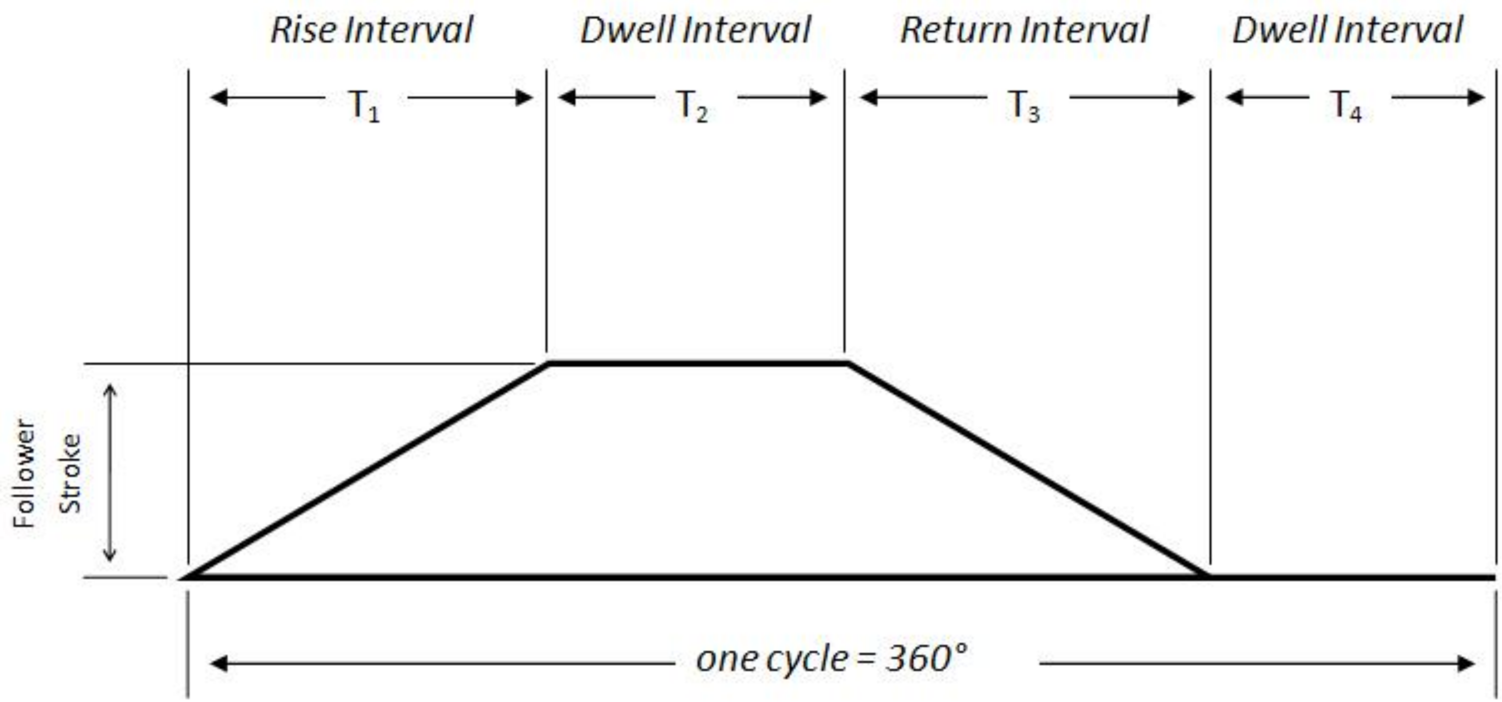
See also tappets and push-rods.

Cams have many similarities with the motion by slider-cranks. However, they have higher control of different motions.



Source:
https://en.wikipedia.org/wiki/Cam#/media/File:Nockenwelle_ani.gif

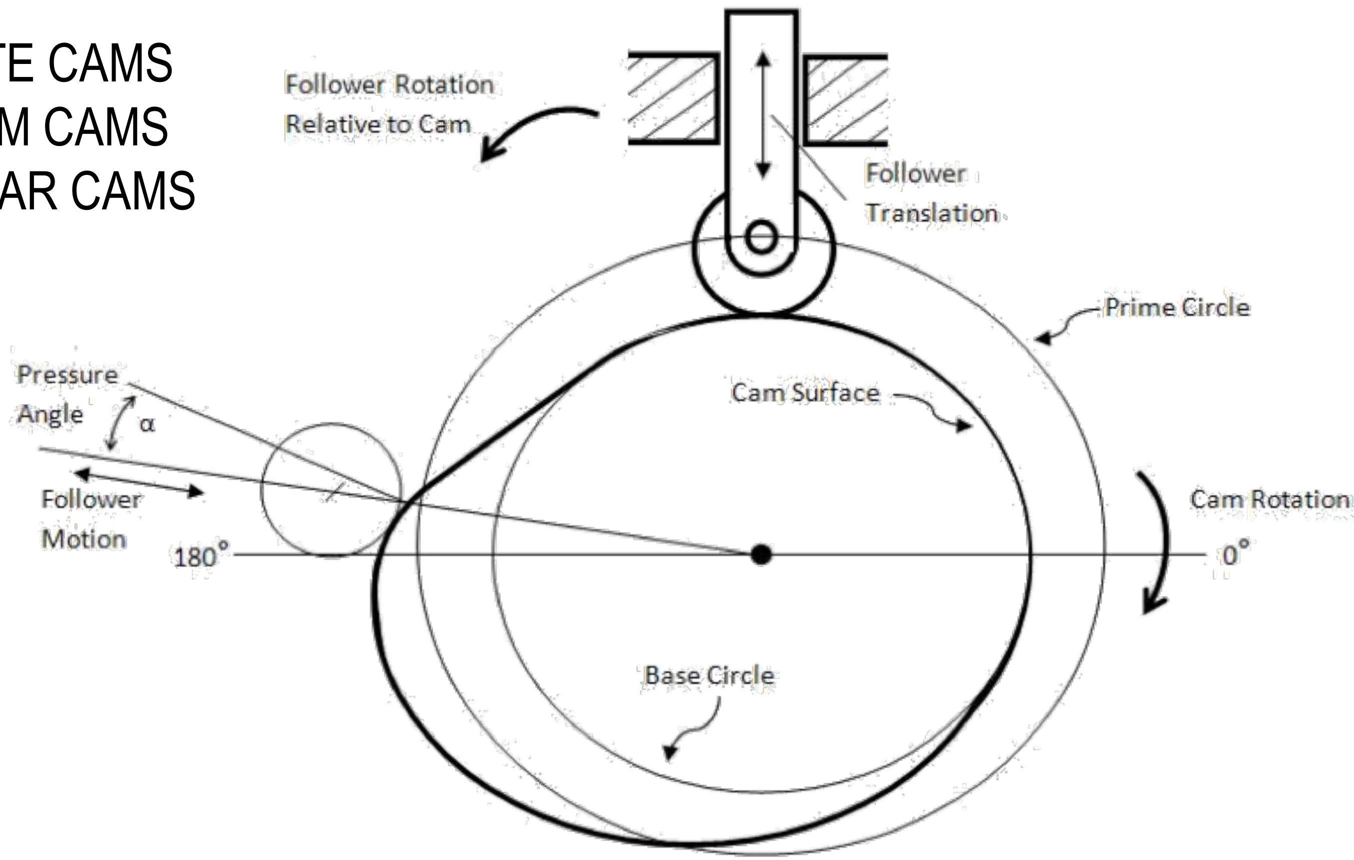
DISPLACEMENT DIAGRAMS



Source:
https://commons.wikimedia.org/wiki/File:Basic_Displacement_Diagram.JPG

TYPES OF CAMS

- ❑ PLATE CAMS
- ❑ DRUM CAMS
- ❑ LINEAR CAMS



Source:
https://en.wikipedia.org/wiki/Cam#/media/File:Cam_Profile.JPG

TYPES OF FOLLOWERS

Position:

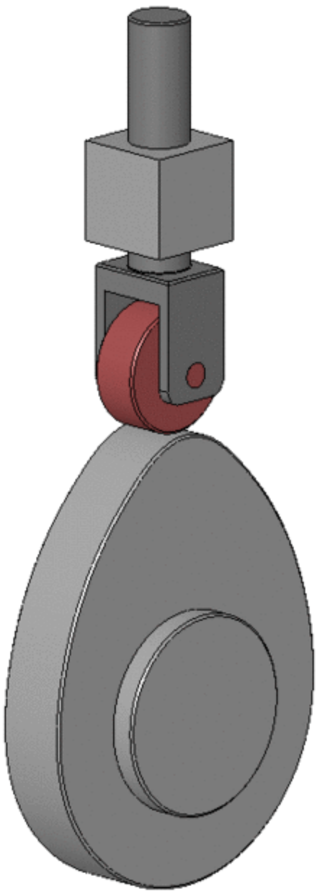
- Inline
- Offset

Motion:

- Pivoted
- Translating

Shape:

- Roller
- Knife edge
- Spherical
- Flat-face



Source:
https://en.wiktionary.org/wiki/tappet#/media/File:Cam-disc-3_3D_animated.gif

STEPS IN DESIGNING A CAM

- Know how much time in each interval. Get the total time per revolution. Solve for angular speed.
- Find the relationship between time and angles.
- Choose type of curve to follow in each motion interval.

EXAMPLE 3

A cam is synthesized according to these motion schemes.

1. Rise 20 mm with harmonic motion in 3 sec.
2. Dwell for 2 sec.
3. Fall in 1.5 sec with constant acceleration.
4. Dwell for 1.5 sec.

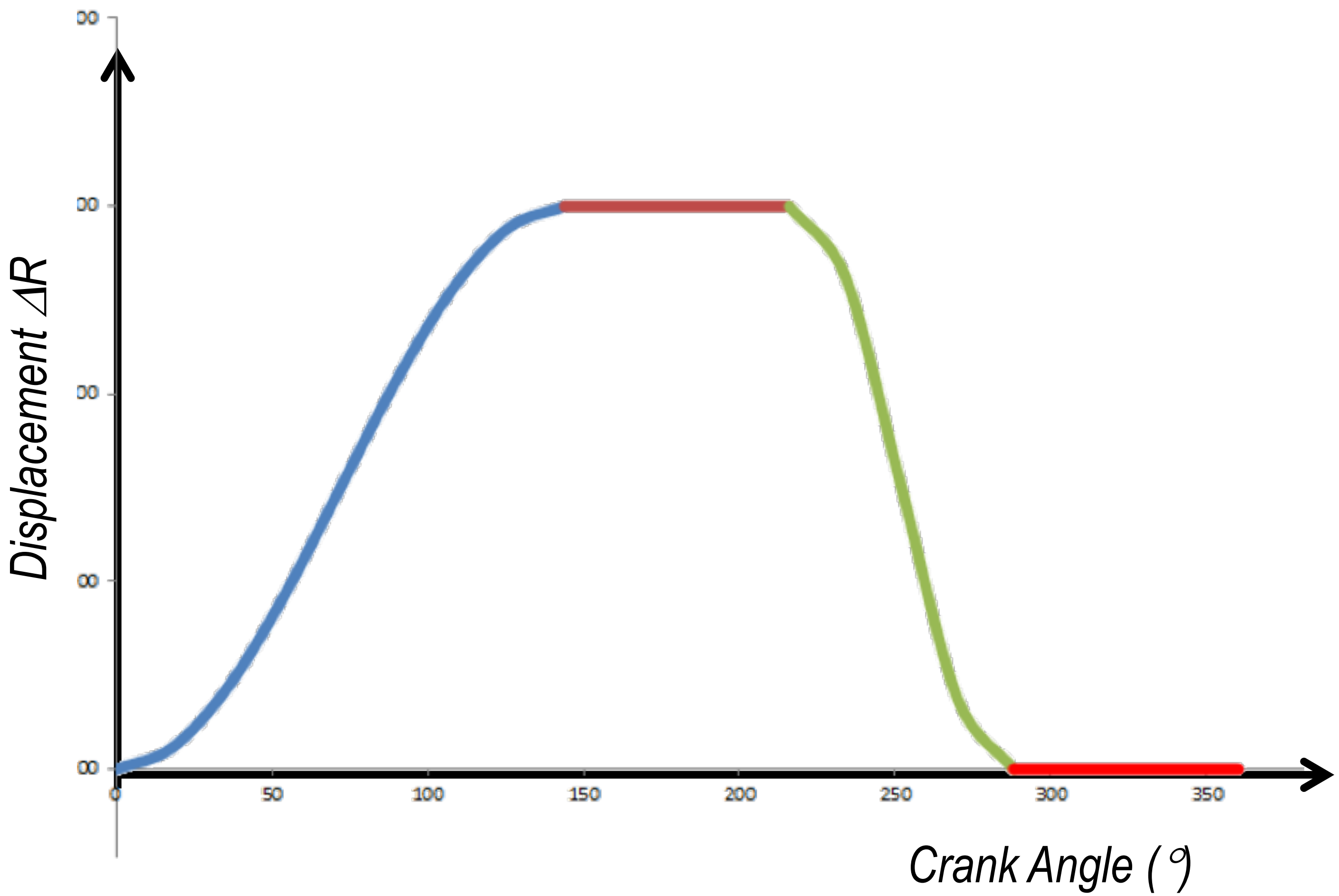
Then do the following.

- Develop a spreadsheet to plot a displacement diagram.
- Construct the profile of the cam.

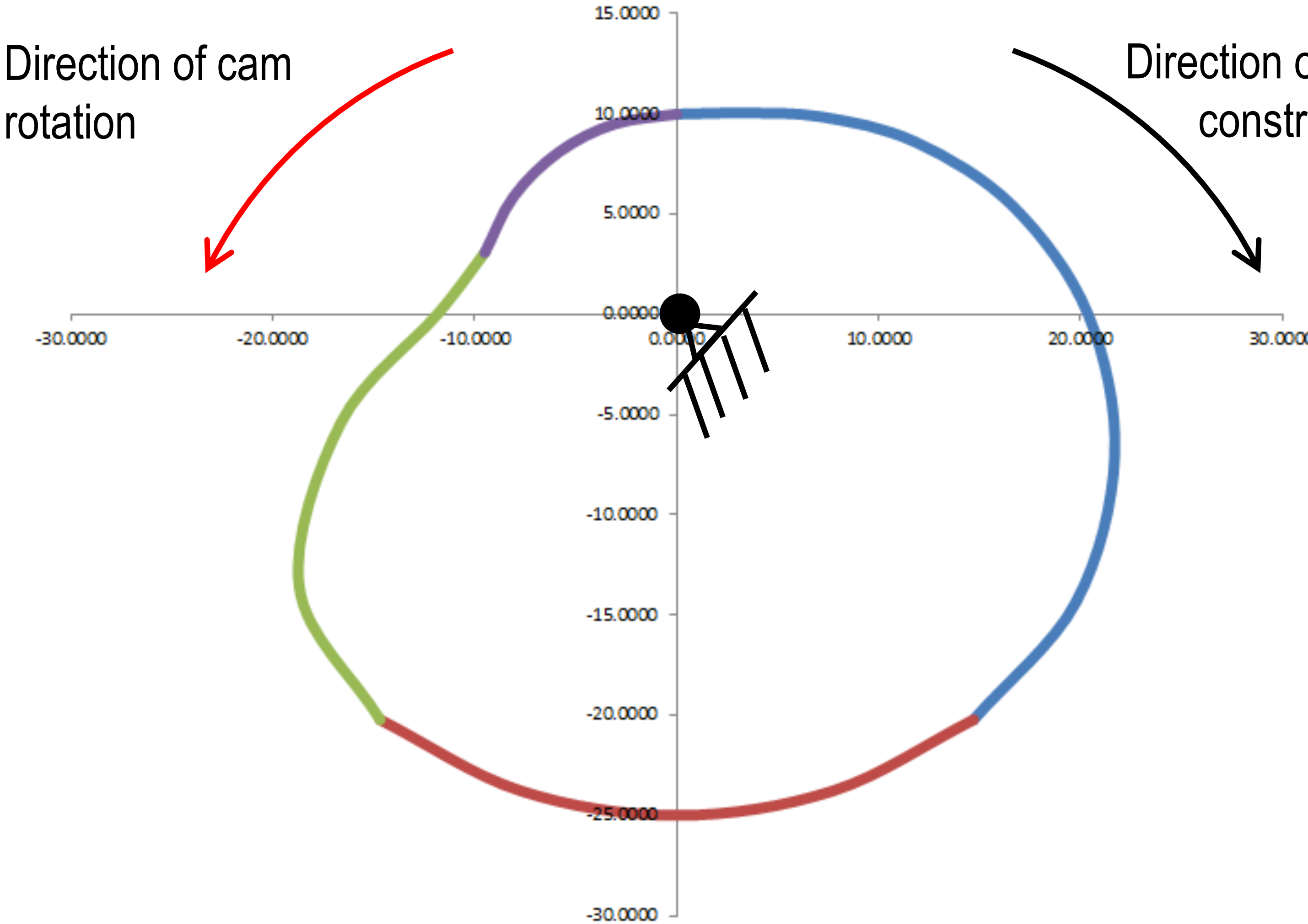
SOLUTION – Spreadsheet

Height, H	15	Base Radius	10	mm				
		Harmonic Motion					PROFILE	
t (s)	Angle (°)	ΔR	ΔR + R	Type of Motion			x	y
0	0	0.000	10.000	Rise 4 sec in harmonic motion			0.0000	10.0000
0.5	18	0.571	10.571				3.2666	10.0535
1	36	2.197	12.197				7.1690	9.8673
1.5	54	4.630	14.630				11.8358	8.5992
2	72	7.500	17.500				16.6435	5.4078
2.5	90	10.370	20.370				20.3701	0.0000
3	108	12.803	22.803				21.6872	-7.0466
3.5	126	14.429	24.429				19.7636	-14.3591
4	144	15.000	25.000				14.6946	-20.2254
		Dwell						
0	144	15.000	25.000	Dwell 2 sec			14.6946	-20.2254
0.5	162	15.000	25.000				7.7254	-23.7764
1	180	15.000	25.000				0.0000	-25.0000
1.5	198	15.000	25.000				-7.7254	-23.7764
2	216	15.000	25.000				-14.6946	-20.2254
		Constant Acceleration Motion						
0	216	15.000	25.000	Fall 2 sec in constant acceleration motion			-14.6946	-20.2254
0.5	234	13.125	23.125				-18.7085	-13.5925
1	252	7.500	17.500				-16.6435	-5.4078
1.5	270	1.875	11.875				-11.8750	0.0000
2	288	0.000	10.000				-9.5106	3.0902
		Dwell						
0	288	0.000	10.000	Dwell 2 sec			-9.5106	3.0902
0.5	306	0.000	10.000				-8.0902	5.8779
1	324	0.000	10.000				-5.8779	8.0902
1.5	342	0.000	10.000				-3.0902	9.5106
2	360	0.000	10.000				0.0000	10.0000

SOLUTION – Displacement Diagram



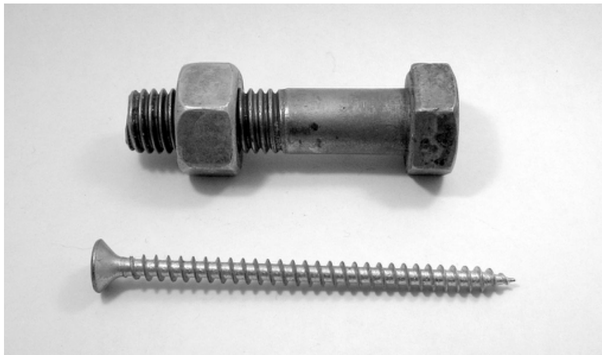
SOLUTION – Cam Profile



MOTION EQUATIONS AND FURTHER READINGS

- http://ocw.metu.edu.tr/pluginfile.php/6886/mod_resource/content/1/ch8/8-3.htm
- https://www.cs.cmu.edu/~rapidproto/mechanisms/chpt6.html
- http://www.camcoindex.com/svcman/moonbook.pdf

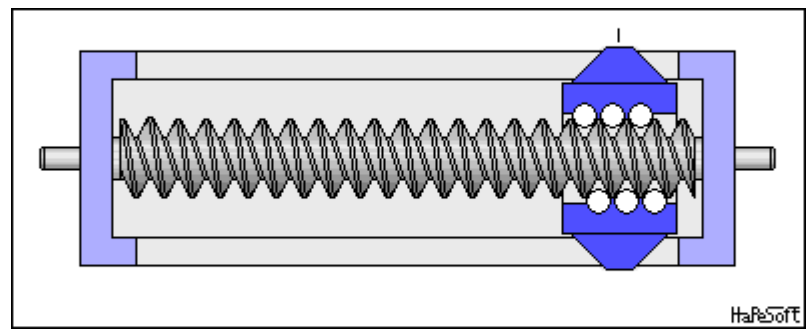
SCREW



(1) FASTENING

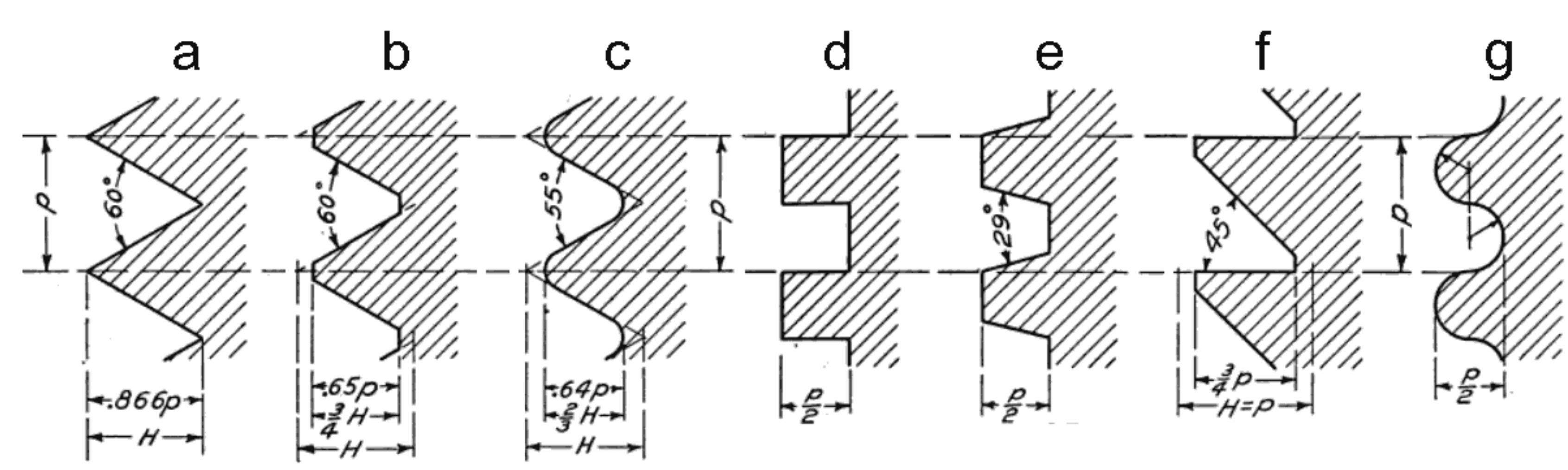
Source:
https://en.wikipedia.org/wiki/Simple_machine#/media/File:BOLT_SCREW_UBT_199.JPG

(2) MOVING OR CREATING MOTION



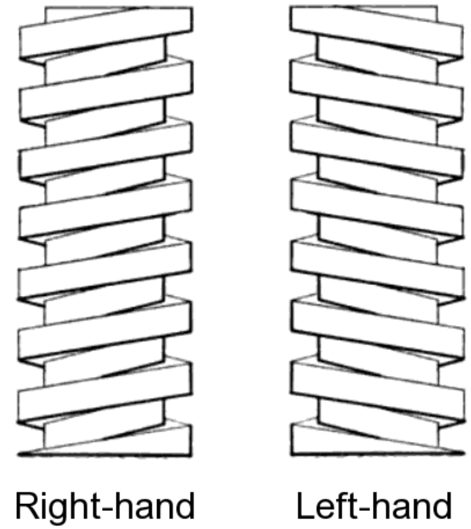
Source:
https://en.wikipedia.org/wiki/Screw_(simple_machine)#/media/File:GearBoxRotLinScrew.gif

SCREW SHAPES

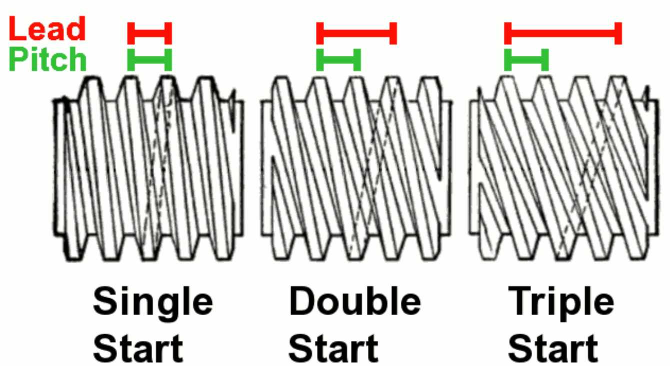


Source:
https://en.wikipedia.org/wiki/Screw_(simple_machine)#/media/File:Screw_thread_forms.png

SCREW BASICS

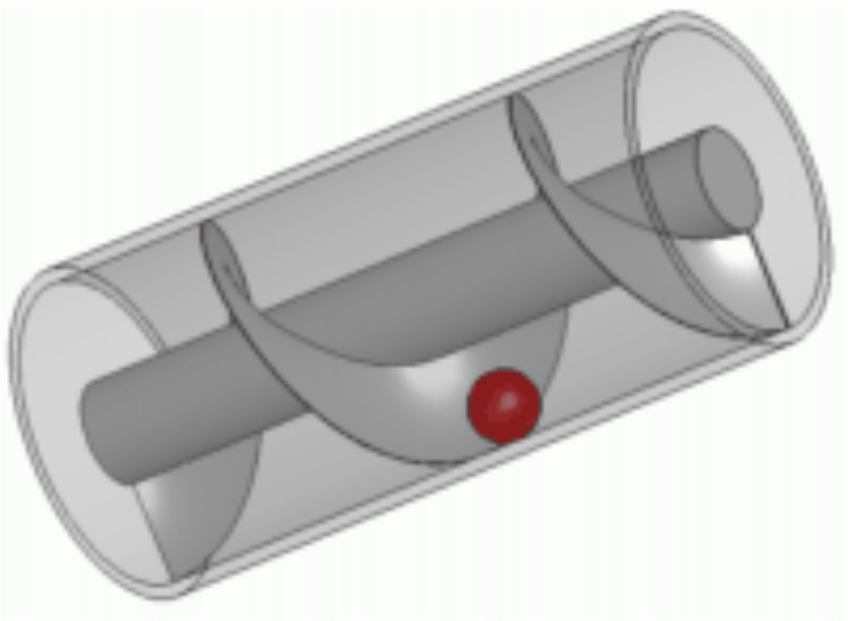


Source:
https://en.wikipedia.org/wiki/Screw_(simple_machine)#/media/File:Screw_thread_handedness.png



Source:
https://commons.wikimedia.org/wiki/File:Lead_and_pitch_in_screws.png

DISTANCE MOVED



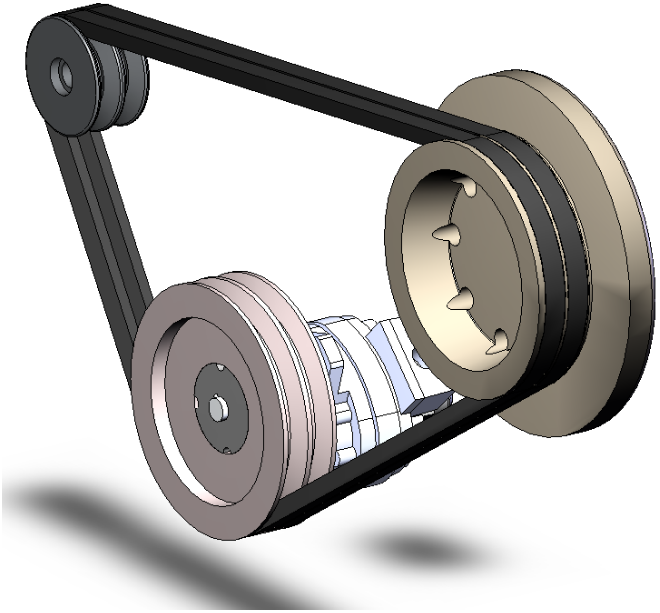
$$s = l\frac{\theta°}{360°}$$

$$l = \{1,2,3\}$$

Source:
https://commons.wikimedia.org/wiki/File:Pump.gif

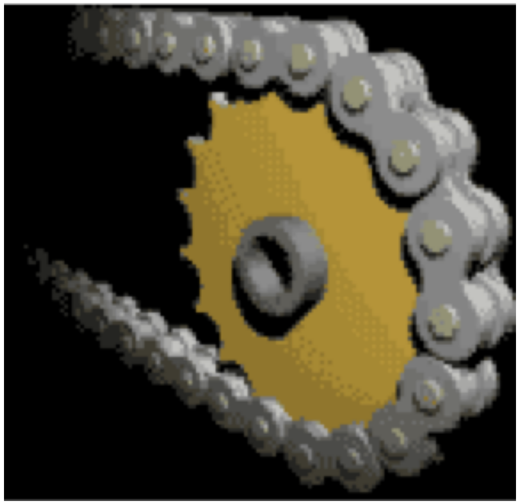
BELT AND CHAIN

Read:
http://www.gatesmectrol.com/common/downloads/files/mectrol/brochure/GatesMectrol_Belt_Pulley_Catalog.pdf



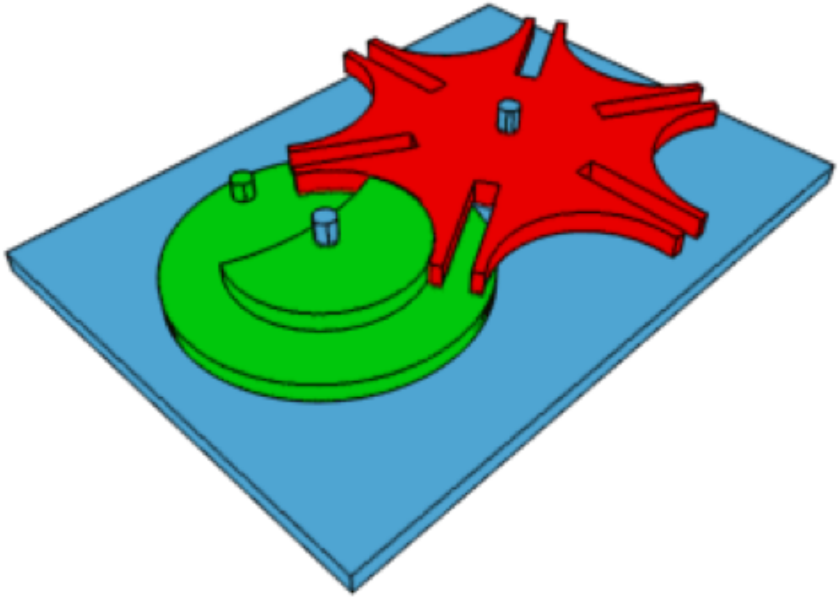
Source:
https://en.wikipedia.org/wiki/Belt_(mechanical)#/media/File:Keilriemen-V-Belt.png

Read:
http://www.bmgworld.net/downloads/fennermanual/04CHAPTER2_CHAINDRIVES.pdf

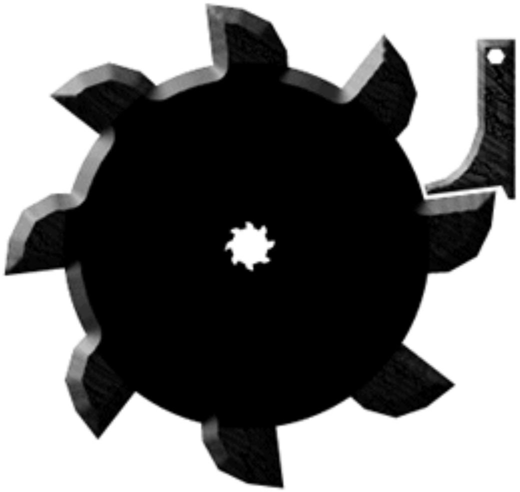


Source:
https://en.wikipedia.org/wiki/Chain_drive#/media/File:Chain.gif

GENEVA CAM & RATCHET



Source:
https://en.wikipedia.org/wiki/Geneva_drive#/media/File:Geneva_mechanism_6spoke_animation.gif



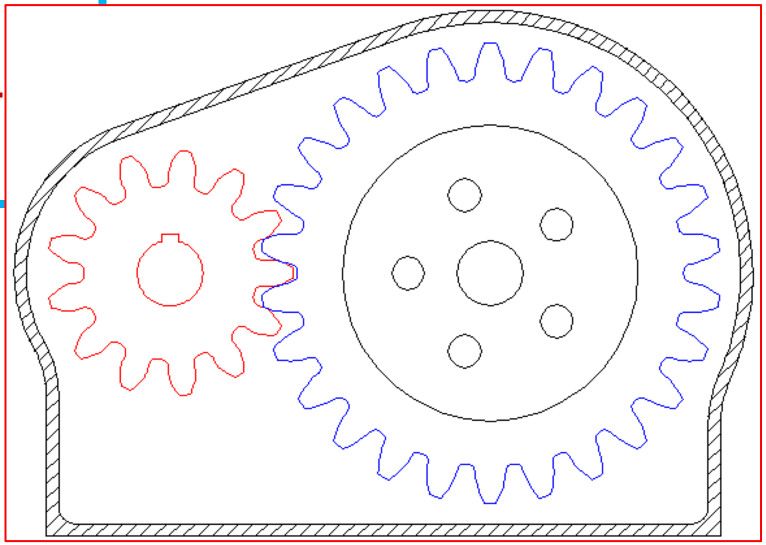
Source:
https://en.wikipedia.org/wiki/Ratchet_(device)#/media/File:Ratchet_example.gif

THANK YOU ☺

Main References:

[1] Myszka, David H. 2012. Machines and Mechanism: Applied Kinematic Analysis, 4th ed., Prentice Hall, New York.

[2] Budynas, Richard G. and Nisbett, J. Keith. 2011. Shigley's Mechanical Engineering Design, 9th ed., McGraw Hill, New York.



Source:
https://en.wikipedia.org/wiki/Transmission_(mechanics)#/media/File:Gear_reducer.gif